Charles University in Prague
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BACHELOR THESIS

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Lagrangians for massive spin one particles

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Field of study: General physics

2006

I would like to thank in the first place Jiří Novotný for many discussions, enlightening a lot of issues in chiral perturbation theory to me and for his time helping me with this work. I also thank Karol Kampf for discussing several problems and Jan Prachař for inspiring talks.

I declare that I wrote the thesis by myself and listed all used sources. I agree with making this thesis publicly available.

V Praze dne 24.7.2006 Jaroslav Trnka

Název práce: Lagrangiány pro hmotné částice se spinem 1
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Abstrakt: V předložené práci studujeme dva možné přístupy k popisu resonancí v rámci chirální poruchové teorie, konkrétně popis pomocí vektorových a antisymetrických tensorových polí. Ukazuje se, že ekvivalence mezi nimi není úplná a při přechodu z jednoho popisu do druhého v daném řádu je nutné dodat členy vyšších řádů pro její zachování. Dále zavádíme smíšený formalismus, který oba předchozí v jistém smyslu spojuje. Pro demonstraci na závěr vypočítáme VVP korelátor ve všech zmíněných popisech a srovnáme získané výsledky s vysokoenergetickými podmínkami.
Klíčová slova: chirální poruchová teorie, vektorové resonance, VVP korelátor

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Abstract: In the present work we study two possible approaches of desctiptions of resonances in context of chiral perturbation theory, concretly, using vector and antisymmetric tensor field formulations. As it turned out the equivalence of these two approaches is not complete. Converting one formalism to the another at definite order we have to add the terms of higher orders to preserve equivalence. Then we introduce mixed formulism which connects both mentioned above. For demonstration we calculate VVP correlator in all descriptions and compare the results with the high energy conditions.
Keywords: chiral perturbation theory, vector resonances, VVP correlator

Contents

Chapter 1

Introduction

In the relativistic quantum field theory spin one particles obey Bose-Einstein statistics. In the Standard model there exist two types of spin one bosons - gauge bosons and vector mesons. Our task is to describe these particles in the framework of quantum field theory. The difficulties will arise when we want to make the particles massive. Unfortunately general theories with massive spin one particles described using Proca fields are not renormalizable in the conventional sence.[1]

Gauge bosons are the mediators of interactions: photons for electromagnetic, gluons for strong and vector bosons $W^{\pm}$, Z^0 for electroweak. First two particles are out of our interest because of their masslessness. This is not true about the vector bosons with their huge masses (80 and 90 GeV). In spite of this problem theory of electroweak interactions is renormalizable thanks to Higgs mechanism which gives masses to vector bosons indirectly, so their mass terms (the origin of the problems) don't appear in the original Lagrangian. After rewriting the Lagrangian in terms of new physical fields they are already generated.

In spite of our conviction that Quantum chromodynamic (shortly QCD) is the fundamental theory of strong interactions we are not able to use it in the low energy region. QCD describes only quarks and gluons as its fundamental degrees of freedom which are not asympotic states due to confinement. This is strictly non-perturbative effect. Regardless perturbative QCD cannot describe hadrons, particles composed of quarks, at low energies. So we have to omit the requirement of renormalizability (in the conventional sence) and try to build the effective theory for vector mesons which doesn't have to satisfy this condition.

In the present thesis we first mention in Chapter 1 some basic issues of

[1] Effective theories are renormalizable in the modern meaning [1].

resonances and Chiral perturbation theory (Section 1), two possible descriptions of vector resonances (Section 2) and their equivalence (Section 3). In Chapter 2 is calculated VVP correlator.

This thesis was inspired by the article

[2] K. Kampf, J. Novotný and J. Trnka, *On different lagrangian formalisms for vector resonances within chiral perturbation theory*, arXiv:hep-ph/0608051.

Notation

We use the same notation as in [2]. All used fields transform under adjoint representation of $SU(3)_V$. Using the normalisation of [3] we have $V_\mu = V_\mu^a T^a$ where $T^a = \lambda^a/\sqrt{2}$ and $T^0 = \mathbf{1}/\sqrt{3}$. The same is true about the antisymmetric tensor fields and pseudoscalar fields, $R_{\mu\nu} = R_{\mu\nu}^a T^a$ and $\phi = \phi^a T^a$ (footnoteThe pseudoscalar mesons transform as an octet so there is no term $\phi^0 T^0$). For sources v and p we have $p = p^a T^a/\sqrt{2}$ and $v_\mu = v_\mu^a T^a/\sqrt{2}$.

The dot in brackets means the contraction of group and tensor indices followed by the trace in group space.

$$
\begin{aligned}
(A \cdot B) &\equiv A_\mu^a B^{a\mu}, \\
(V \cdot K \cdot V) &\equiv V_\mu^a K^{ab\mu\nu} V_\nu^b.
\end{aligned}
$$

For generic tensors we employ " : " for a pair of contracted antisymmetric indices, i.e.

$$R : J \equiv R_{\mu\nu} J^{\mu\nu}.$$

We also use the symbol $\widehat{V}$ for an antisymmetric derivative of the vector field V, id. $\widehat{V} = D^\mu V^\nu - D^\nu V^\mu$ and W for a derivative of the antisymmetric tensor field $W^{a\beta} = D_\alpha^{ab} R^{b\alpha\beta}$.

Chapter 2

Theoretical background

2.1 Spin one particles and χPT

In the beginning of the thesis we remind some fundamentals of spin one resonances and Chiral perturbation theory.

2.1.1 Description of spin one particles

There are two main possibilities how to describe spin one particles in the framework of quantum field theory. Either we can use the formalism of vector fields or antisymmetric tensor fields. In the first case we can write the free field Lagrangian in the form

$$\mathcal{L}_V = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A_\mu A^\mu, \tag{2.1}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. Classical equation of motion gives

$$\partial^2 A_\mu + m^2 A_\mu - \partial_\mu(\partial \cdot A) = 0.$$

Taking the divergence we get

$$\partial \cdot A = 0$$

and hence

$$\left(\partial^2 + m^2\right) A_\mu = 0.$$

A real vector field satisfying these two equations can be expressed as Fourier transform

$$A_\mu(x) = \frac{1}{(2\pi)^{3/2}} \sum_{\sigma=-1}^{\sigma=1} \frac{d^3\mathbf{p}}{\sqrt{2E}} \left\{B_\mu(\mathbf{p},\sigma)e^{ip\cdot x} + B_\mu^*(\mathbf{p},\sigma)e^{-ip\cdot x}\right\}$$

where $E = \sqrt{\mathbf{p}^2 + m^2}$. Comming to quantum field theory we substitute operators for functions, id. $B_\mu \to \widehat{B}_\mu$. Separating the tensor structure we can write

$$A_\mu(x) = \frac{1}{(2\pi)^{3/2}} \sum_{\sigma=-1}^{\sigma=1} \frac{d^3\mathbf{p}}{\sqrt{2E}} \left\{ \varepsilon^\mu(\mathbf{p},\sigma) a(\mathbf{p},\sigma) e^{ip\cdot x} + \varepsilon^{\mu *}(\mathbf{p},\sigma) a^\dagger(\mathbf{p},\sigma) e^{-ip\cdot x} \right\},$$

where $\varepsilon^\mu(\mathbf{p},\sigma)$ are three independent polarization vectors satisfying

$$\begin{aligned} \sum_{\sigma=-1}^{\sigma=1} \varepsilon^\mu(\mathbf{p},\sigma)\varepsilon^{\nu *}(\mathbf{p},\sigma) &= -g^{\mu\nu} + \frac{p^\mu p^\nu}{m^2}, \\ \varepsilon_\mu(\mathbf{p},\sigma)\varepsilon^\mu(\mathbf{p},\sigma') &= -\delta_{\sigma\sigma'}, \\ p_\mu \varepsilon^\mu(\mathbf{p},\sigma) &= 0. \end{aligned}$$

and $a(\mathbf{p},\sigma)$, $a^\dagger(\mathbf{p},\sigma)$ are anihilation and creation operators that satisfy commutation relations

$$\begin{aligned} \left[a(\mathbf{p},\sigma), a^\dagger(\mathbf{p}',\sigma')\right] &= \delta^3(\mathbf{p}' - \mathbf{p})\delta_{\sigma,\sigma'}, \\ \left[a(\mathbf{p},\sigma), a(\mathbf{p}',\sigma')\right] &= \left[a^\dagger(\mathbf{p},\sigma), a^\dagger(\mathbf{p}',\sigma')\right] = 0. \end{aligned}$$

Then fields $A_\mu(x)$ transforms under $(1/2, 1/2)$ representation of Lorentz group. The 2-point Green function we call propagator of the field

$$i\Delta_F^V(x-y)_{\mu\nu} \equiv \langle 0|T[A_\mu(x)A_\nu(y)]|0\rangle. \tag{2.2}$$

Direct calculation gives

$$i\Delta_F^V(x-y)_{\mu\nu} = \int \frac{d^4p}{(2\pi)^4} i\Delta_F(p)_{\mu\nu} e^{-ip\cdot(x-y)}$$

where

$$i\Delta_F^V(p)_{\mu\nu} = \frac{-i}{p^2 - m^2 + i\epsilon}\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{m^2}\right) \tag{2.3}$$

is the covariant propagator in the momentum representation.

For the description using antisymmetric tensor formalism we have the free field Lagrangian

$$\mathcal{L}_T = -\frac{1}{2}W_\mu W^\mu + \frac{1}{4}m^2 R_{\mu\nu}R^{\mu\nu}. \tag{2.4}$$

where $W_\mu = \partial^\alpha R_{\alpha\mu}$. Classical equation of motion has the form

$$\partial_\mu \partial^\alpha R_{\alpha\nu} - \partial_\nu \partial^\alpha R_{\alpha\mu} + m^2 R_{\mu\nu} = 0.$$

Applying the derivative ∂^ν we obtain for $\partial^\alpha R_{\alpha\mu}$ (multiplied by m because of proper dimension of the field)

$$\left(\partial^2 + m^2\right)(m\partial^\alpha R_{\alpha\mu}) = 0.$$

The condition of transversality is satisfied identically due to antisymmetry of $R_{\mu\nu}$. So we obtain again Proca field equation and it is possible to write for $\partial^\alpha R_{\alpha\mu}$ the same expression (using the same creation and anihilation operators!). Guessing the general form of the expansion for $R_{\mu\nu}$ we get[1]

$$R_{\mu\nu}(x) = \frac{1}{(2\pi)^{3/2}} \sum_{\sigma=-1}^{\sigma=1} \frac{d^3\mathbf{p}}{\sqrt{2E}} \left\{ A_{\mu\nu}(\mathbf{p},\sigma) a(\mathbf{p},\sigma) e^{ip\cdot x} + B_{\mu\nu}(\mathbf{p},\sigma) a^\dagger(\mathbf{p},\sigma) e^{-ip\cdot x} \right\}.$$

Applying the derivative in the momentum space we obtain

$$\begin{aligned} ip^\mu A_{\mu\nu} &= m\varepsilon^\mu(\mathbf{p},\sigma), \\ -ip^\mu B_{\mu\nu} &= m\varepsilon^{\mu *}(\mathbf{p},\sigma). \end{aligned}$$

Easy calculation using the relation $p_\mu \varepsilon^\mu(\mathbf{p},\sigma) = 0$ gives the result

$$\begin{aligned} R_{\mu\nu}(x) = & \frac{1}{(2\pi)^{3/2}} \sum_{\sigma=-1}^{\sigma=1} \frac{d^3\mathbf{p}}{\sqrt{2E}} \frac{i}{m} \Big\{ \left(p_\nu \varepsilon_\mu(\mathbf{p},\sigma) - p_\mu \varepsilon_\nu(\mathbf{p},\sigma)\right) a(\mathbf{p},\sigma) e^{ip\cdot x} + \\ & \left(p_\nu \varepsilon^*_\mu(\mathbf{p},\sigma) - p_\mu \varepsilon^*_\nu(\mathbf{p},\sigma)\right) a^\dagger(\mathbf{p},\sigma) e^{-ip\cdot x} \Big\}. \end{aligned}$$

The covariant propagator of the field is then

$$i\Delta^T_F(x-y)_{\alpha\beta\mu\nu} \equiv \langle 0|T[R_{\alpha\beta}(x) R_{\mu\nu}(y)]|0\rangle = \int \frac{\mathrm{d}^4 p}{(2\pi)^4} i\Delta_F(p)_{\alpha\beta\mu\nu} e^{-ip\cdot(x-y)} \tag{2.5}$$

where

$$i\Delta^T_F(p)_{\alpha\beta\mu\nu} = \frac{-i}{p^2 - m^2 + i\epsilon} \frac{1}{m^2} \left((m^2 - p^2) g_{\alpha\mu} g_{\beta\nu} + g_{\alpha\mu} p_\beta p_\nu - g_{\alpha\nu} p_\beta p_\mu - (\mu \leftrightarrow \nu)\right). \tag{2.6}$$

2.1.2 Chiral perturbation theory

In the low energy region the hadron sector can be described with the lightest particles - pseudoscalar mesons. The effective theory for them is called

[1] Actually, this is not a guess. Antisymmetric tensor field transforms under $(1,0)+(0,1)$ representation of Lorentz group which guaranties the possibility of the expansion of the field in this form [1], [4].

Chiral perturbation theory.[2] It is based on the hypothesis that the lightest pseudoscalar mesons are the only degrees of freedom at the low energy and the contribution of all other hadrons is effectively incorporated into constants of chiral Lagrangian which is built phenomenologically from the chiral building blocks satisfying the same symmetry properties as QCD. The original QCD Lagrangian is in chiral limit (which corresponds to the massless quarks) invariant under group $SU(3)_L \times SU(3)_R$ (we call it chiral symmetry) but the vacuum is invariant only under its subgroup $SU(3)_V$ - the symmetry is spontaneously broken. So according to Goldstone theorem there exist eight massless Goldstone bosons which we identify with pseudoscalar mesons[3].

As in every effective theory also in χPT the Lagrangian can be decomposed in powers of small physical quantity. Here it is the external momenta p which should be much smaller than an energy scale $\Lambda \approx 1\,\mathrm{GeV}$ which is related to typical (nongoldstone) hadron masses. Another small quantities are the quark masses (in quark mass matrix) first order of which corresponds with second order in momenta[4], $\mathcal{M} \sim \mathcal{O}(p^2)$.

Expansion of the Lagrangian in terms of p has the form (according to symmetry conditions only even terms can contribute)

$$\mathcal{L}_\chi = \mathcal{L}_\chi^{(2)} + \mathcal{L}_\chi^{(4)} + \mathcal{L}_\chi^{(6)} + ... \tag{2.7}$$

where $\mathcal{L}^{(n)}$ stands for part of Lagrangian which is of n-th order in p, ie. $\mathcal{L}^{(n)} = \mathcal{O}(p^n)$. The lowest order Lagrangian is then

$$\mathcal{L}_\chi^{(2)} = \frac{F_0^2}{4}\mathrm{Tr}[u_\mu u^\mu + \chi_+] \tag{2.8}$$

where

$$\begin{aligned} u_\mu &= i[u^\dagger(\partial_\mu - ir_\mu)u - u(\partial_\mu - il_\mu)u^\dagger], \\ \chi_\pm &= u^\dagger \chi u^\dagger \pm u\chi^\dagger u, \quad \chi = 2B_0(s+ip) \end{aligned}$$

are the chiral building blocks. Pseudocalar mesons come into Lagrangian through $u(\phi)$ what is

$$u(\phi) = \exp\left\{ i\frac{\phi}{\sqrt{2}F_0} \right\}$$

[2]The fundamental papers of χPT are [5],[6] and [7]. The classification of $\mathcal{O}(p^6)$ Lagrangians can be found in [8]. Good introduction is provided in [9] and [10].

[3]Precisely the chiral symmetry is broken explicitly due to nonzero quark masses and we identify pseudoscalar mesons with massive pseudogoldstone bosons.

[4]There is another approach, based on assumption $\mathcal{M} \sim \mathcal{O}(p)$, which is called Generalized chiral perturbation theory.

where ϕ is the parametrization of octet of pseudoscalar mesons fields (π^0, $\pi^\pm$, $K^\pm$, K^0, $\overline{K}^0$, η). It transforms under adjoint representation of $SU(3)_V$, ie. $\phi = \phi^a T^a$ where $T^a = \lambda^a/\sqrt{2}$. External sources $s, p, v_\mu = (r_\mu + l_\mu)/2, a_\mu = (r_\mu - l_\mu)/2$ are the classical sources for scalar, pseudoscalar, vector and axial vector currents (densities)

$$
\begin{aligned}
S^a &= \frac{1}{\sqrt{2}}\overline{q}T^a q, \\
P^a &= \frac{1}{\sqrt{2}}\overline{q}i\gamma_5 T^a q, \\
V^{a\mu} &= \frac{1}{\sqrt{2}}\overline{q}\gamma_\mu T^a q, \\
A^{a\mu} &= \frac{1}{\sqrt{2}}\overline{q}\gamma_\mu\gamma_5 T^a q.
\end{aligned}
$$

These currents (densities) are the interpolating fields for the external particles entering the process, coupled to quark mass matrix and so on. For instance the process $\pi^0 \to 2\gamma$ corresponds to 3-point Green function composed from two vector currents and one pseudoscalar density.

There exist more chiral building blocks in higher orders Lagrangians. For our next calculation we need

$$f_\pm^{\mu\nu} = u f_L^{\mu\nu} u^\dagger \pm u^\dagger f_R^{\mu\nu} u$$

where

$$
\begin{aligned}
f_L^{\mu\nu} &= \partial^\mu l^\nu - \partial^\nu l^\mu - i[l^\mu, l^\nu], \\
f_R^{\mu\nu} &= \partial^\mu r^\nu - \partial^\nu r^\mu - i[r^\mu, r^\nu].
\end{aligned}
$$

We see that the second order Lagrangian contains only two unknown constants F_0 and B_0 (in chiral limit). But it is not true for higher orders. The number of constants rapidly grows, e.g. $\mathcal{L}_\chi^{(4)}$ has 10 constants, $\mathcal{L}_\chi^{(6)}$ has already 90 constants. The generating functional for χPT is

$$Z_{\chi PT}[s,p,v,a] = \int \mathcal{D}u \exp\left\{ i \int d^4x \mathcal{L}_\chi \right\} \tag{2.9}$$

Because chiral perturbation theory is the effective theory of QCD we have

$$
\begin{aligned}
&Z_{\chi PT}[s,p,v,a] = Z_{QCD}[s,p,v,a] \\
&= \int \mathcal{D}q\mathcal{D}\overline{q}\mathcal{D}G \exp\left\{ i \int d^4x \left[\mathcal{L}_{QCD} + \overline{q}\gamma_\mu(v^\mu + \gamma_5 a^\mu)q - \overline{q}(s - i\gamma_5 p)q\right] \right\}
\end{aligned}
$$

Unfortunately we don't know this function from the first principles so the constants in χPT cannot be computed directly from QCD.

2.1.3 Resonances in effective field theory

The spectroscopy experiments reveal existence of mesons at the energy 1 GeV. Their lifetime is very short therefore we call them resonances. In [11] was first done the systematic effective field theory for resonances in the context of chiral perturbation theory.

There are more types of resonance (axial, scalar and so on) but we restrict ourselves to the vector resonances 1^{--}. They are described by the Lagrangian containing the free field part and the interaction part. The simplest form with one external source (in vector formulation) is

$$\mathcal{L}_V = -\frac{1}{4}(\widehat{V} : \widehat{V}) + \frac{1}{2}m^2(V \cdot V) + (J_1 \cdot V) \tag{2.10}$$

where $V_\mu = V_\mu^a T^a$ stand for nonet of vector resonances (ω, ϕ, ρ^0, $\rho^\pm$, $K^{*\pm}$, K^{*0}, $\overline{K}^{*0}$) and transform under adjoint representation of $SU(3)_V$, J_1 is the external source which is constructed from the chiral building blocks. Generally there could be infinite number of terms coupled resonances with J_i sources. We can get on-shell matrix elements from correlators using LSZ formulas. The correlators can be obtained from generating functional expanding in terms of external sources s, p, v^μ, a^μ. They enter the Lagrangian through chiral building blocks (also pseudoscalar mesons enter the process through this blocks) being in J_i sources. For example in our simple model

$$J_{1\mu\nu} = -\frac{f_V}{2\sqrt{2}} f_{+\mu\nu} + i g_V [u_\mu, u_\nu] \tag{2.11}$$

where f_V, g_V are the constants and $f_{+\mu\nu}, u_\mu$ are usual building blocks (for example [9] and [10]). In the low energy region we can get rid of the resonances and the dynamics is fully determined by pseudoscalar mesons, id. it is the domain of pure χPT. It means that integrating out the resonances we get effective chiral Lagrangian which can be decomposed into standard terms of $\mathcal{L}_\chi$. Some constants in chiral Lagrangian are then saturated by vector resonances.

For complete description of processes with resonances we have to add chiral Lagrangian at the lowest order. The odd intrinsic parity sector requires to take into account also the Wess-Zumino term [12]. So the complete Lagrangian of the theory is

$$\mathcal{L} = \mathcal{L}_V + \mathcal{L}_\chi^{(2)} + \mathcal{L}_{WZ}^{(4)} \tag{2.12}$$

where

$$\mathcal{L}_{WZ}^{(4)} = -\frac{\sqrt{2}N_C}{8\pi^2 F}\epsilon_{\mu\nu\alpha\beta}\left\langle \phi \left\{ \partial^\mu v^\nu, \partial^\alpha v^\beta \right\} \right\rangle . \tag{2.13}$$

2.2 Two descriptions of vector resonances

There are two main possibilities how to describe vector resonances in the field theory. As was mentioned we can use the description of the vector fields. Moreover we can do the same with the antisymmetric tensor fields.

2.2.1 Vector field formalism

Taking into account just terms linear and quadratic in the resonance fields we write for the general Lagrangian in the vector formalism

$$\mathcal{L}_V = -\frac{1}{4}(\widehat{V}:\widehat{V}) + \frac{1}{2}m^2(V\cdot V) + (J_1\cdot V) + (J_2:\widehat{V}) + \frac{1}{2}(V\cdot K\cdot V) + (V\cdot J_3:\widehat{V}) \tag{2.14}$$

The sources J_i and K are constructed from the usual chiral building blocks (for example [9]). The chiral orders are

$$\begin{aligned} J_1 &= \mathcal{O}(p^3), \\ J_2 &= \mathcal{O}(p^2), \\ J_3 &= \mathcal{O}(p), \\ K &= \mathcal{O}(p^2). \end{aligned}$$

The concrete form of the sources we show later in Section 3. Classical equation of motion gives the solution to the lowest order

$$V = -\frac{1}{m^2}(J_1 - 2D\cdot J_2).$$

We see that the chiral order of the vector field V has to start at order $\mathcal{O}(p^3)$, the derivative of the field $\widehat{V}$ is then of order $\mathcal{O}(p^4)$. So we can decompose the resonance Lagrangian in powers of p

$$\mathcal{L}_V = \mathcal{L}_V^{(6)} + \mathcal{L}_V^{(8)} \tag{2.15}$$

where

$$\begin{aligned} \mathcal{L}_V^{(6)} &= \frac{1}{2}m^2(V\cdot V) + (J_1\cdot V) + (J_2:\widehat{V}), \\ \mathcal{L}_V^{(8)} &= -\frac{1}{4}(\widehat{V}:\widehat{V}) + \frac{1}{2}(V\cdot K\cdot V) + (V\cdot J_3:\widehat{V}). \end{aligned}$$

Substituing for the solution of classical equation of motion which is equivalent to Gaussian integration (neglecting loops and after further expansion) we get the resonance contribution to chiral Lagrangian up to $\mathcal{O}(p^6)$.

After integration by parts the result is

$$\mathcal{L}_{\chi,V}^{(6)} = -\frac{1}{2}m^2(J_1 \cdot J_1) + \frac{2}{m^2}(D \cdot J_2 \cdot J_1) + \frac{2}{m^2}(D \cdot J_2 \cdot J_2 \cdot \overleftarrow{D}). \qquad (2.16)$$

The contributions of higher orders are generated of course too but there is none of order $\mathcal{O}(p^4)$.

2.2.2 Antisymmetric tensor field formalism

Lagrangian in the antisymmetric tensor formulation has the following form

$$\mathcal{L}_T = \frac{1}{4}m^2(R : R) - \frac{1}{2}(W \cdot W) + (J_1 \cdot W) + (J_2 : R) + (W \cdot J_3 : R) + (R : J_4 : R) + (R : J_5 \cdot D : R). \qquad (2.17)$$

The chiral order of the sources are

$$\begin{aligned} J_1 &= \mathcal{O}(p^3), \\ J_2 &= J_2^{(2)} + J_2^{(4)} = \mathcal{O}(p^2) + \mathcal{O}(p^4), \\ J_3 &= \mathcal{O}(p), \\ J_4 &= \mathcal{O}(p^2), \\ J_5 &= \mathcal{O}(p). \end{aligned}$$

The solution of the classical equation of motion at lowest order is

$$R = -\frac{2}{m^2}J_2^{(2)}.$$

Antisymmetric tensor field starts the expansion at second order $R = \mathcal{O}(p^2)$. Organizing the Lagrangian in powers of p we obtain

$$\mathcal{L}_T = \mathcal{L}_T^{(4)} + \mathcal{L}_T^{(6)} \qquad (2.18)$$

where

$$\begin{aligned} \mathcal{L}_T^{(4)} &= \frac{1}{4}m^2(R : R) + (J_2^{(2)} : R), \\ \mathcal{L}_T^{(6)} &= -\frac{1}{2}(W \cdot W) + (J_2^{(4)} : R) + (W \cdot J_3 : R) + (R : J_4 : R) + (R : J_5 \cdot D : R). \end{aligned}$$

For the effective chiral Lagrangian up to $\mathcal{O}(p^6)$ we have

$$\mathcal{L}_{\chi,T} = \mathcal{L}_{\chi,T}^{(4)} + \mathcal{L}_{\chi,T}^{(6)} \qquad (2.19)$$

where

$$\mathcal{L}^{(4)}_{\chi,T} = -\frac{1}{m^2}(J_2^{(2)} : J_2^{(2)}),$$

$$\mathcal{L}^{(6)}_{\chi,T} = -\frac{2}{m^2}(J_2^{(2)} : J_2^{(4)}) + \frac{2}{m^4}(D \cdot J_2^{(2)} \cdot J_2^{(2)} \cdot \overleftarrow{D}) - \frac{2}{m^2}(D \cdot J_2^{(2)} \cdot J_1) + \frac{4}{m^4}(D \cdot J_2^{(2)} \cdot J_3 : J_2^{(2)}) + \frac{4}{m^4}(J_2^{(2)} : J_4 : J_2^{(2)}) + \frac{4}{m^4}(J_2^{(2)} : J_5 \cdot D : J_2^{(2)}).$$

2.3 Equivalence of both approaches

As it was recognized in [13] the naive correspondence connecting free vector and antisymmetric tensor fields

$$R \leftrightarrow \frac{1}{m}\widehat{V}, \qquad V \leftrightarrow -\frac{1}{m}W$$

doesn't relate the Lagrangians properly. Simplest way how to show the difference we start with the simple antisymmetric tensor Lagrangian

$$\mathcal{L}_T = \frac{1}{4}m^2(R : R) - \frac{1}{2}(W \cdot W) + (J_2 : R).$$

From the naive correspondence we obtain

$$\mathcal{L}_T \to \mathcal{L}_V = -\frac{1}{4}(\widehat{V} : \widehat{V}) + \frac{1}{2}m^2(V \cdot V) + \frac{1}{m}(J_2 : \widehat{V}).$$

However, the contributions to the effective chiral Lagrangians up to $\mathcal{O}(p^6)$ are not identical (as can be shown from last section). For instance to restore equality up to $O(p^4)$ we have to add the contact term

$$\mathcal{L}_T \to \mathcal{L}_V - \frac{1}{m^2}(J_2^{(2)} : J_2^{(2)}). \tag{2.20}$$

Therefore the naive substitution into the interaction terms with the sources J_i doesn't ensure the equivalence of both formulations.

The correspondence of these two formulations was studied in the past (cf. references [11], [13], [14], [15], [16], [17], [18]).

2.3.1 Vector → tensor correspondence

In this subsection we start with the vector field Lagrangian $\mathcal{L}_V$ and try to construct the antisymmetric tensor field Lagrangian $\mathcal{L}_T^{eff}$ which is equivalent to $\mathcal{L}_V$. Let us consider the Goldstone boson effective action $\Gamma_V[J_i, K]$ defined as

$$Z_V[J_i, K] = \exp i\Gamma_V[J_i, K] = \int \mathcal{D}V \exp\left(i\int d^4x \mathcal{L}_V\right).$$

Equivalence of $\mathcal{L}_V$ and $\mathcal{L}_T^{eff}$ means the equivalence of the contributions to the effective action $\Gamma_V[J_i, K]$

$$Z_V[J_i, K] = \exp i\Gamma_V[J_i, K] = \int \mathcal{D}R \exp\left(i\int d^4x \mathcal{L}_T^{eff}\right).$$

Introducing an auxiliary antisymmetric tensor field R we can write

$$\begin{aligned} Z_V[J_i, K] &= \int \mathcal{D}V \exp\left(i\int d^4x \mathcal{L}_V\right) \\ &= \frac{\int \mathcal{D}V\mathcal{D}R \exp\left(i\int d^4x \left(\frac{1}{4}m^2(R:R) + \mathcal{L}_V\right)\right)}{\int \mathcal{D}R \exp\left(i\int d^4x \frac{1}{4}m^2(R:R)\right)}. \end{aligned}$$

The auxiliary field R is merely an integration variable, it can be therefore freely redefined. In the following we try to integrate out the vector field and get the expression for the effective Lagrangian $\mathcal{L}_T^{eff}$ which is completely equivalent to $\mathcal{L}_V$. The detailed calculation is done in [2]. The result is an infinite series in powers of p which can be found in the same article. The antisymmetric tensor field Lagrangian $\mathcal{L}_T^{eff(\leq 6)}$ is not completely equivalent to original $\mathcal{L}_V$ but is equivalent up to $\mathcal{O}(p^6)$ and gives the same chiral Lagrangian $\mathcal{O}(p^6)$. After integration out the vector fields the Lagrangian $\mathcal{L}_T^{eff(\leq 6)}$ can be obtained by the truncation of the infinite series

$$\begin{aligned} \mathcal{L}_T^{eff(\leq 6)} = \frac{1}{4}m^2(R:R) - \frac{1}{2}(W\cdot W) + \left(J_1^{eff}\cdot W\right) + (J_2^{eff}:R) + (W\cdot J_3^{eff}:R) \\ +(R:J_4^{eff}:R) + (R:J_5^{eff}\cdot D:R) + \mathcal{L}_T^{eff,(\leq 6)contact}. \end{aligned} \tag{2.21}$$

where

$$\begin{aligned} J_1^{eff} &= -\frac{1}{m}J_1, \\ J_2^{eff} &= mJ_2 - \frac{2}{m}J_2:J_3\cdot J_3 - \frac{1}{m}J_1\cdot J_3, \\ J_3^{eff} &= -J_3, \\ J_4^{eff} &= -\frac{1}{2}J_3\cdot J_3, \\ J_5^{eff} &= 0, \end{aligned}$$

and the contact term

$$\mathcal{L}_T^{eff(\leq 6),contact} = (J_2 : J_2) - \frac{1}{2m^2}(J_1 \cdot J_1) - \frac{2}{m^2}(J_2 : J_3 \cdot J_1) - \frac{2}{m^2}(J_2 : J_3 \cdot J_3 : J_2).$$

The equivalence between the vector and the effective antisymmetric tensor Lagrangian up to the order $\mathcal{O}(p^6)$ cannot be complete unless $K = J_3 = 0$ in the original model. Then we have explicitly $\mathcal{L}_T^{eff(\geq 8)} = 0$ and the infinite series reduces to $\mathcal{L}_T^{eff(\leq 6)}$. This condition is satisfied in the vector field formulation so the equivalence is guaranteed.

2.3.2 Tensor → vector correspondence

Analogously we want to find the effective vector Lagrangian $\mathcal{L}_V^{eff}$ which is completely equivalent to the antisymmetric tensor Lagrangian $\mathcal{L}_T$. Detailed calculation is done again in [2]. The result up to $\mathcal{O}(p^6)$ is then

$$\mathcal{L}_V^{eff(\leq 6)} = -\frac{1}{4}(\widehat{V} : \widehat{V}) + \frac{1}{2}m^2(V \cdot V) + \left(J_1^{eff} \cdot V\right) + (J_2^{eff} : \widehat{V}) + \frac{1}{2}(V \cdot K^{eff} \cdot V) + (V \cdot J_3^{eff} : \widehat{V}) + \mathcal{L}_V^{eff(\leq 6),contact} \quad (2.22)$$

where

$$\begin{aligned} J_1^{eff} &= mJ_1, \\ J_2^{eff} &= -\frac{1}{m}J_2^{(2)}, \\ K^{eff} &= J_3^{eff} = 0. \end{aligned}$$

and the contact term

$$\begin{aligned} \mathcal{L}_V^{eff(\leq 6),contact} = {} & \frac{1}{2}(J_1 \cdot J_1) - \frac{1}{m^2}(J_2^{(2)} : J_2^{(2)}) - \frac{2}{m^2}(J_2^{(2)} : J_2^{(4)}) + \\ & \frac{4}{m^2}(J_2^{(2)} : J_4 : J_2^{(2)}) + \frac{4}{m^4}(J_2^{(2)} : D \cdot J_5 : J_2^{(2)}). \end{aligned}$$

The equivalence between the antisymmetric tensor and the effective vector Lagrangian up to order $\mathcal{O}(p^6)$ cannot be complete unless $J_3 = J_4 = J_5 = 0$ in the original model. But the concrete forms of the sources J_i in the antisymmetric tensor field formulation don't satisfy these conditions so the infinite series doesn't generally reduce to the finite number of terms.

2.3.3 Mixed formalism

In last two subsections we have tried to prove the equivalence between the vector and the antisymmetric tensor formulation up to $\mathcal{O}(p^6)$. But we have failed! These two approaches are not generally equivalent. It is seen already from the effective chiral Lagrangians which neither start at the same order nor all terms are analogous. The antisymmetric tensor formulation seems to be better (and it really is) but as it was mentioned in [21] and [20] it does not create the contact term

$$\mathcal{L}_\chi^{J_1} = -\frac{1}{2}(J_1 \cdot J_1) \tag{2.23}$$

in the effective chiral $\mathcal{O}(p^6)$ Lagrangian after integrating out the resonances. So it must be added by hand as in [21]. All these problems lead us to find another formulation from which can be derived both previous and will be more general. Because of the reasons mentioned in [2] we start with the following first order Lagrangian

$$\mathcal{L}_{VT} = \frac{1}{4}m^2(R:R) + \frac{1}{2}m^2(V\cdot V) - \frac{1}{2}m(R:\widehat{V}) + \frac{1}{2}(V\cdot K\cdot V) + (J_1\cdot V)$$
$$+(J_2:R) + (V\cdot J_3:R) + (R:J_4:R) + (R:J_5\cdot D:R). \tag{2.24}$$

The solutions of the equations of motion to the lowest order are

$$\begin{aligned} R &= \frac{2}{m^2}J_2^{(2)}, \\ V &= -\frac{1}{m^2}\left(J_1 - \frac{2}{m^3}J_3 : J_2^{(2)} - \frac{2}{m}D\cdot J_2^{(2)}\right) \end{aligned}$$

indicate the chiral counting $R = \mathcal{O}(p^2)$ and $V = \mathcal{O}(p^3)$, we have therefore

$$\mathcal{L}_{VT} = \mathcal{L}_{VT}^{(4)} + \mathcal{L}_{VT}^{(6)} + \mathcal{L}_{VT}^{(8)} \tag{2.25}$$

where

$$\begin{aligned} \mathcal{L}_{VT}^{(4)} &= \frac{1}{4}m^2(R:R) + (J_2^{(2)}:R), \\ \mathcal{L}_{VT}^{(6)} &= \frac{1}{2}m^2(V\cdot V) - \frac{1}{2}m(R:\widehat{V}) + (J_1\cdot V) + (J_2^{(4)}:R) + (V\cdot J_3:R) \\ &\quad +(R:J_4:R) + (R:J_5\cdot D:R), \\ \mathcal{L}_{VT}^{(8)} &= \frac{1}{2}(V\cdot K\cdot V). \end{aligned}$$

The corresponding effective chiral Lagrangian up to $\mathcal{O}(p^6)$ is

$$\mathcal{L}_{\chi,VT} = \mathcal{L}_{\chi,VT}^{(4)} + \mathcal{L}_{\chi,VT}^{(6)} \tag{2.26}$$

where

$$
\begin{aligned}
\mathcal{L}^{(4)}_{\chi,VT} &= -\frac{1}{m^2}(J_2^{(2)} : J_2^{(2)}), \\
\mathcal{L}^{(6)}_{\chi,VT} &= -\frac{1}{2m^2}(J_1 \cdot J_1) - \frac{2}{m^2}(J_2^{(2)} : J_2^{(4)}) + \frac{2}{m^4}(D \cdot J_2^{(2)} \cdot J_2^{(2)} \cdot \overleftarrow{D}) \\
&\quad + \frac{2}{m^3}(D \cdot J_2^{(2)} \cdot J_1) - \frac{4}{m^5}(D \cdot J_2^{(2)} \cdot J_3 : J_2^{(2)}) + \frac{4}{m^4}(J_2^{(2)} : J_4 : J_2^{(2)}) \\
&\quad + \frac{4}{m^4}(J_2^{(2)} : J_5 \cdot D : J_2^{(2)}) - \frac{2}{m^6}(J_2^{(2)} : J_3 \cdot J_3 : J_2^{(2)}) + \frac{2}{m^4}(J_1 \cdot J_3 : J_2^{(2)}).
\end{aligned}
$$

As was shown in [2] it is possible to integrate out the vector or the antisymmetric tensor fields and to derive the corresponding effective vector or antisymmetric tensor Lagrangians up to $\mathcal{O}(p^6)$. Their effective chiral Lagrangians are already identical (including the contact terms which were not present in the previous approaches).

Chapter 3

VVP correlator

As an explicit example of the mentioned theoretical description of vector resonances we calculate one off-shell process in all five formulations - vector, antisymmetric tensor, effective vector, effective antisymmetric tensor and mixed. Concretely it will be VVP correlator in the chiral limit, ie. the process with two vector external sources and one pseudoscalar. This can describe the processes $\pi^0 \to 2\gamma$, $\omega \to \pi\gamma$ and many others.

The result for the antisymmetric tensor method was already published in [3] and for the vector description in [19]. Formally we compute the correlator

$$(\Pi_{VVP})^{abc}_{\mu\nu}(p,q) = \int d^4x d^4y e^{i(q\cdot x + p\cdot y)} \left\langle 0 | T[V^a_\mu(x) V^b_\nu(y) P^c(0)|0 \right\rangle$$

where vector and pseudoscalar currents are

$$\begin{aligned} V^a_\mu(x) &= \frac{1}{\sqrt{2}} \overline{q} \gamma_\mu T^a q, \\ P^a &= \frac{1}{\sqrt{2}} \overline{q} i \gamma_5 T^a q. \end{aligned}$$

The invariances of QCD under P and T transformations and Ward identities allow us to extract group and tensor structure

$$(\Pi_{VVP})^{abc}_{\mu\nu}(p,q) = \epsilon_{\mu\nu\alpha\beta} p^\alpha q^\beta d^{abc} \Pi_{VVP}(p^2, q^2, r^2)$$

with $r = -(p+q)$. The QCD calculations within the OPE framework give in the chiral limit the high energy conditions [22], [19]

$$\lim_{\lambda\to\infty} \Pi_{VVP}((\lambda p)^2, (\lambda q)^2, (\lambda p + \lambda q)^2) = -\frac{B_0}{2\lambda^4} \frac{p^2+q^2+r^2}{p^2 q^2 r^2} + \mathcal{O}\left(\frac{1}{\lambda^6}\right)$$

Therefore our task is not only to compute the correlator but also to verify whether the high energy conditions are satisfied. As was already said due to suppresing loops in large N_C limit [23] we can take into account only tree level diagrams. Following calculations are organized in the same way for all formalisms:

- Lagrangians in definite formalism - Here we repeat the used Lagrangian, write concrete forms of sources and divide Lagrangian into particular terms contributing to vertices.
- Feynman rules - Here we derive Feynman rules for the vertices in definite model.
- Feynman diagrams - We construct all possible diagrams from vertices determined in preceding item.
- Result - We write the complete contribution to VVP correlator, compute the high energy behaviour and determine if it satisfies the high energy conditions or not.

3.1 Vector formalism

Lagrangian in vector formalism

Complete Lagrangian in the vector formalism is

$$\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_V + \mathcal{L}_{WZ}.$$

with the resonance Lagrangian of the form

$$\mathcal{L}_V = -\frac{1}{4}(\widehat{V} : \widehat{V}) + \frac{1}{2}m^2 (V \cdot V) + (J_1 \cdot V) + (J_2 : \widehat{V}) + \frac{1}{2}(V \cdot K \cdot V) + (V \cdot J_3 : \widehat{V}).$$

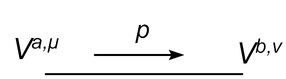
The sources that contribute to our Green function has the form

$$\begin{aligned}
J_1^{a\mu} &= h_V \epsilon_{\mu\nu\alpha\beta} \langle T^a \left\{ u^\nu, f_+^{\alpha\beta} \right\} \rangle, \\
J_2^{a\mu\nu} &= -\frac{1}{2} f_V \langle T^a f_+^{\mu\nu} \rangle, \\
J_3^{a\alpha\mu\nu} &= \frac{1}{2} \sigma_V \epsilon^{\alpha\beta\mu\nu} \langle \{T^a, T^b\} u_\beta \rangle, \\
K &= 0.
\end{aligned}$$

The expansion of the Lagrangian into particular contributions gives

Vector propagator - The kinetic and mass terms form the vector propagator.

$$i\Delta_V(p)^{ab}_{\mu\nu} = -\frac{i}{p^2 - m^2} \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{m^2} \right) \delta^{ab}$$



$\underline{J_1 \cdot V}$ - There is a contribution of the form $Vv\phi$.

$$\mathcal{L}_1 = h_V \epsilon_{\mu\nu\alpha\beta} \left\langle V^\mu \{u^\nu, f_+^{\alpha\beta}\} \right\rangle \sim -\frac{4\sqrt{2}h_V}{F} d^{abc} \epsilon_{\mu\nu\alpha\beta} V^{\mu,a} \partial^\nu \phi^b \partial^\alpha v^{\beta,c}.$$

$\underline{J_2 : \widehat{V}}$ - There is a contribution of the form Vv.

$$\mathcal{L}_2 = -\frac{f_V}{2\sqrt{2}} \langle (D_\mu V_\nu - D_\nu V_\mu) f_+^{\mu\nu} \rangle \sim -f_V \delta^{ab} \partial_\mu V_\nu^a (\partial_\mu v_\nu^b - \partial_\nu v_\mu^b).$$

$\underline{V \cdot J_3 : \widehat{V}}$ - There is a contribution of the form $VV\phi$.

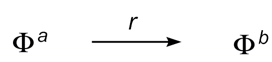
$$\mathcal{L}_3 = \frac{1}{2} \sigma_V \epsilon_{\alpha\beta\mu\nu} \left\langle \{D^\mu V^\nu - D^\nu V^\mu, V^\alpha\} u^\beta \right\rangle \sim -\frac{2\sigma_V}{F} d^{abc} \epsilon_{\alpha\beta\mu\nu} \partial^\mu V^{\nu,a} V^{\alpha,b} \partial^\beta \phi^c.$$

Chiral and Wess-Zumino - There are contributions of the form $VV\phi$, $P\phi$ and pseudoscalar propagator.

$$\mathcal{L}_{WZ}^{(4)} = -\frac{\sqrt{2}N_C}{8\pi^2 F} d^{abc} \epsilon_{\mu\nu\alpha\beta} \left\langle \Phi \partial^\mu v^\nu \partial^\alpha v^\beta \right\rangle \sim -\frac{N_C}{16\pi^2 F} \epsilon_{\mu\nu\alpha\beta} \partial^\mu v^{\nu,a} \partial^\alpha v^{\beta,b} \phi^c,$$
$$\mathcal{L}_\chi^{(2)} = \frac{F^2}{4} \langle u_\mu u^\mu + \chi_+ \rangle \sim \frac{1}{2} \partial_\mu \phi^a \partial^\mu \phi^a + F B_0 \delta^{ab} P^a \phi^b.$$

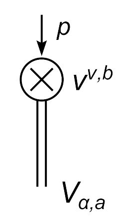
Pseudoscalar propagator - The kinetic term comes from $\mathcal{L}_\chi^{(2)}$, there is no mass term because pseudoscalar mesons are massless particles in the chiral limit.

$$i\Delta_P(r)_{\mu\nu}^{ab} = \frac{i}{r^2} \delta^{ab}.$$



Feynman rules in vector formulation

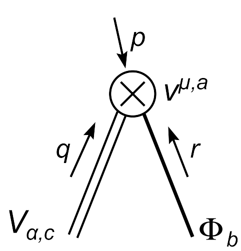
Vertex 1 : vector source - resonance



There contributes $\mathcal{L}_2$ to this vertex. The Feynman rule is

$$(V_1)_{\alpha\nu}^{ab} = -if_V (ip_\rho) g_{\alpha\sigma} \delta^{ab} [(-ip^\rho) g_\nu^\sigma - (-ip^\sigma) g_\nu^\rho]$$
$$= -if_V \delta^{ab} (p^2 g_{\alpha\nu} - p_\alpha p_\nu).$$

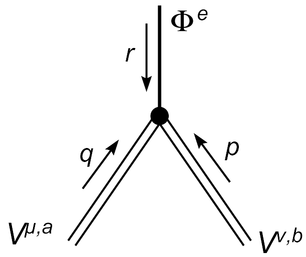
Vertex 2 : vector source - resonance - pseudoscalar



There contributes $\mathcal{L}_1$ to this vertex. The Feynman rule is

$$(V_2)_{\mu\alpha}^{abc} = -i\frac{4\sqrt{2}h_V}{F} d^{abc} \epsilon_{\rho\sigma\kappa\lambda} g_\alpha^\rho (-ir^\sigma)(-ip^\kappa) g_\mu^\lambda$$
$$= i\frac{4\sqrt{2}h_V}{F} d^{abc} \epsilon_{\alpha\sigma\beta\mu} r^\sigma p^\beta.$$

Vertex 3 : resonance - resonance - pseudoscalar



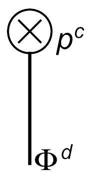
There contributes $\mathcal{L}_3$ to this vertex. The Feynman rule is

$$\left(V_3^{'}\right)_{\mu\nu}^{abe} = -i\frac{2\sigma_V}{F} d^{abe} \epsilon_{\rho\sigma\kappa\lambda}(-iq^\kappa) g_\mu^\lambda g_\nu^\rho (-ir^\sigma)$$
$$= i\frac{2\sigma_V}{F} d^{abe} \epsilon_{\nu\sigma\rho\mu} q^\rho r^\sigma.$$

Due to Bose statistics we have

$$(V_3)_{\mu\nu}^{abc} = i\frac{2\sigma_V}{F} d^{abc} \epsilon_{\nu\sigma\rho\mu}(q^\rho - p^\rho) r^\sigma.$$

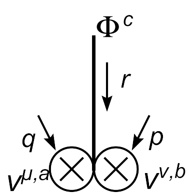
Vertex χ : pseudoscalar source - pseudoscalar



There contributes $\mathcal{L}_\chi^{(2)}$ to this vertex. The Feynman rule is

$$(V_\chi)^{cd} = iFB_0\delta^{cd}.$$

Vertex WZ : vector source - vector source - pseudoscalar



There contributes the anomaly term $\mathcal{L}_{WZ}^{(4)}$. The Feynman rule is

$$(V_{WZ}^{'})_{\mu\nu}^{abc} = -i\frac{N_C}{16\pi^2 F} d^{abc} \epsilon_{\mu\nu\alpha\beta} p^\alpha q^\beta.$$

Due to Bose statistics we get

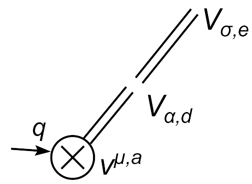
$$(V_{WZ})_{\mu\nu}^{abc} = -i\frac{N_C}{8\pi^2 F} d^{abc} \epsilon_{\mu\nu\alpha\beta} p^\alpha q^\beta.$$

Last two vertices exist in every formulation.

Feynman diagrams

First we determine two subdiagrams which will be useful in the following computation.

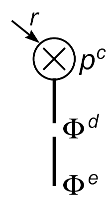
Subdiagram 1: vertex 1 - tensor



This subdiagram consists of vector propagator and vertex 1.

$$\begin{aligned}(S_1)^{ae}_{\mu\rho\sigma} &= (V_1)^{ad}_{\mu\alpha} i \left(\Delta_V(q)\right)^{\alpha\sigma,de} \\ &= \frac{f_V \delta^{ae}}{m^2 - q^2}(p^2 g_{\sigma\mu} - p_\sigma p_\mu).\end{aligned}$$

Subdiagram χ: vertex χ - pseudoscalar

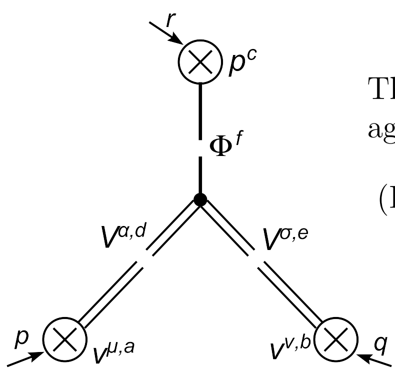


This subdiagram consists of pseudoscalar propagator and vertex χ.

$$(S_2)^{ce} = (V_\chi)^{cd} i(\Delta_P(r))^{de} = -\frac{FB_0}{r^2}\delta^{ce}$$

Next we calculate all Feynman diagrams contributing to VVP correlator (on the tree level, of course).

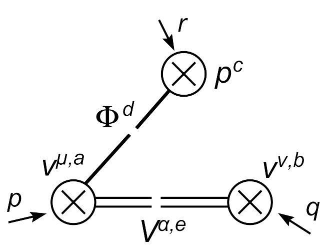
Diagram 1



The diagram consists of vertex 3 and three subdiagrams. Taking all together we find

$$\begin{aligned}(\Pi_1)^{abc}_{\mu\nu} &= (V_3)^{\alpha\sigma,def}(S_2)^{fc}(S_1)^{ad}_{\mu\alpha}(S_1)^{be}_{\nu\sigma} \\ &= \frac{4iB_0\sigma_V f_V^2 p^2 q^2}{r^2(p^2-m^2)(q^2-m^2)} d^{abc}\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta.\end{aligned}$$

Diagram 2



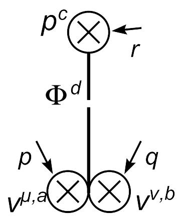
The diagram consists of vertex 2 and two subdiagrams. Taking all together we find

$$\begin{aligned}(\Pi_2)^{abc}_{\mu\nu} &= (V_2)^{dae}_{\alpha\mu}(S_2)^{dc}(S_1)^{\alpha,be}_{\nu} \\ &= -\frac{4\sqrt{2} i p^2 B_0 h_V F_V}{r^2(p^2-m^2)} d^{abc}\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta.\end{aligned}$$

After changing $v^a_\mu \leftrightarrow v^b_\nu$ and $p \leftrightarrow q$ we get similar diagram. Its contribution to the correlator is

$$(\Pi_3)^{abc}_{\mu\nu} = -\frac{4\sqrt{2} i q^2 B_0 h_V f_V}{r^2(q^2-m^2)} d^{abc}\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta.$$

Diagram χ



The diagram consists of vertex WZ and subdiagram χ. Taking it together we find

$$\begin{aligned}(\Pi_4)^{abc}_{\mu\nu} &= (V_{WZ})^{abd}_{\mu\nu}(S_2)^{dc} \\ &= \frac{iB_0 N_C}{8\pi^2 r^2} d^{abc}\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta.\end{aligned}$$

Results

Summary

After multiplication by i from the Feynman rule (see Appendix) we get for the complete VVP correlator

$$\begin{aligned}(\Pi_{VVP})^{abc}_{\mu\nu}(p,q) &= \int d^4x \int d^4y e^{i(p.x+q.y)} \left\langle 0|T\left[V^a_\mu(x)V^b_\nu(y)P^c(0)\right]|0\right\rangle \\ &= \epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta d^{abc}\frac{B_0}{r^2}\Big\{-\frac{4\sigma_V f_V^2 p^2 q^2}{(p^2-m^2)(q^2-m^2)} \\ &\quad +\frac{4\sqrt{2}p^2 h_V f_V}{(p^2-m^2)} + \frac{4\sqrt{2}q^2 h_V f_V}{(q^2-m^2)} - \frac{N_C}{8\pi^2}\Big\}.\end{aligned}$$

High energy conditions

The high energy behaviour of VVP correlator in the vector formulation is

$$\begin{aligned}\lim_{\lambda\to\infty} \quad \Pi_{VVP}((\lambda p)^2,(\lambda q)^2,(\lambda r)^2) &= \frac{B_0}{\lambda^2 r^2}\left[-\frac{N_C}{8\pi^2} + 8\sqrt{2}f_V h_V - 4f_V^2\sigma_V\right] \\ &+\frac{B_0 m^2}{\lambda^4}\left[\frac{4\sqrt{2}f_V h_V - 4f_V^2\sigma_V}{p^2r^2} + \frac{4\sqrt{2}f_V h_V - 4f_V^2\sigma_V}{q^2r^2}\right] + \mathcal{O}(\frac{1}{\lambda^6})\end{aligned}$$

The term proportional to $1/\lambda^2$ has to vanish, id.

$$-\frac{N_C}{8\pi^2} + 8\sqrt{2}f_V h_V - 4f_V^2\sigma_V = 0.$$

However at order $1/\lambda^4$ we are not able to satisfy the conditions.

The result in the vector formulation is not consistent with the high energy conditions.

3.2 Tensor formalism

Lagrangian in tensor formalism

Complete Lagrangian in the antisymmetric tensor formalism is

$$\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_T + \mathcal{L}_{WZ}.$$

Chiral and Wess-Zumino terms are the same as in the vector case. The resonance Lagrangian up to $\mathcal{O}(p^6)$ is

$$\begin{aligned}\mathcal{L}_T &= -\frac{1}{2}(W\cdot W) + \frac{1}{4}m^2(R:R) + (J_1\cdot W) + (J_2:R)\\ &\quad +(W\cdot J_3:R) + (R:J_4:R) + (R:J_5\cdot D:R).\end{aligned}$$

The sources contributing to VVP correlator has the form

$$\begin{aligned}(J_2)^a_{\mu\nu} &= \langle T^a J_{2\mu\nu}\rangle,\\ (J_4)^{ab}_{\mu\nu\alpha\beta} &= \langle\{T^a,T^b\}J_{4\mu\nu\alpha\beta}\rangle,\\ (J_5)^{ab\alpha}_{\mu\nu\rho\sigma} &= \langle\{T^a,T^b\}J^\alpha_{5\mu\nu\rho\sigma}\rangle\end{aligned}$$

where

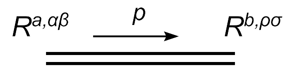
$$\begin{aligned}J^{(2)}_{2\mu\nu} &= \frac{1}{2\sqrt{2}}F_V f_{+\mu\nu},\\ J^{(4)}_{2\mu\nu} &= \epsilon_{\mu\kappa\rho\sigma}\frac{c_1}{m}\left\{f_+^{\rho\alpha},D_\alpha u^\sigma\right\}g^\kappa_\nu + \frac{c_2}{m}\left\{f_+^{\rho\sigma},D_\nu u^\kappa\right\}\\ &\quad +i\frac{c_3}{m}\left\{f_+^{\rho\sigma},\chi_-\right\}g^\kappa_\nu + i\frac{c_4}{m}\left[f_-^{\rho\sigma},\chi_+\right]g^\kappa_\nu - \frac{c_5}{m}D_\lambda\left\{f_+^{\rho\lambda},u^\sigma\right\}g^\kappa_\nu\\ &\quad -\frac{c_6}{m}D_\nu\left\{f_+^{\rho\sigma},u^\kappa\right\} - \frac{c_7}{m}D^\sigma\left\{f_+^{\rho\lambda},u_\lambda\right\}g^\kappa_\nu,\\ J_{4\mu\nu\alpha\beta} &= \frac{1}{4}d_1(\epsilon_{\mu\nu\alpha\sigma}D_\beta u^\sigma - \epsilon_{\mu\nu\beta\sigma}D_\alpha u^\sigma + \epsilon_{\alpha\beta\mu\sigma}D_\nu u^\sigma - \epsilon_{\alpha\beta\nu\sigma}D_\mu u^\sigma)\\ &\quad +\epsilon_{\mu\nu\alpha\beta}d_2\chi_-,\\ J^\alpha_{5\mu\nu\rho\sigma} &= \epsilon_{\rho\sigma\mu\lambda}(d_3 u^\lambda g^\alpha_\nu + d_4 u_\nu g^{\alpha\lambda}).\end{aligned}$$

The source J_2 is parted into $\mathcal{O}(p^2)$ and $\mathcal{O}(p^4)$ terms. Sources J_1 and J_3 don't contribute to VVP correlator.

For this Lagrangians and sources we get in approximation the particular contributions:

Tensor propagator - The kinetic and mass terms form the tensor propagator.

$$i(\Delta_R(p))^{ab}_{\alpha\beta,\rho\sigma} = -\frac{i\delta^{ab}}{m^2(p^2-m^2)}[(m^2-p^2)g_{\alpha\rho}g_{\beta\sigma}+g_{\alpha\rho}p_\beta p_\sigma-g_{\alpha\sigma}p_\beta p_\rho-(\alpha\leftrightarrow\beta)].$$



$J_2 : R$ - There are contributions of the form Rv, $Rv\phi$ and Rvp. Last three terms are integrated by parts in order to simplify their form.

$$\begin{aligned}
\mathcal{L}_1 &= \frac{F_V}{2\sqrt{2}}\langle R_{\mu\nu}f_+^{\mu\nu}\rangle \sim F_V\delta^{ab}R^{\mu\nu,a}\partial_\mu v^b_\nu,\\
\mathcal{L}_2 &= \frac{c_1}{m}\epsilon_{\mu\nu\rho\sigma}\langle\{R^{\mu\nu},f_+^{\rho\alpha}\}D_\alpha u^\sigma\rangle \sim -\frac{2\sqrt{2}c_1}{mF}d^{abc}\epsilon_{\mu\nu\rho\sigma}R^{\mu\nu,a}\left(\partial_\rho v^b_\alpha-\partial_\alpha v^b_\rho\right)\partial^\alpha\partial^\sigma\phi^c,\\
\mathcal{L}_3 &= \frac{c_2}{m}\epsilon_{\mu\nu\rho\sigma}\langle\{R^{\mu\alpha},f_+^{\rho\sigma}\}D_\alpha u^\nu\rangle \sim -\frac{4\sqrt{2}c_2}{mF}d^{abc}\epsilon_{\mu\nu\rho\sigma}R^{\mu\lambda,a}\partial^\rho v^{\sigma,b}\partial_\lambda\partial^\nu\phi^c,\\
\mathcal{L}_4 &= \frac{ic_3}{m}\epsilon_{\mu\nu\rho\sigma}\langle\{R^{\mu\nu},f_+^{\rho\sigma}\}\chi_-\rangle \sim -\frac{8\sqrt{2}c_3B_0}{m}d^{abc}\epsilon_{\mu\nu\rho\sigma}R^{\mu\nu,a}\partial^\rho v^{\sigma,b}P^c,\\
\mathcal{L}_5 &= \frac{c_4}{m}\epsilon_{\mu\nu\rho\sigma}\langle R^{\mu\alpha}[f_-^{\rho\sigma},\chi_+]\rangle \sim 0,\\
\mathcal{L}_6 &= \frac{c_5}{m}\epsilon_{\mu\nu\rho\sigma}\langle\{D_\alpha R^{\mu\nu},f_+^{\rho\alpha}\}u^\sigma\rangle \sim -\frac{2\sqrt{2}c_5}{mF}d^{abc}\epsilon_{\mu\nu\rho\sigma}\partial_\lambda R^{\mu\nu,a}(\partial^\rho v^{\lambda,b}-\partial^\lambda v^{\rho,b})\partial^\sigma\phi^c,\\
\mathcal{L}_7 &= \frac{c_6}{m}\epsilon_{\mu\nu\rho\sigma}\langle\{D_\alpha R^{\mu\alpha},f_+^{\rho\sigma}\}u^\nu\rangle \sim -\frac{4\sqrt{2}c_6}{mF}d^{abc}\epsilon_{\mu\nu\rho\sigma}\partial_\lambda R^{\mu\lambda,a}\partial^\rho v^{\sigma,b}\partial^\nu\phi^c,\\
\mathcal{L}_8 &= \frac{c_7}{m}\epsilon_{\mu\nu\rho\sigma}\langle\{D^\sigma R^{\mu\nu},f_+^{\rho\alpha}\}u_\alpha\rangle \sim -\frac{2\sqrt{2}c_7}{mF}d^{abc}\epsilon_{\mu\nu\rho\sigma}\partial^\sigma R^{\mu\nu,a}(\partial^\rho v^{\lambda,b}-\partial^\lambda v^{\rho,b})\partial_\lambda\phi^c.
\end{aligned}$$

$R : J_4 : R$ - There are contributions of the form $RR\phi$ and RRp.

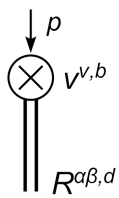
$$\begin{aligned}
\mathcal{L}_9 &= \frac{1}{4}d_1\left\langle\left\{R^{\mu\nu},R^{\alpha\beta}\right\}(\epsilon_{\mu\nu\alpha\sigma}D_\beta u^\sigma-\epsilon_{\mu\nu\beta\sigma}D_\alpha u^\sigma+\epsilon_{\alpha\beta\mu\sigma}D_\nu u^\sigma-\epsilon_{\alpha\beta\nu\sigma}D_\mu u^\sigma)\right\rangle\\
&\sim -\frac{2}{F}d^{abc}\epsilon_{\mu\nu\rho\sigma}R^{\mu\nu,a}R^{\rho\lambda,b}\partial_\lambda\partial^\sigma\phi^c,\\
\mathcal{L}_{10} &= id_2\epsilon_{\mu\nu\rho\sigma}\langle\{R^{\mu\nu},R^{\rho\sigma}\}\chi_-\rangle \sim -4B_0d^{abc}\epsilon_{\mu\nu\rho\sigma}R^{\mu\nu,a}R^{\rho\sigma,b}P^c.
\end{aligned}$$

$R : J_5 \cdot D : R$ - There are contributions of the form $RR\phi$.

$$\begin{aligned}
\mathcal{L}_{11} &= d_3\epsilon_{\mu\nu\rho\sigma}\langle\{D_\alpha R^{\mu\nu},R^{\rho\alpha}\}u^\sigma\rangle \sim -\frac{2}{F}d^{abc}\epsilon_{\mu\nu\rho\sigma}\partial_\lambda R^{\mu\nu,a}R^{\rho\lambda,b}\partial^\sigma\phi^c,\\
\mathcal{L}_{12} &= d_4\epsilon_{\mu\nu\rho\sigma}\langle\{D^\sigma R^{\mu\nu},R^{\rho\alpha}\}u_\alpha\rangle \sim -\frac{2}{F}d^{abc}\epsilon_{\mu\nu\rho\sigma}\partial^\sigma R^{\mu\nu,a}R^{\rho\lambda,b}\partial_\lambda\phi^c.
\end{aligned}$$

Feynman rules in tensor formulation

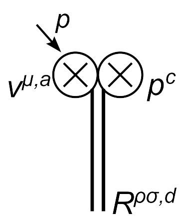
Vertex 1 : vector source - resonance



There contributes only $\mathcal{L}_1$ to this vertex. The Feynman rule is

$$
\begin{aligned}
(V_1)^{bd}_{\alpha\beta\nu} &= i\frac{F_V}{2}(g^\rho_\alpha g^\sigma_\beta - g^\rho_\beta g^\sigma_\alpha)(-ip_\rho)g_{\sigma\nu}\delta^{bd} \\
&= \frac{F_V}{2}\delta^{bd}(p_\alpha g_{\beta\nu} - p_\beta g_{\alpha\nu}).
\end{aligned}
$$

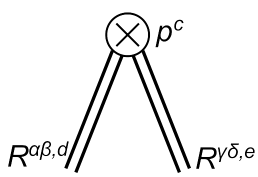
Vertex 2 : vector source - pseudoscalar source - resonance



There contributes $\mathcal{L}_4$ to this vertex. The Feynman rule is

$$
\begin{aligned}
(V_2)^{acd}_{\rho\sigma\mu} &= -i\frac{c_3}{2m}8\sqrt{2}B_0 d^{dac}\epsilon^{\alpha\lambda\pi\beta}(g_{\alpha\rho}g_{\lambda\sigma} - g_{\alpha\sigma}g_{\lambda\rho})g_{\beta\mu}(-ip_\pi) \\
&= -\frac{c_3}{m}8\sqrt{2}B_0 d^{dac}\epsilon_{\rho\sigma\lambda\mu}p^\lambda.
\end{aligned}
$$

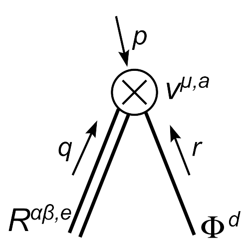
Vertex 3 : pseudoscalar source - resonance - resonance



There contributes $\mathcal{L}_{10}$ to this vertex. The Feynman rule is

$$
\begin{aligned}
(V_3)^{cde}_{\alpha\beta\gamma\delta} &= -2id_2B_0d^{dec}\epsilon^{\kappa\lambda\pi\theta}(g_{\kappa\alpha}g_{\lambda\beta} - g_{\kappa\beta}g_{\lambda\alpha})(g_{\pi\gamma}g_{\theta\delta} - g_{\pi\delta}g_{\theta\gamma}) \\
&= -8id_2B_0d^{dec}\epsilon_{\alpha\beta\gamma\delta}.
\end{aligned}
$$

Vertex 4 : vector source - resonance - pseudoscalar

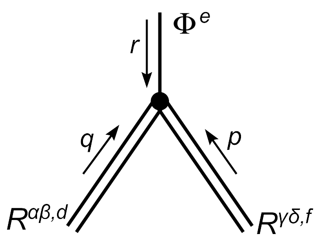


There contribute $\mathcal{L}_2$, $\mathcal{L}_3$, $\mathcal{L}_6$, $\mathcal{L}_7$ and $\mathcal{L}_8$ to this vertex. For particular terms we find these Feynman rules

$$
\begin{aligned}
(V_4^1)^{ade}_{\mu\alpha\beta} &= -c_1\frac{2\sqrt{2}}{mF}d^{ead}\epsilon_{\alpha\beta\mu\sigma}r^\sigma p_\rho r^\rho + c_1\frac{2\sqrt{2}}{mF}d^{ead}\epsilon_{\alpha\beta\rho\sigma}p^\rho r^\sigma r_\mu, \\
(V_4^2)^{ade}_{\mu\alpha\beta} &= c_2\frac{2\sqrt{2}}{mF}d^{ead}(\epsilon_{\alpha\lambda\rho\mu}p^\rho r_\beta r^\lambda - \epsilon_{\beta\lambda\rho\mu}p^\rho r_\alpha r^\lambda), \\
(V_4^3)^{ade}_{\mu\alpha\beta} &= -c_5\frac{2\sqrt{2}}{mF}d^{ead}\epsilon_{\alpha\beta\mu\delta}r^\delta p_\rho q^\rho + c_5\frac{2\sqrt{2}}{mF}d^{ead}\epsilon_{\alpha\beta\lambda\delta}q_\mu p^\lambda r^\delta,
\end{aligned}
$$

$$
\begin{aligned}
(V_4^4)^{ade}_{\mu\alpha\beta} &= c_6 \frac{2\sqrt{2}}{mF} d^{ead} (\epsilon_{\alpha\rho\lambda\mu} q_\beta p^\lambda r^\rho - \epsilon_{\beta\rho\lambda\mu} q_\alpha p^\lambda r^\rho), \\
(V_4^5)^{ade}_{\mu\alpha\beta} &= -c_7 \frac{2\sqrt{2}}{mF} d^{ead} \epsilon_{\alpha\beta\mu\sigma} q^\sigma p_\rho r^\rho + c_7 \frac{2\sqrt{2}}{mF} d^{ead} \epsilon_{\alpha\beta\lambda\rho} q^\rho p^\lambda r_\mu.
\end{aligned}
$$

Vertex 5 : resonance - resonance - pseudoscalar



There contribute $\mathcal{L}_9$, $\mathcal{L}_{11}$ and $\mathcal{L}_{12}$ to this vertex. For particular terms we find these Feynmane rules

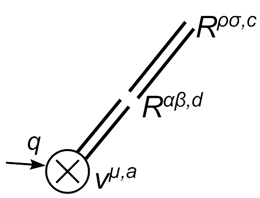
$$
\begin{aligned}
(V_5^1)^{def}_{\alpha\beta\gamma\delta} &= d_1 \frac{i}{F} d^{dfe} (\epsilon_{\alpha\beta\gamma\sigma} r^\sigma r_\delta - \epsilon_{\alpha\beta\delta\sigma} r^\sigma r_\gamma), \\
(V_5^2)^{def}_{\alpha\beta\gamma\delta} &= d_3 \frac{2i}{F} d^{dfe} \epsilon_{\alpha\beta\gamma\sigma} q_\delta r^\sigma, \\
(V_5^3)^{def}_{\alpha\beta\gamma\delta} &= d_4 \frac{i}{F} d^{dfe} (\epsilon_{\alpha\beta\gamma\sigma} q^\sigma r_\delta - \epsilon_{\alpha\beta\delta\sigma} q^\sigma r_\gamma).
\end{aligned}
$$

Vertices χ and WZ are the same as in the vector case.

Feynman diagrams

First we determine the subdiagram which will be useful in the future. It is very similar as in vector case

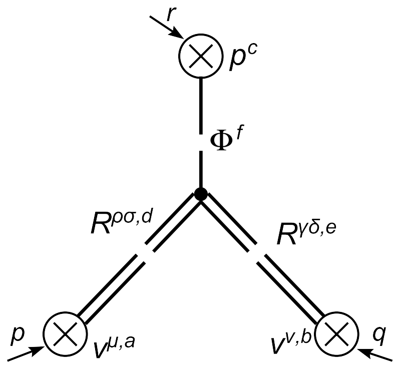
Subdiagram 1: vertex 1 - tensor



$$
\begin{aligned}
(S_1)^{ac}_{\mu\rho\sigma} &= (V_1)^{ac}_{\mu\alpha\beta} i \left(\Delta_T(q)\right)^{\alpha\beta\rho\sigma} \\
&= \frac{iF_V \delta^{ad}}{m^2 - q^2} (g_{\mu\sigma} q_\rho - g_{\mu\rho} q_\sigma).
\end{aligned}
$$

We label $(S_2)^{ab}$ the second subdiagram which is the same as in the vector case (vertex χ - pseudoscalar).

Diagram 1



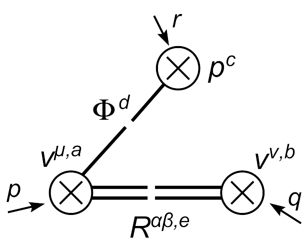
The diagram consists of vertex 5 and three subdiagrams. Taking all together we find

$$(\Pi_1)^{abc}_{\mu\nu} = (V_5)^{def}_{\rho\sigma\gamma\delta}(S_2)^{fc}(S_1)^{da}_{\mu\rho\sigma}(S_1)^{eb}_{\gamma\delta\nu}$$

The term with d_4 coming from vertex 5 vanish so we obtain

$$(\Pi_1)^{abc}_{\mu\nu} = -\frac{4iF_V^2 d^{abc}}{r^2(m^2-q^2)(m^2-p^2)}\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta[(d_1-d_3)r^2 + d_3(p^2+q^2)].$$

Diagram 2



The diagram consists of vertex 4 and two subdiagrams. Taking all together we find

$$(\Pi_2)^{abc}_{\mu\nu} = (V_4)^{be}_{\mu\alpha\beta}(S_1)^{be}_{\alpha\beta\nu}(S_2)^{cd}.$$

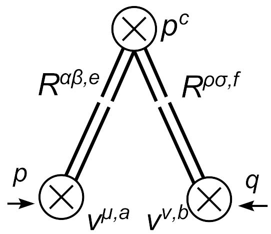
The term with c_7 vanish so the result is

$$\begin{aligned}(\Pi_2)^{abc}_{\mu\nu} &= \frac{2i\sqrt{2}F_V B_0}{r^2(m^2-p^2)}d^{abc}\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta[p^2(-c_1+c_2+c_5-2c_6) \\ &\quad +q^2(c_1-c_2+c_5)+r^2(c_1+c_2-c_5)].\end{aligned}$$

Changing $v^a_\mu \leftrightarrow v^b_\nu$ and $p \leftrightarrow q$ we get very similar diagram

$$\begin{aligned}(\Pi_3)^{abc}_{\mu\nu} &= \frac{2i\sqrt{2}F_V B_0}{r^2(m^2-q^2)}d^{abc}\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta[q^2(-c_1+c_2+c_5-2c_6) \\ &\quad +p^2(c_1-c_2+c_5)+r^2(c_1+c_2-c_5)].\end{aligned}$$

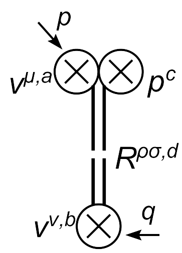
Diagram 3



The diagram consists of vertex 3 and two subdiagrams. Taking together we find

$$
\begin{aligned}
(\Pi_4)^{abc}_{\mu\nu} &= (V_3)^{def}_{\rho\sigma\alpha\beta}(S_1)^{\rho\sigma,f}(S_1)^{\alpha\beta,e} \\
&= -\frac{32iF_V^2 d_2 B_0}{(m^2-p^2)(m^2-q^2)} d^{abc}\epsilon_{\alpha\beta\mu\nu}p^\alpha q^\beta.
\end{aligned}
$$

Diagram 4



The diagram consists of vertex 2 and one subdiagram. Taking it together we find

$$
\begin{aligned}
(\Pi_5)^{abc}_{\mu\nu} &= (S_1)^{bd}_{\rho\sigma\nu}(V_2)^{\rho\sigma,acd}_{\mu} \\
&= \frac{16i\sqrt{2}B_0F_Vc_3}{m(m^2-p^2)} d^{abc}\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta.
\end{aligned}
$$

Changing $v^a_\mu \leftrightarrow v^b_\nu$ and $p \leftrightarrow q$ we get similar contribution.

$$
(\Pi_6)^{abc}_{\mu\nu} = \frac{16i\sqrt{2}B_0F_Vc_3}{m(m^2-q^2)} d^{abc}\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta.
$$

Finally we add diagram χ which is the same as in vector formulation.

Results

Summary

After multiplication by i from the Feynman rule we get for the complete VVP correlator in the antisymmetric tensor formulation

$$
\begin{aligned}
&(\Pi_{VVP})^{abc}_{\mu\nu}(p,q) = \int d^4x \int d^4y e^{i(p.x+q.y)} \left\langle 0|T\left[V^a_\mu(x)V^b_\nu(y)P^c(0)\right]|0\right\rangle \\
= \;& \epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta d^{abc}B_0\Big\{4F_V^2\frac{(d_1-d_3)r^2+d_3(p^2+q^2)}{(m^2-p^2)(m^2-q^2)r^2} \\
&-2\sqrt{2}\frac{F_V}{m}\frac{r^2(c_1+c_2-c_5)+p^2(-c_1+c_2+c_5-2c_6)+q^2(c_1-c_2+c_5)}{(m^2-p^2)r^2} \\
&-2\sqrt{2}\frac{F_V}{m}\frac{r^2(c_1+c_2-c_5)+q^2(-c_1+c_2+c_5-2c_6)+p^2(c_1-c_2+c_5)}{(m^2-q^2)r^2} \\
&+\frac{32F_V^2d_2}{(m^2-p^2)(m^2-q^2)} - \frac{16\sqrt{2}F_Vc_3}{m(m^2-p^2)} - \frac{16\sqrt{2}F_Vc_3}{m(m^2-q^2)} - \frac{N_C}{8\pi^2r^2}\Big\}.
\end{aligned}
$$

High energy behaviour

The short distance behaviour of VVP correlator in the antisymmetric tensor formulation is

$$\lim_{\lambda\to\infty}\Pi_{VVP}((\lambda p)^2,(\lambda q)^2,(\lambda r)^2)=\frac{B_0}{\lambda^2 r^2}\Big\{-\frac{N_C}{8\pi^2 r^2}+2\sqrt{2}\frac{F_V}{mp^2q^2r^2}\times$$
$$\Big[(c_1+c_2-c_5)(q^2+p^2)+2p^2q^2(-c_1+c_2+c_5-2c_6)+(c_1-c_2+c_5)(p^4+q^4)\Big]$$
$$+\frac{16\sqrt{2}F_Vc_3(p^2+q^2)}{mp^2q^2}\Big\}+\frac{B_0}{\lambda^4 r^2}\Big\{\frac{4F_V^2(d_1-d_3)}{p^2q^2}+\frac{4F_V^2(p^2+q^2)d_3}{p^2q^2r^2}$$
$$-\frac{2\sqrt{2}F_V}{mp^2q^2r^2}[2m^2r^2(c_1+c_2-c_5)+(p^2+q^2)m^2(-c_1+c_2+c_5-2c_6)$$
$$+m^2(p^2+q^2)(c_1-c_2+c_5)]+\frac{32F_V^2d_2}{p^2q^2}-\frac{32\sqrt{2}F_Vc_3m}{p^2q^2}\Big\}+\mathcal{O}(\frac{1}{\lambda^6}).$$

To fulfill the high energy conditions we have to demand

$$\begin{aligned}
c_1 &= -4c_3\\
c_2 &= -4c_3+c_5\\
c_6 &= c_5-\frac{N_Cm}{64\sqrt{2}\pi^2F_V}\\
d_1 &= -8d_2-\frac{N_Cm^2}{64\pi^2F_V^2}+\frac{F^2}{4F_V^2}\\
d_3 &= -\frac{N_Cm^2}{64\pi^2F_V^2}+\frac{F^2}{4F_V^2}.
\end{aligned}$$

The result in the antisymmetric tensor formulation is consistent with the high energy conditions. In this case the expression simplifies to

$$(\Pi_{VVP})^{abc}_{\mu\nu}(p,q)=\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta d^{abc}\frac{B_0^2F_0^2}{2}\frac{p^2+q^2+r^2-\frac{N_Cm^4}{4\pi^2F_0^2}}{(p^2-m^2)(q^2-m^2)r^2}.$$

3.3 Effective tensor formalism

Lagrangians

In tensor-vector correspondence we start with vector Lagrangian and derive the effective tensor Lagrangian [2] up to $\mathcal{O}(p^6)$

$$\begin{aligned}
\mathcal{L}_T^{eff} &= -\frac{1}{2}(W\cdot W)+\frac{1}{4}m^2\,(R:R)+(J_1^{eff}\cdot W)+(J_2^{eff}:R)+(W\cdot J_3^{eff}:R)\\
&\quad+(R:J_4^{eff}:R)+(R:J_5^{eff}\cdot D:R)+\mathcal{L}_T^{eff,contact},
\end{aligned}$$

where the effective sources (using the sources J_1, J_2, J_3 from the vector case)

$$\begin{aligned}
J_1^{eff} &= -\frac{1}{m}J_1 - \frac{2}{m}J_2 : J_3, \\
J_2^{eff} &= mJ_2 - \frac{1}{m}J_2 : J_3 \cdot J_3 - \frac{1}{m}J_1 \cdot J_3 \sim mJ_2, \\
J_3^{eff} &= -J_3, \\
J_4^{eff} &= -\frac{1}{2}J_3 \cdot J_3 \sim 0.
\end{aligned}$$

Effective source J_5^{eff} vanish identically. The contact term is

$$\mathcal{L}_T^{eff,contact} = (J_2 : J_2) - \frac{1}{2m^2}(J_1 \cdot J_1) - \frac{1}{m^2}(J_2 : J_3 \cdot J_1) - \frac{1}{2m^2}(J_2 : J_3 \cdot J_3 : J_2).$$

Substituing for the effective sources into Lagrangian and taking into account only relevant terms we get (no contact term contributes to VVP correlator)

$$\mathcal{L}_T^{eff} \sim -\frac{1}{2}(W \cdot W) + \frac{1}{4}m^2(V : V) - \frac{1}{m}(J_1 \cdot V) - \frac{1}{m}(J_2 : J_3 \cdot W) + m(J_2 : V) - (W \cdot J_3 : V).$$

We are able to rewrite Lagrangian into particular terms as in previous cases.

$J_1 \cdot W$ - There is a contribution of the form $Rv\phi$.

$$\mathcal{L}_1 = -\frac{1}{m}h_V \epsilon_{\mu\nu\alpha\beta} \left\langle W^\mu \{u^\nu, f_+^{\alpha\beta}\} \right\rangle \sim \frac{4\sqrt{2}h_V}{mF} d^{abc} \epsilon_{\mu\nu\alpha\beta} \partial_\sigma V^{\sigma\mu,a} \partial^\nu \phi^b \partial^\alpha v^{\beta,c}.$$

$J_2 : J_3 \cdot W$ - There is a contribution of the form $Rv\phi$.

$$\mathcal{L}_2 = -\frac{2}{m}\left(-\frac{f_V}{2\sqrt{2}}\right)\frac{1}{2}\sigma_V \epsilon^{\alpha\beta\mu\nu} \left\langle \{f_{\mu\nu}^+, W_\alpha\} u_\beta \right\rangle \sim -\frac{2 f_V \sigma_V}{mF} \epsilon_{\alpha\beta\mu\nu} d^{abc} \partial^\mu v^{\nu,a} \partial_\sigma R^{\sigma\alpha,b} \partial^\beta \phi^c.$$

$J_2 : R$ - There is a contribution of the form Rv.

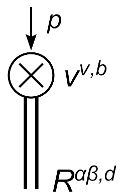
$$\mathcal{L}_3 = m\left(-\frac{1}{2\sqrt{2}}\right) f_V \left\langle f_{\mu\nu}^+ R^{\mu\nu} \right\rangle \sim -m f_V \delta^{ab} R^{\mu\nu,a} \partial_\mu v_\nu^b.$$

$W \cdot J_3 : R$ - There is a contribution of the form $RR\phi$.

$$\mathcal{L}_4 = -\frac{1}{2}\sigma_V \epsilon^{\alpha\beta\mu\nu} \langle \{W_\alpha, R_{\mu\nu}\} u_\beta \rangle \sim \frac{\sigma_V}{F} \epsilon_{\alpha\beta\mu\nu} d^{abc} R^{\mu\nu} \partial_\sigma R^{\sigma\alpha,b} \partial^\beta \phi^c.$$

Feynman rules for vertices

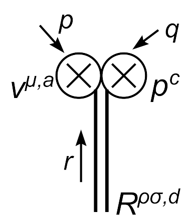
Vertex 1 : vector source - resonance



There contributes only $\mathcal{L}_3$ to this vertex. The Feynman rule is

$$\begin{aligned}(V_1)^{db}_{\alpha\beta\nu} &= -if_V m \frac{1}{2}(g^\sigma_\alpha g^\lambda_\beta - g^\sigma_\beta g^\lambda_\alpha)(-ip_\sigma)g_{\lambda\nu}\delta^{db} \\ &= -\frac{f_V m}{2}\delta^{db}(p_\alpha g_{\beta\nu} - p_\beta g_{\alpha\nu}).\end{aligned}$$

Vertex 2 : vector source - resonance - pseudoscalar source



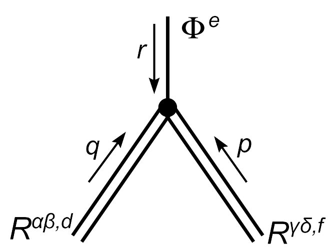
There contribute $\mathcal{L}_1$ and $\mathcal{L}_2$ to this vertex. Particular terms give

$$\begin{aligned}\left(V_2^1\right)^{ade}_{\mu\alpha\beta} &= -\frac{2\sqrt{2}h_V}{mF}d^{ade}(\epsilon_{\beta\nu\lambda\mu}q_\alpha r^\nu p^\lambda - \epsilon_{\alpha\nu\lambda\mu}q_\beta r^\nu p^\lambda), \\ \left(V_2^2\right)^{ade}_{\mu\alpha\beta} &= \frac{f_V\sigma_V}{mF}d^{ade}(\epsilon_{\beta\lambda\rho\mu}q_\alpha r^\lambda p^\rho - \epsilon_{\alpha\lambda\rho\mu}q_\beta r^\lambda p^\rho)\end{aligned}$$

The Feynman rule for this vertex is

$$(V_2)^{ade}_{\mu\alpha\beta} = -\frac{(4\sqrt{2}h_V - 2f_V\sigma_V)}{2mF}p^\lambda r^\nu(\epsilon_{\mu\nu\beta\lambda}q_\alpha - \epsilon_{\mu\nu\alpha\lambda}q_\beta).$$

Vertex 3 : resonance - resonance - pseudoscalar



There contributes $\mathcal{L}_4$ to this vertex. The Feynman rule is

$$\begin{aligned}\left(V_3^{'}\right)^{def}_{\alpha\beta\gamma\delta} &= \frac{i\sigma_V}{F}d^{def}\epsilon_{\kappa\lambda\mu\nu}\frac{1}{4}(-iq_\epsilon)(g^\epsilon_\alpha g^\kappa_\beta - g^\epsilon_\beta g^\kappa_\alpha) \times \\ &\qquad (g^\mu_\gamma g^\nu_\delta - g^\mu_\delta g^\nu_\gamma)(-ir^\lambda) \\ &= \frac{i\sigma_V}{2F}r^\lambda(\epsilon_{\alpha\lambda\gamma\delta}q_\beta - \epsilon_{\beta\lambda\gamma\delta}q_\alpha).\end{aligned}$$

Due to Bose statistics we have

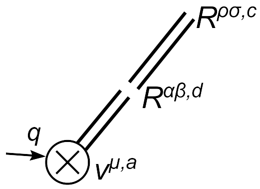
$$(V_3)^{def}_{\alpha\beta\gamma\delta} = \frac{i\sigma_V}{2F}r^\lambda(\epsilon_{\alpha\lambda\gamma\delta}q_\beta - \epsilon_{\beta\lambda\gamma\delta}q_\alpha + \epsilon_{\gamma\lambda\alpha\beta}p_\delta - \epsilon_{\delta\lambda\alpha\beta}p_\lambda).$$

Furthermore we have the chiral vertex $(V_\chi)^{ab}$.

Diagrams

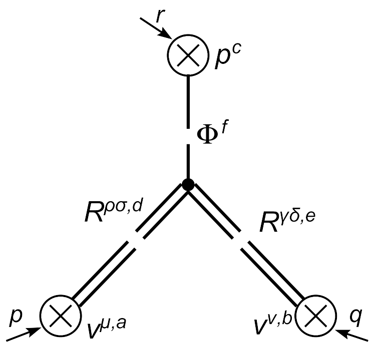
First we determine useful subdiagram as in previous cases.

Subdiagram 1: vertex 1 - tensor



$$
\begin{aligned}
(S_1)^{ac}_{\mu\rho\sigma} &= (V_1)^{ad}_{\mu\alpha\beta} i\,(\Delta_T(q))^{\alpha\beta\rho\sigma,cd} \\
&= \frac{i f_V m}{q^2-m^2}\delta^{ac}(q_\rho g_{\sigma\mu} - q_\sigma g_{\rho\mu}).
\end{aligned}
$$

Diagram 1

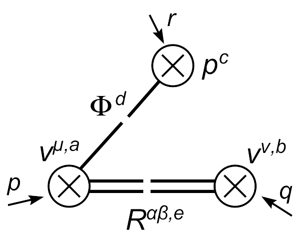


The diagram consists of vertex 3 and three subdiagrams. Taking all together we find

$$
(\Pi_1)^{abc}_{\mu\nu} = (V_2)^{\rho\sigma\gamma\delta,def}(S_1)^{ad}_{\mu\rho\sigma}(S_1)^{be}_{\gamma\delta\nu}(S_\chi)^{cf}
$$

$$
= \frac{2iB_0\sigma_V f_V^2 m^2(p^2+q^2)}{r^2(p^2-m^2)(q^2-m^2)} d^{abc}\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta.
$$

Diagram 2



The diagram consists of vertex 2 and two subdiagrams. Taking all together we obtain

$$
(\Pi_2)^{abc}_{\mu\nu} = (V_2)^{ade}_{\mu\alpha\beta}(S_1)^{\alpha\beta,be}_{\nu}(S_\chi)^{cd}
$$

$$
(\Pi_2)^{abc}_{\mu\nu} = -d^{abc}\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta \left(\frac{iB_0 f_V(4\sqrt{2}h_V - 2f_V\sigma_V)p^2}{r^2(p^2-m^2)}\right).
$$

We get similar contribution changing $v^a_\mu \leftrightarrow v^b_\nu$ and $p \leftrightarrow q$

$$
(\Pi_3)^{abc}_{\mu\nu} = -d^{abc}\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta \left(\frac{iB_0 f_V(4\sqrt{2}h_V - 2f_V\sigma_V)q^2}{r^2(q^2-m^2)}\right).
$$

Results

Complete VVP correlator in effective tensor formalism is

$$\begin{aligned}(\Pi_{VVP})^{abc}_{\mu\nu}(p,q) &= \int d^4x \int d^4y e^{i(p.x+q.y)} \left\langle 0|T\left[V^a_\mu(x)V^b_\nu(y)P^c(0)\right]|0\right\rangle \\ &= \epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta d^{abc}B_0\Big\{-\frac{4\sigma_V f_V^2 p^2 q^2}{r^2(p^2-m^2)(q^2-m^2)} \\ &\quad +\frac{4\sqrt{2}p^2 h_V f_V}{r^2(p^2-m^2)}+\frac{4\sqrt{2}q^2 h_V f_V}{r^2(q^2-m^2)}-\frac{N_C}{8\pi^2 r^2}\Big\}.\end{aligned}$$

The final result is identical with the one in the vector case. It means that the structure of $(\Pi_{VVP})^{abc}_{\mu\nu}$ is fully reconstructed in the effective tensor formalism up to $\mathcal{O}(p^6)$.

3.4 Effective vector formalism

Lagrangians

In vector-tensor correspondence we start with the tensor Lagrangian and derive the effective vector Lagrangian up to $\mathcal{O}(p^6)$

$$\begin{aligned}\mathcal{L}^{eff}_V = \quad & -\frac{1}{4}(\widehat{V}:\widehat{V})+\frac{1}{2}m^2(V\cdot V)+(J_1^{eff}\cdot V)+(J_2^{eff}:\widehat{V}) \\ & +\frac{1}{2}(V\cdot K^{eff}\cdot V)+(V\cdot J_3^{eff}:V)+\mathcal{L}_V^{eff,contact},\end{aligned}$$

where the effective sources are (using the sources J_i from tensor case)

$$\begin{aligned}J_1^{eff} &= mJ_1, \\ J_2^{eff} &= -\frac{1}{m}J_2^{(2)}, \\ K^{eff} &= J_3^{eff}=0.\end{aligned}$$

Only J_2^{eff} contributes to VVP correlator. The contact term is

$$\begin{aligned}\mathcal{L}_V^{eff,contact} &= \frac{1}{2}(J_1\cdot J_1)-\frac{1}{m^2}\left(J_2^{(2)}:J_2^{(2)}\right)-\frac{2}{m^2}\left(J_2^{(2)}:J_2^{(4)}\right) \\ &\quad +\frac{4}{m^4}(J_2^{(2)}:J_4:J_2^{(2)})+\frac{4}{m^4}(J_2^{(4)}:J_5\cdot D:J_2^{(2)})\end{aligned}$$

The complete effective Lagrangian up to $\mathcal{O}(p^6)$ that is relevant for VVP correlator is then

$$\begin{aligned}\mathcal{L}^{eff}_V \sim \quad & -\frac{1}{4}(\widehat{V}:\widehat{V})+\frac{1}{2}m^2(V\cdot V)-\frac{1}{m}(J_2^{(2)}:\widehat{V})-\frac{2}{m^2}(J_2^{(2)}:J_2^{(4)}) \\ & +\frac{4}{m^4}(J_2^{(2)}:J_4:J_2^{(2)})+\frac{4}{m^4}(J_2^{(2)}:J_5\cdot D:J_2^{(2)})\end{aligned}$$

The kinetic term is of order $\mathcal{O}(p^8)$ but it doesn't matter. We organize the Lagrangian in particular terms as usual.

$\underline{J_2^{(2)} : \hat{V}}$ - That is the analogue of the term in tensor case substituing $R^{\mu\nu} \to \partial^\mu V^\nu - \partial^\nu V^\mu$. So the term is of the form Vv.

$$\mathcal{L}_1 = -\frac{F_V}{m}\partial_\mu v_\nu^a(\partial^\mu V^{\nu,b} - \partial^\nu V^{\mu,b})\delta^{ab}$$

$\underline{J_2^{(2)} : J_2^{(4)}}$ - The terms are of form $vv\phi$ and vvp.

$$\begin{aligned}
\mathcal{L}_2 &= \frac{4\sqrt{2}F_V c_1}{m^3 F} d^{abc}\epsilon_{\mu\nu\rho\sigma}\partial^\mu v^{\nu,a}(\partial^\rho v^{\alpha,b} - \partial^\alpha v^{\rho,b})\partial_\alpha\partial^\sigma\phi^c \\
\mathcal{L}_3 &= \frac{4\sqrt{2}F_V c_2}{m^3 F} d^{abc}\epsilon_{\mu\alpha\rho\sigma}\left(\partial^\mu v^{\nu,a} - \partial^\nu v^{\mu,a}\right)\partial^\rho v^{\sigma,b}\partial^\alpha\partial_\nu\phi^c \\
\mathcal{L}_4 &= \frac{16\sqrt{2}B_0 F_V c_3}{m^3} d^{abc}\epsilon_{\mu\nu\rho\sigma}\partial^\mu v^{\nu,a}\partial^\rho v^{\sigma,b}P^c \\
\mathcal{L}_5 &= \frac{4\sqrt{2}F_V c_5}{m^3 F} d^{abc}\epsilon_{\mu\nu\rho\sigma}\partial_\lambda\partial^\mu v^{\nu,a}(\partial^\rho v^{\lambda,b} - \partial^\lambda v^{\rho,b})\partial^\sigma\phi^c \\
\mathcal{L}_6 &= \frac{4\sqrt{2}F_V c_6}{m^3 F} d^{abc}\epsilon_{\mu\alpha\rho\sigma}\left(\partial_\nu\partial^\mu v^{\nu,a} - \partial^2 v^{\mu,a}\right)\partial^\rho v^{\sigma,b}\partial_\alpha\phi^c \\
\mathcal{L}_7 &= \frac{4\sqrt{2}F_V c_7}{m^3 F} d^{abc}\epsilon_{\mu\nu\rho\sigma}\partial^\sigma\partial^\mu v^{\nu,a}(\partial^\rho v^{\lambda,b} - \partial^\lambda v^{\rho,b})\partial_\lambda\phi^c
\end{aligned}$$

$\underline{J_2^{(2)} : J_4 : J_2^{(2)}}$ - The terms are of form $vv\phi$ and vvp.

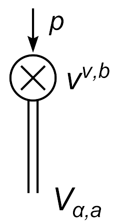
$$\begin{aligned}
\mathcal{L}_8 &= -\frac{4F_V^2 d_1}{m^4 F} d^{abc}\epsilon_{\mu\nu\rho\sigma}\partial^\mu v^{\nu,a}(\partial^\rho v^{\alpha,b} - \partial^\alpha v^{\rho,b})\partial_\alpha\partial^\sigma\phi^c \\
\mathcal{L}_9 &= -\frac{16F_V^2 B_0 d_2}{m^4} d^{abc}\epsilon_{\mu\nu\rho\sigma}\partial^\mu v^{\nu,a}\partial^\rho v^{\sigma,b}P^c
\end{aligned}$$

$\underline{J_2^{(2)} : J_5 \cdot D : J_2^{(2)}}$ - The terms are of form $vv\phi$.

$$\begin{aligned}
\mathcal{L}_{10} &= -\frac{4F_V^2 d_3}{m^4 F}\epsilon_{\mu\nu\rho\sigma}\partial_\alpha\partial^\mu v^{\nu,a}\left(\partial^\alpha v^{\rho,b} - \partial^\rho v^{\alpha,b}\right)\partial^\sigma\phi^c \\
\mathcal{L}_{11} &= -\frac{4F_V^2 d_4}{m^4 F}\epsilon_{\mu\nu\rho\sigma}\partial_\sigma\partial^\mu v^{\nu,a}\left(\partial^\alpha v^{\rho,b} - \partial^\rho v^{\alpha,b}\right)\partial^\alpha\phi^c
\end{aligned}$$

Feynman rules for vertices

Vertex 1 : vector source-resonance

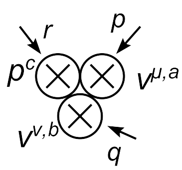


There contributes only one term to this vertex, $\mathcal{L}_1$. The Feynman rule is

$$(V_1)^{ab}_{\nu\alpha} = -i\frac{F_V}{m}(-ip_\kappa)g_{\lambda\nu}[(-ip^\kappa)g^\lambda_\alpha - (-ip^\lambda)g^\kappa_\alpha] = \frac{iF_V}{m}(p^2 g_{\alpha\nu} - p_\nu p_\alpha)$$

We will see that this vertex is not part of any diagram.

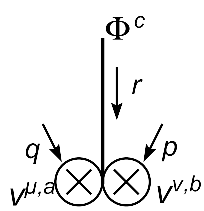
Vertex 2 : vector source-vector source-pseudoscalar source



There contribute $\mathcal{L}_4$ and $\mathcal{L}_9$ to this vertex. The Feynman rule is (respecting Bose statistics)

$$\begin{aligned}(V_3)^{abc}_{\mu\nu} &= \frac{32iF_V B_0}{m^4} d^{abc}\epsilon_{\kappa\lambda\rho\sigma}(-ip^\kappa)g^\lambda_\mu(-iq^\rho)g^\sigma_\nu(F_V d_2 - \sqrt{2}c_3 m) \\ &= -\frac{32iF_V B_0}{m^4} d^{abc}\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta \left(F_V d_2 - \sqrt{2}c_3 m\right)\end{aligned}$$

Vertex 3 : vector source-vector source-pseudoscalar



There contribute $\mathcal{L}_2$, $\mathcal{L}_3$, $\mathcal{L}_6$, $\mathcal{L}_7$, $\mathcal{L}_8$ and $\mathcal{L}_{10}$ to this vertex. It is originally the Wess-Zumino term, the new contributions come from contact terms. Terms coming from $\mathcal{L}_5$ and $\mathcal{L}_{11}$ vanish. The Feynman rule is then

$$\begin{aligned}
\left(V_5^{\prime 1}\right)^{abc}_{\mu\nu} &= \frac{4iF_V^2 d_1}{m^4 F} d^{abc}\epsilon_{\kappa\mu\nu\sigma}q^\kappa r^\sigma p^\alpha r_\alpha \\
\left(V_5^{\prime 2}\right)^{abc}_{\mu\nu} &= \frac{4F_V^2 d_3}{m^4 F} d^{abc}\epsilon_{\rho\nu\mu\lambda}p^\rho r^\lambda p_\beta q^\beta \\
\left(V_5^{\prime 3}\right)^{abc}_{\mu\nu} &= -\frac{4\sqrt{2}iF_V c_1}{m^3 F} d^{abc}\epsilon_{\kappa\mu\nu\sigma}r^\sigma q^\kappa p_\alpha r^\alpha \\
\left(V_5^{\prime 4}\right)^{abc}_{\mu\nu} &= -\frac{4\sqrt{2}iF_V c_2}{m^3 F} d^{abc}\epsilon_{\mu\kappa\rho\nu}r^\kappa p^\rho q_\alpha r^\alpha \\
\left(V_5^{\prime 5}\right)^{abc}_{\mu\nu} &= -\frac{4\sqrt{2}iF_V c_5}{m^3 F} d^{abc}\epsilon_{\alpha\mu\nu\sigma}q^\alpha r^\sigma p_\alpha q^\alpha \\
\left(V_5^{\prime 6}\right)^{abc}_{\mu\nu} &= -\frac{4\sqrt{2}iF_V c_6 q^2}{m^3 F} d^{abc}\epsilon_{\mu\kappa\rho\nu}p^\rho r^\kappa
\end{aligned}$$

Due to Bose statistics and after simplification we get

$$(V_5)^{abc}_{\mu\nu} = \frac{4iF_V^2}{m^4 F} d^{abc}\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta[d_1(p.r+q.r)+2d_3(p.q)]$$
$$- \frac{4\sqrt{2}iF_V}{m^3 F} d^{abc}\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta[c_1(p.r+q.r)+c_2(p.r+q.r)+2c_5(p.q)+c_6(p^2+q^2)]$$

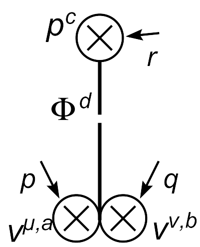
Diagrams

Diagram 1

It consists of vertex 2 only. So we have

$$(\Pi_1)_{\mu\nu} = (V_2)^{abc}_{\mu\nu} = -\frac{32iF_V B_0}{m^4} d^{abc}\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta \left(F_V d_2 - \sqrt{2}c_3 m\right)$$

Diagram 2



The diagram consists of vertex 3 and subdiagram χ.

$$(\Pi_2)^5_{\mu\nu} = (V_5)^{abd}_{\mu\nu}(S_\chi)^{cd}.$$

After substitution for vertex functions and propagator we find

$$\begin{aligned}(\Pi_2)^{abc}_{\mu\nu} &= \frac{4iB_0F_V^2}{m^4r^2} d^{abc}\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta[d_1(p.r)+d_3(p.q)] + (p\leftrightarrow q)\\ &- \frac{4\sqrt{2}iB_0F_V}{m^3r^2} d^{abc}\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta[c_1(p.r)+c_2(q.r)+c_5(p.q)+c_6q^2] + (p\leftrightarrow q)\end{aligned}$$

Expanding the scalar products we find

$$\begin{aligned}(\Pi_2)^{abc}_{\mu\nu} &= -i\frac{4F_V^2B_0}{m^4r^2} d^{abc}\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta[(d_1-d_3)\,r^2 + d_3(p^2+q^2)]\\ &\quad +i\frac{2\sqrt{2}F_VB_0}{m^3r^2} d^{abc}\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta[p^2(c_1-c_2+c_5)\\ &\quad +q^2(-c_1+c_2+c_5-2c_6)+r^2(c_1+c_2-c_5)] + (p\leftrightarrow q)\end{aligned}$$

Finally we have to add $\chi - WZ$ diagram.

Results

The VVP correlator in the effective vector formalism up to $\mathcal{O}(p^6)$ has the form

$$
\begin{aligned}
& (\Pi_{VVP})^{abc}_{\mu\nu}(p,q) = \int d^4x \int d^4y e^{i(p.x+q.y)} \left\langle 0 \right| T\left[V^a_\mu(x) V^b_\nu(y) P^c(0)\right] \left| 0 \right\rangle \\
= \quad & \epsilon_{\mu\nu\alpha\beta} p^\alpha q^\beta d^{abc} B_0 \Big\{ 4F_V^2 \frac{(d_1-d_3)r^2 + d_3(p^2+q^2)}{m^4 r^2} \\
& -2\sqrt{2}\frac{F_V}{m} \frac{r^2(c_1+c_2-c_5) + p^2(-c_1+c_2+c_5-2c_6) + q^2(c_1-c_2+c_5)}{m^2 r^2} \\
& -2\sqrt{2}\frac{F_V}{m} \frac{r^2(c_1+c_2-c_5) + q^2(-c_1+c_2+c_5-2c_6) + p^2(c_1-c_2+c_5)}{m^2 r^2} \\
& + \frac{32F_V^2 d_2}{m^4} - \frac{32\sqrt{2}F_V c_3}{m^3} - \frac{N_C}{8\pi^2 r^2} \Big\}.
\end{aligned}
$$

The result is similar as in tensor case. But the structure is not fully reconstructed. At order $\mathcal{O}(p^6)$ the antisymmetric tensor description is not equivalent with the effective vector case.

We can see that the result is the low energy limit of the one calculated in the antisymmetric tensor formalism.

$$
(m^2 - p^2) \approx m^2 \qquad (m^2 - q^2) \approx m^2.
$$

The last chance is to add the terms of higher orders. We will see that the result calculated in the tensor formalism is fully reconstructed if we consider the effective vector Lagrangian up to $\mathcal{O}(p^{10})$.

3.5 Effective vector formalism up to $\mathcal{O}(p^{10})$

Lagrangians

Up to $\mathcal{O}(p^{10})$ we have to add to the previous Lagrangian a lot of terms but only few of them contribute. The relevant terms are

$$
\begin{aligned}
\mathcal{L}_V^{eff(8)} &= -\frac{1}{m}\left(J_2^{(4)} : \widehat{V}\right) + \frac{4}{m^3}\left(J_2^{(2)} : J_4 : \widehat{V}\right) \\
& \quad + \frac{2}{m^3}\left(J_2^{(2)} : J_5 \cdot D : \widehat{V}\right) + \frac{2}{m^3}\left(\widehat{V} : J_5 \cdot D : J_2^{(2)}\right) \\
\mathcal{L}_V^{eff(10)} &= \frac{1}{m^3}\left(\widehat{V} : J_4 : \widehat{V}\right) + \frac{1}{m^2}\left(\widehat{V} : J_5 \cdot D : \widehat{V}\right)
\end{aligned}
$$

We organize the Lagrangian in the particular terms as usual.

$\underline{J_2^{(4)} : \widehat{V}}$ - The terms are of the form $v\phi V$ and vpV.

$$
\begin{aligned}
\mathcal{L}_1 &= \frac{4\sqrt{2}c_1}{m^2F} d^{abc}\epsilon_{\mu\nu\rho\sigma}\partial^\mu V^{\nu,a}(\partial^\rho v^{\alpha,b} - \partial^\alpha v^{\rho,b})\partial_\alpha\partial^\sigma\phi^c \\
\mathcal{L}_2 &= \frac{4\sqrt{2}c_2}{m^2F} d^{abc}\epsilon_{\mu\nu\rho\sigma}(\partial^\mu V^{\lambda,a} - \partial^\lambda V^{\mu,a})\partial^\rho v^{\sigma,b}\partial_\lambda\partial^\nu\phi^c \\
\mathcal{L}_3 &= \frac{16\sqrt{2}c_3B_0}{m^2} d^{abc}\epsilon_{\mu\nu\rho\sigma}\partial^\mu V^{\nu,a}\partial^\rho v^{\sigma,b}P^c \\
\mathcal{L}_4 &= \frac{4\sqrt{2}c_5}{m^2F} d^{abc}\epsilon_{\mu\nu\rho\sigma}\partial_\lambda\partial^\mu V^{\nu,a}(\partial^\rho v^{\lambda,b} - \partial^\lambda v^{\rho,b})\partial^\sigma\phi^c \\
\mathcal{L}_5 &= \frac{4\sqrt{2}c_6}{m^2F} d^{abc}\epsilon_{\mu\nu\rho\sigma}\partial_\lambda(\partial^\mu V^{\lambda,a} - \partial^\lambda V^{\mu,a})\partial^\rho v^{\sigma,b}\partial^\nu\phi^c \\
\mathcal{L}_6 &= \frac{4\sqrt{2}c_7}{m^2F} d^{abc}\epsilon_{\mu\nu\rho\sigma}\partial^\sigma\partial^\mu V^{\nu,a}(\partial^\rho v^{\lambda,b} - \partial^\lambda v^{\rho,b})\partial_\lambda\phi^c
\end{aligned}
$$

$\underline{J_2^{(2)} : J_4 : \widehat{V}}$ - The terms are of the form $v\phi V$ and vpV.

$$
\begin{aligned}
\mathcal{L}_7 &= -\frac{8F_Vd_1}{m^3F}\epsilon_{\mu\nu\alpha\sigma}d^{abc}\partial^\mu v^{\nu,a}\partial^\beta\partial^\sigma\phi^b(\partial_\alpha V^c_\beta - \partial_\beta V^c_\alpha) \\
\mathcal{L}_8 &= -\frac{32B_0F_Vd_2}{m^3}\epsilon_{\mu\nu\alpha\beta}d^{abc}\partial^\mu v^{\nu,a}p^b\partial_\alpha V^c_\beta
\end{aligned}
$$

$\underline{J_2^{(2)} : J_5 \cdot D : \widehat{V}}$ - The terms are of the form $v\phi V$.

$$
\begin{aligned}
\mathcal{L}_9 &= -\frac{4F_Vd_3}{m^3F}\epsilon_{\rho\sigma\mu\lambda}d^{abc}(\partial^\mu v^{\nu,a} - \partial^\nu v^{\mu,a})\partial^\lambda\phi^b\partial_\nu\partial^\rho V^{\sigma,c} \\
\mathcal{L}_{10} &= -\frac{4F_Vd_4}{m^3F}\epsilon_{\rho\sigma\mu\lambda}d^{abc}(\partial^\mu v^{\nu,a} - \partial^\nu v^{\mu,a})\partial^\nu\phi^b\partial_\lambda\partial^\rho V^{\sigma,c}
\end{aligned}
$$

$\underline{\widehat{V} : J_5 \cdot D : J_2^{(2)}}$ - The terms are of the form $v\phi V$.

$$
\begin{aligned}
\mathcal{L}_{11} &= -\frac{4F_Vd_3}{m^3F}\epsilon_{\rho\sigma\mu\lambda}d^{abc}(\partial^\mu V^{\nu,a} - \partial^\nu V^{\mu,a})\partial^\lambda\phi^b\partial_\nu\partial^\rho v^{\sigma,c} \\
\mathcal{L}_{12} &= -\frac{4F_Vd_4}{m^3F}\epsilon_{\rho\sigma\mu\lambda}d^{abc}(\partial^\mu V^{\nu,a} - \partial^\nu)V^{\mu,a}\partial^\nu\phi^b\partial_\lambda\partial^\rho v^{\sigma,c}
\end{aligned}
$$

$\underline{\widehat{V} : J_4 : \widehat{V}}$ - The terms are of the form $VV\phi$ and $Vp\phi$.

$$
\begin{aligned}
\mathcal{L}_{13} &= -\frac{4d_1}{m^2F}\epsilon_{\mu\nu\alpha\sigma}d^{abc}\partial^\mu V^{\nu,a}\partial_\beta\partial^\sigma\phi^b(\partial^\alpha V^{\beta,c} - \partial^\beta V^{\alpha,c}) \\
\mathcal{L}_{14} &= \frac{16iB_0d_2}{m^2}\epsilon_{\mu\nu\alpha\beta}d^{abc}\partial^\mu V^{\nu,a}p^b\partial^\alpha V^{\beta,c}
\end{aligned}
$$

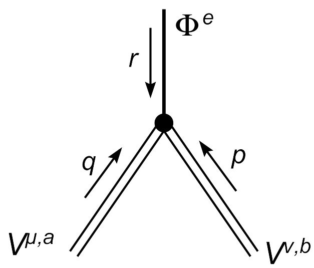
$\underline{\widehat{V} : J_5 \cdot D : \widehat{V}}$ - The terms are of the form $VV\phi$.

$$
\begin{aligned}
\mathcal{L}_{15} &= -\frac{4d_3}{m^2F}\epsilon_{\rho\sigma\mu\lambda}d^{abc}(\partial^\mu V^{\nu,a} - \partial^\nu V^{\mu,a})\partial_\lambda\phi^b\partial^\nu\partial^\rho V^{\sigma,c} \\
\mathcal{L}_{16} &= -\frac{4d_4}{m^2F}\epsilon_{\rho\sigma\mu\lambda}d^{abc}(\partial^\mu V^{\nu,a} - \partial^\nu V^{\mu,a})\partial_\nu\phi^b\partial^\lambda\partial^\rho V^{\sigma,c}
\end{aligned}
$$

Feynman rules for vertices

In addition to the previous case we have four more vertices coming from new terms in Lagrangian.

Vertex 4 : resonance-pseudoscalar-resonance



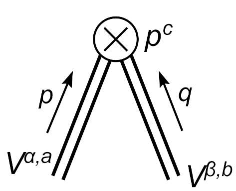
There contributes only one term to this vertex, $\mathcal{L}_{13}$, $\mathcal{L}_{15}$ and $\mathcal{L}_{16}$. Due to Bose statistics we obtain the Feynman rule

$$
\begin{aligned}
(V_4^1)^{abe}_{\mu\nu} &= -\frac{4id_1r^2}{m^2F}\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta \\
(V_4^2)^{abe}_{\mu\nu} &= \frac{8id_3}{m^2F}\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta(p.q)
\end{aligned}
$$

Last term vanishes due to antisymmetry of ϵ. Taking together we find

$$
(V_4)^{abe}_{\mu\nu} = -\frac{4i}{m^2F}\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta[d_1r^2 - 2d_3(p.q)]
$$

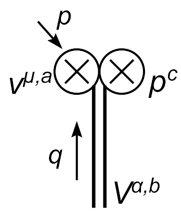
Vertex 5 : resonance-resonance-pseudoscalar source



There contributes $\mathcal{L}_{14}$ to this vertex. The Feynman rule is (respecting Bose statistics)

$$
\begin{aligned}
(V_5)^{abc}_{\mu\nu} &= -\frac{16B_0d_2}{m}d^{abc}\epsilon_{\mu\nu\rho\sigma}(-ip^\mu)g^\alpha_\nu(-iq^\rho)g^\sigma_\beta \\
&= \frac{16B_0d_2}{m^2}\epsilon_{\mu\alpha\rho\beta}p^\mu q^\rho
\end{aligned}
$$

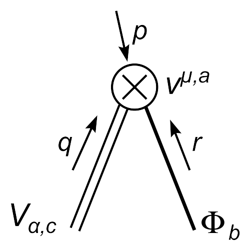
Vertex 6 : vector source-pseudoscalar source-resonance



There contribute $\mathcal{L}_3$ and $\mathcal{L}_8$ to this vertex. The Feynman rule is then

$$
\begin{aligned}
(V_6)^{abc}_{\mu\alpha} &= \frac{16\sqrt{2}iB_0}{m^3}\epsilon_{\beta\nu\rho\sigma}(-iq^\beta)g^\nu_\alpha(-ip^\rho)g^\sigma_\mu(mc_3-\sqrt{2}d_2) \\
&= -\frac{16\sqrt{2}iB_0}{m^3}\epsilon_{\beta\alpha\rho\mu}p^\rho q^\beta(mc_3-\sqrt{2}d_2)
\end{aligned}
$$

Vertex 7 : vector source-pseudoscalar source-resonance



There contribute $\mathcal{L}_1$, $\mathcal{L}_2$, $\mathcal{L}_4$, $\mathcal{L}_5$, $\mathcal{L}_6$, $\mathcal{L}_7$, $\mathcal{L}_9$, $\mathcal{L}_{10}$, $\mathcal{L}_{11}$ and $\mathcal{L}_{12}$ to this vertex. The Feynman rule is then

$$
\begin{aligned}
\left(V_7^1\right)^{abc}_{\mu\alpha} &= \frac{4\sqrt{2}ic_1}{m^2F}\epsilon_{\rho\alpha\mu\sigma}p^\sigma q^\rho(p.r) \\
\left(V_7^2\right)^{abc}_{\mu\alpha} &= \frac{4\sqrt{2}ic_2}{m^2F}\epsilon_{\alpha\nu\rho\mu}p^\rho q^\nu(q.r) \\
\left(V_7^3\right)^{abc}_{\mu\alpha} &= \frac{4\sqrt{2}ic_5}{m^2F}\epsilon_{\rho\alpha\mu\sigma}p^\sigma q^\rho(p.q) \\
\left(V_7^4\right)^{abc}_{\mu\alpha} &= \frac{4\sqrt{2}ic_6}{m^2F}\epsilon_{\alpha\nu\rho\mu}p^\rho q^\nu q^2 \\
\left(V_7^5\right)^{abc}_{\mu\alpha} &= \frac{4iF_Vd_1r^2}{m^3F}\epsilon_{\rho\alpha\mu\sigma}p^\rho q^\sigma \\
\left(V_7^6\right)^{abc}_{\mu\alpha} &= -\frac{4iF_Vd_3}{m^3F}\epsilon_{\rho\alpha\mu\sigma}p^\sigma q^\rho(p.q) \\
\left(V_7^7\right)^{abc}_{\mu\alpha} &= -\frac{4iF_Vd_3}{m^3F}\epsilon_{\rho\mu\alpha\sigma}p^\rho q^\sigma(p.q)
\end{aligned}
$$

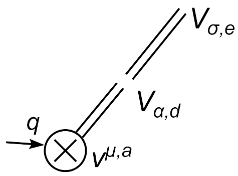
Other terms vanish. Taking together we get

$$
\begin{aligned}
(V_7)^{abc}_{\mu\alpha} &= \frac{4\sqrt{2}i}{m^2F}\epsilon_{\mu\alpha\rho\sigma}p^\rho q^\sigma[c_1(p.r)+c_2(q.r)+c_5(p.q)+c_6q^2] \\
&+\frac{4iF_V}{m^3F}\epsilon_{\mu\alpha\rho\sigma}p^\rho q^\sigma[d_1r^2-2d_3(p.q)]
\end{aligned}
$$

Diagrams

First we determine the subdiagram which will be useful in the future. It is very similar as in the vector case.

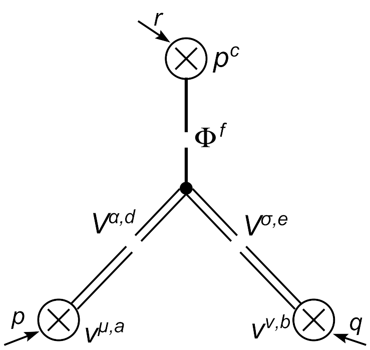
Subdiagram 1: vertex 1 - resonance



$$
\begin{aligned}
(S_1)^{ae}_{\mu\sigma} &= (V_1)^{ad}_{\mu\alpha} i\,(\Delta_V(q))^{\alpha\sigma,ed} \\
&= \frac{F_V \delta^{ae}}{m(p^2 - m^2)}(p^2 g_{\mu\sigma} - q_\mu q_\sigma).
\end{aligned}
$$

Next we draw Feynman diagrams constructed from all vertices coming from the effective vector Lagrangian up to $\mathcal{O}(p^{10})$.

Diagram 3



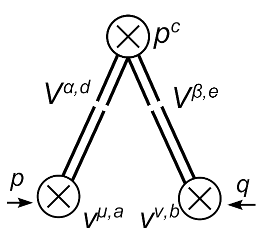
The diagram consists of vertex 4 and three subdiagrams.

$$(\Pi_3)^{abc}_{\mu\nu} = (V_4)^{def}_{\alpha\sigma}(S_1)^{\mu\alpha,ad}(S_1)^{\nu\sigma,be}(S_\chi)^{ce}.$$

After substitution for vertex functions and propagator we find

$$(\Pi_3)^{abc}_{\mu\nu} = -\frac{4iF_V^2 B_0 p^2 q^2}{m^4 r^2 (p^2 - m^2)(q^2 - m^2)}[d_1 r^2 - 2d_3(p.q)]d^{abc}\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta$$

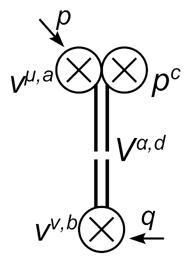
Diagram 4



The diagram consists of vertex 5 and three subdiagrams.

$$
\begin{aligned}
(\Pi_4)^{abc}_{\mu\nu} &= (V_5)^{de}_{\alpha\beta}(S_1)^{\mu\alpha,ad}(S_1)^{\nu\beta,be} \\
&= -\frac{32iB_0F_V^2 d_2 p^2 q^2}{m^4(p^2 - m^2)(q^2 - m^2)}d^{abc}\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta
\end{aligned}
$$

Diagram 5



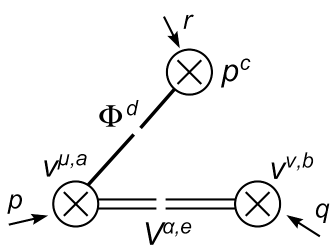
The diagram consists of vertex 6 and one subdiagram.

$$\begin{aligned}(\Pi_5)^{abc}_{\mu\nu} &= (V_6)^{acd}_{\alpha\mu}(S_1)^{\nu\alpha,bd} \\ &= -\frac{16\sqrt{2}iB_0F_Vp^2}{m^4(p^2-m^2)}d^{abc}\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta(mc_3-\sqrt{2}d_2)\end{aligned}$$

Changing the sources we get similar diagram.

$$(\Pi_6)^{abc}_{\mu\nu} = -\frac{16\sqrt{2}iB_0F_Vq^2}{m^4(q^2-m^2)}d^{abc}\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta(mc_3-\sqrt{2}d_2)$$

Diagram 6



The diagram consists of vertex 7 and two subdiagrams.

$$\begin{aligned}(\Pi_7)^{abc}_{\mu\nu} &= (V_7)^{ade}_{\alpha\mu}(S_1)^{\nu\alpha,be}(S_\chi)^{cd} \\ &= -\frac{4\sqrt{2}iF_VB_0q^2}{m^3r^2(q^2-m^2)}d^{abc}\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta[c_1(p.r)+c_2(q.r)+c_5(p.q)+c_6q^2] \\ &\quad -\frac{4iF_VB_0q^2}{m^4r^2(q^2-m^2)}d^{abc}\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta[d_1r^2-2d_3(p.q)]\end{aligned}$$

Changing the sources we get similar diagram.

$$\begin{aligned}(\Pi_8)^{abc}_{\mu\nu} &= -\frac{4\sqrt{2}iF_VB_0p^2}{m^3r^2(p^2-m^2)}d^{abc}\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta[c_1(q.r)+c_2(p.r)+c_5(p.q)+c_6p^2] \\ &\quad -\frac{4iF_VB_0p^2}{m^4r^2(p^2-m^2)}d^{abc}\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta[d_1r^2-2d_3(p.q)]\end{aligned}$$

Results

The VVP correlator in the effective vector formalism up to $\mathcal{O}(p^{10})$ is

$$(\Pi_{VVP})^{abc}_{\mu\nu}(p,q) = \int d^4x\int d^4y e^{i(p.x+q.y)}\left\langle 0|T\left[V^a_\mu(x)V^b_\nu(y)P^c(0)\right]|0\right\rangle$$

$$
\begin{aligned}
= & \ \epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta d^{abc}B_0\Big\{4F_V^2\frac{(d_1-d_3)r^2+d_3(p^2+q^2)}{(m^2-p^2)(m^2-q^2)r^2} \\
& -2\sqrt{2}\frac{F_V}{m}\frac{r^2(c_1+c_2-c_5)+p^2(-c_1+c_2+c_5-2c_6)+q^2(c_1-c_2+c_5)}{(m^2-p^2)r^2} \\
& -2\sqrt{2}\frac{F_V}{m}\frac{r^2(c_1+c_2-c_5)+q^2(-c_1+c_2+c_5-2c_6)+p^2(c_1-c_2+c_5)}{(m^2-q^2)r^2} \\
& +\frac{32F_V^2d_2}{(m^2-p^2)(m^2-q^2)}-\frac{16\sqrt{2}F_Vc_3}{m(m^2-p^2)}-\frac{16\sqrt{2}F_Vc_3}{m(m^2-q^2)}-\frac{N_C}{8\pi^2r^2}\Big\}.
\end{aligned}
$$

We find out that this result is identical with the one in the tensor case. So the structure is fully reconstructed if we consider all terms up to $\mathcal{O}(p^{10})$.

3.6 Mixed formalism

Lagrangians

The resonance Lagrangian in the mixed formalism has the form

$$
\begin{aligned}
\mathcal{L}_{TV} = & \ \frac{1}{4}m^2(R:R)+\frac{1}{2}m^2(V\cdot V)-\frac{1}{2}m(R:\widehat{V})+(J_1\cdot V)+\frac{1}{2}(V\cdot K\cdot V) \\
& +(J_2:R)+(V\cdot J_3:R)+(R:J_4:R)+(R:J_5\cdot D:R),
\end{aligned}
$$

where the sources are

$$
\begin{aligned}
(J_1)^a_\mu &= \langle T^aJ_{1\mu}\rangle, \\
(J_2)^a_{\mu\nu} &= \langle T^aJ_{2\mu\nu}\rangle, \\
(J_4)^{ab}_{\mu\nu\alpha\beta} &= \langle\{T^a,T^b\}J_{4\mu\nu\alpha\beta}\rangle, \\
(J_5)^{ab\alpha}_{\mu\nu\rho\sigma} &= \langle\{T^a,T^b\}J^\alpha_{5\mu\nu\rho\sigma}\rangle
\end{aligned}
$$

where

$$
\begin{aligned}
J_{1\mu} &= \frac{1}{\sqrt{2}}f_VD^\nu f_{+\mu\nu}+h_V\epsilon_{\mu\nu\alpha\beta}u^\nu f_+^{\alpha\beta} \\
J^{(2)}_{2\mu\nu} &= \frac{1}{2\sqrt{2}}F_Vf_{+\mu\nu} \\
J^{(4)}_{2\mu\nu} &= \epsilon_{\mu\kappa\rho\sigma}\frac{c_1}{m}\left\{f_+^{\rho\alpha},D_\alpha u^\sigma\right\}g^\kappa_\nu+\frac{c_2}{m}\left\{f_+^{\rho\sigma},D_\nu u^\kappa\right\}+i\frac{c_3}{m}\left\{f_+^{\rho\sigma},\chi_-\right\}g^\kappa_\nu+ \\
& \quad i\frac{c_4}{m}\left[f_-^{\rho\sigma},\chi_+\right]g^\kappa_\nu-\frac{c_5}{m}D_\lambda\left\{f_+^{\rho\lambda},u^\sigma\right\}g^\kappa_\nu-\frac{c_6}{m}D_\nu\left\{f_+^{\rho\sigma},u^\kappa\right\} \\
& \quad -\frac{c_7}{m}D^\sigma\left\{f_+^{\rho\lambda},u_\lambda\right\}g^\kappa_\nu
\end{aligned}
$$

$$
\begin{aligned}
J_{3\alpha\mu\nu} &= \frac{1}{2} m\sigma_V \epsilon_{\alpha\beta\mu\nu} u^\beta \\
J_{4\mu\nu\alpha\beta} &= \frac{1}{4} d_1 (\epsilon_{\mu\nu\alpha\sigma} D_\beta u^\sigma - \epsilon_{\mu\nu\beta\sigma} D_\alpha u^\sigma + \epsilon_{\alpha\beta\mu\sigma} D_\nu u^\sigma - \epsilon_{\alpha\beta\nu\sigma} D_\mu u^\sigma) + \epsilon_{\mu\nu\alpha\beta} d_2 \chi_- \\
J^\alpha_{5\mu\nu\rho\sigma} &= \epsilon_{\rho\sigma\mu\lambda} (d_3 u^\lambda g^\alpha_\nu + d_4 u_\nu g^{\alpha\lambda})
\end{aligned}
$$

Sources J_2, J_4 and J_5 come from the antisymmetric tensor formalism and couple only to antisymmetric tensor field. So we have all contributions as in the antisymmetric tensor case. Moreover we have some new terms from sources J_1 and J_3.

$\underline{J_1 \cdot V}$ - There are contributions of the form Vv, $Vv\phi$.

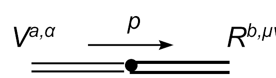
$$
\begin{aligned}
\mathcal{L}_1 &= \frac{f_V}{\sqrt{2}} \langle \partial_\nu f_+^{\mu\nu} V_\mu \rangle \sim f_V \delta^{ab} \partial_\nu (\partial^\mu v^{\nu,a} - \partial^\nu v^{\mu,a}) V^b_\mu \\
\mathcal{L}_2 &= h_V \epsilon_{\mu\nu\alpha\beta} \left\langle \{u^\nu, f_+^{\alpha\beta}\} V^\mu \right\rangle \sim -\frac{4\sqrt{2} h_V}{F} d^{abc} \epsilon_{\mu\nu\alpha\beta} \partial^\nu \phi^a \partial^\alpha v^{\beta,b} V^{\mu,c}
\end{aligned}
$$

$\underline{V \cdot J_3 : R}$ - There is a contribution of the form $RV\phi$.

$$
\mathcal{L}_3 = \frac{1}{2} m\sigma_V \epsilon_{\alpha\beta\mu\nu} \left\langle \{V^\alpha, u^\beta\} R^{\mu\nu} \right\rangle \sim -\frac{m\sigma_V}{F} \epsilon_{\kappa\beta\mu\nu} d^{abc} \partial^\beta \phi^a V^{\kappa,b} R^{\mu\nu,c}
$$

Mixed propagator

$$
(i\Delta_{RV}(p))^{ab}_{\alpha\mu\nu} = \frac{\delta^{ab}}{m(p^2 - m^2)} [g_{\alpha\mu} p_\nu - g_{\alpha\nu} p_\mu]
$$

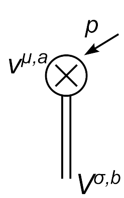


Moreover we have vector propagator and antisymmetric tensor propagator as in preceding sections.

Feynman rules for vertices

We have all vertices mentioned in tensor case. Moreover there are two vertices with vector and one with vector-tensor mixing.

Vertex 1 : vector source - vector

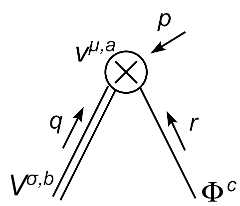


There contributes $\mathcal{L}_1$ to this vertex. The Feynman rule is

$$
\begin{aligned}
(V_1)^{ab}_{\sigma\mu} &= -if_V (ip_\rho) g_{\sigma\beta} \delta^{ab} [(-ip^\rho) g^\beta_\mu - (-ip^\beta) g^\rho_\mu] \\
&= -if_V \delta^{ab} (p^2 g_{\sigma\mu} - p_\sigma p_\mu).
\end{aligned}
$$

It is the analogue of vertex 1 in the vector formulation.

Vertex 2 : vector source - vector - pseudoscalar

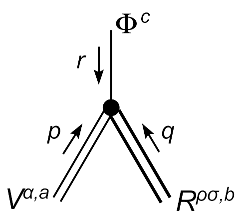


There contributes $\mathcal{L}_2$ to this vertex. The Feynman rule is then

$$
\begin{aligned}
(V_2)^{abc}_{\mu\sigma} &= -\frac{4\sqrt{2}ih_V}{F} d^{abc}\epsilon_{\rho\sigma\kappa\lambda} g^{\rho}_{\alpha}(-ir^{\alpha})(-ip^{\kappa})g^{\lambda}_{\mu} \\
&= \frac{4\sqrt{2}ih_V}{F} d^{abc}\epsilon_{\alpha\sigma\beta\mu} r^{\alpha}p^{\beta}.
\end{aligned}
$$

It is analogue of vertex 2 in the vector formulation.

Vertex 3 : tensor - vector - pseudoscalar



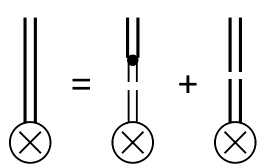
There contributes $\mathcal{L}_3$ to this vertex. The Feynman rule is

$$
\begin{aligned}
(V_3)^{abc}_{\alpha\rho\sigma} &= -i\frac{m\sigma_V}{F}\epsilon_{\kappa\beta\mu\nu}(-ir^{\beta})g^{\kappa\alpha}\frac{1}{2}(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}) \\
&= -\frac{m\sigma_V}{F}\epsilon_{\alpha\beta\rho\sigma}r^{\beta}.
\end{aligned}
$$

Diagrams

Our goal is not to compute the contributions of all possible diagrams. We use another way to obtain the final result. We know the structure of diagrams in the antisymmetric tensor case and that is our starting point. First we analyze the vector source - tensor propagator subdiagram in mixing formalism.

Subdiagram T: vector source - tensor propagator

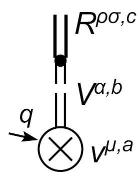


It is subdiagram 1 in the antisymmetric tensor formalism . In the mixed formalism we have an extra term due to vector-tensor mixing. So the total contribution to this subdiagram is

$$
(S_T)^{ab}_{\mu\rho\sigma} = (S^T_T)^{ab}_{\mu\rho\sigma} + (S^{TV}_T)^{ab}_{\mu\rho\sigma}
$$

where

$$
(S^T_T)_{\mu\rho\sigma} = \frac{iF_V\delta^{ad}}{m^2 - q^2}(g_{\mu\sigma}q_{\rho} - g_{\mu\rho}q_{\sigma})
$$



and second term

$$(S_T^{TV})^{ac}_{\mu\rho\sigma} = (V_1)^{ab}_{\mu\alpha} i \,(\Delta_{RV}(q))^{\alpha,bc}_{\rho\sigma} = \frac{i f_V q^2 \delta^{ac}}{m(q^2 - m^2)}(g_{\sigma\mu}p_\rho - g_{\rho\mu}p_\sigma)$$

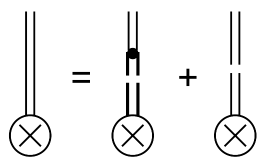
Taking these two terms together we find

$$(S_T)^{ac}_{\mu\rho\sigma} = \frac{i\delta^{ac}}{m^2 - q^2}(g_{\sigma\mu}q_\rho - g_{\rho\mu}q_\sigma)\left(F_V - \frac{f_V q^2}{m}\right)$$

It means that everywhere in the tensor result we have to substitute

$$F_V \to F_V - \frac{f_V q^2}{m}.$$

Subdiagram V: vector source - vector propagator



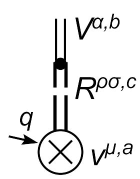
We have two possibilities how to construct this subdiagram. First is analogue of subdiagram 1 in the vector formulation whereas the second one comes from vector-tensor mixing.

$$(S_V)^{ab}_{\mu\alpha} = (S_V^{TV})^{ab}_{\mu\alpha} + (S_V^V)^{ab}_{\mu\alpha}$$

where

$$(S_V^V)^{ab}_{\mu\alpha} = \frac{f_V \delta^{ab}}{m^2 - q^2}(q^2 g_{\mu\alpha} - q_\mu q_\alpha)$$

and second term



$$(S_V^{TV})^{ab}_{\mu\alpha} = (V_1)^{ac}_{\mu\rho\sigma} i \,(\Delta_{RV}(q))^{\rho\sigma,bc}_{\alpha} = \frac{F_V \delta^{ab}}{m(q^2 - m^2)}(q^2 g_{\alpha\mu} - q_\alpha q_\mu)$$

where the sign in tensor-vector propagator is opposite (due to opposite direction of q).

Taking these two terms together we find

$$(S_V)^{ab}_{\mu\alpha} = \frac{\delta^{ab}}{m^2 - q^2}(q^2 g_{\mu\alpha} - q_\mu q_\alpha)(f_V - \frac{F_V}{m})$$

Diagram 1

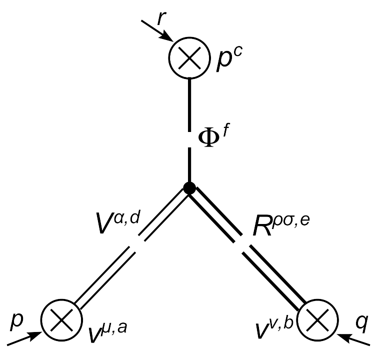
This is the analogue of diagram 2 with the substitution

$$f_V \to f_V - \frac{F_V}{m}.$$

So the result is

$$
\begin{aligned}
(\Pi_1)^{abc}_{\mu\nu} &= -\frac{4\sqrt{2}ip^2 B_0 h_V}{r^2(p^2-m^2)}\left(f_V - \frac{F_V}{m}\right) d^{abc}\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta \\
&\quad -\frac{4\sqrt{2}iq^2 B_0 h_V}{r^2(q^2-m^2)}\left(f_V - \frac{F_V}{m}\right) d^{abc}\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta
\end{aligned}
$$

Diagram 2



It consists of vertex 3, subdiagram χ, subdiagram V and subdiagram T. The Feynman rule is then

$$
\begin{aligned}
(\Pi_2)^{abc}_{\mu\nu} &= (V_3)^{\alpha\rho\sigma,def}(S_\chi)^{cf}(S_V)^{ad}_{\mu\alpha}(S_T)^{be}_{\rho\sigma\nu} \\
&= -\frac{2imB_0\sigma_V}{r^2(p^2-m^2)(q^2-m^2)}\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta \left(f_V - \frac{F_V}{m}\right)\left(F_V - f_V\frac{p^2}{m}\right) q^2
\end{aligned}
$$

Changing the sources we get

$$
(\Pi_3)^{abc}_{\mu\nu} = -\frac{2imB_0\sigma_V}{r^2(p^2-m^2)(q^2-m^2)}\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta \left(f_V - \frac{F_V}{m}\right)\left(F_V - f_V\frac{q^2}{m}\right) p^2
$$

Finally we add complete solution in tensor formulation with the substitution $F_V \to F_V - f_V q^2/m^2$.

Results

Summary

The complete VVP correlator in mixing formalism is

$$
\begin{aligned}
&(\Pi_{VVP})^{abc}_{\mu\nu}(p,q) = \int d^4x \int d^4y e^{i(p.x+q.y)} \left\langle 0 | T\left[V^a_\mu(x) V^b_\nu(y) P^c(0)\right] | 0 \right\rangle \\
= \ & \epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta d^{abc} B_0 \Big\{ 4\left(F_V - \frac{f_V q^2}{m}\right)\left(F_V - \frac{f_V p^2}{m}\right)\frac{(d_1-d_3)r^2 + d_3(p^2+q^2)}{(m^2-p^2)(m^2-q^2)r^2} \\
&-\frac{2\sqrt{2}}{m}\left(F_V - \frac{f_V p^2}{m}\right)\frac{r^2(c_1+c_2-c_5)+p^2(-c_1+c_2+c_5-2c_6)+q^2(c_1-c_2+c_5)}{(m^2-p^2)r^2}
\end{aligned}
$$

$$
-\frac{2\sqrt{2}}{m}\left(F_V-\frac{f_V q^2}{m}\right)\frac{r^2(c_1+c_2-c_5)+q^2(-c_1+c_2+c_5-2c_6)+p^2(c_1-c_2+c_5)}{(m^2-q^2)r^2}
$$
$$
+\frac{32d_2}{(m^2-p^2)(m^2-q^2)}\left(F_V-\frac{f_V q^2}{m}\right)\left(F_V-\frac{f_V p^2}{m}\right)-\frac{16\sqrt{2}c_3}{m(m^2-p^2)}\left(F_V-\frac{f_V p^2}{m}\right)
$$
$$
-\frac{16\sqrt{2}c_3}{m(m^2-q^2)}\left(F_V-\frac{f_V q^2}{m}\right)+\frac{2m\sigma_V}{r^2(p^2-m^2)(q^2-m^2)}\left(f_V-\frac{F_V}{m}\right)\times
$$
$$
\left[\left(F_V-f_V\frac{q^2}{m}\right)p^2+\left(F_V-f_V\frac{p^2}{m}\right)q^2\right]-\frac{4\sqrt{2}p^2h_V}{r^2(m^2-p^2)}\left(f_V-\frac{F_V}{m}\right)
$$
$$
-\frac{4\sqrt{2}q^2h_V}{r^2(q^2-m^2)}\left(f_V-\frac{F_V}{m}\right)-\frac{N_C}{8\pi^2r^2}\Big\}.
$$

Note that for $f_V = h_V = \sigma_V = 0$ we reproduce the tensor result, while $F_V = c_i = d_i = 0$ we recover the vector result.

High energy behaviour

We know from the antisymmetric tensor formalism that it is possible to fulfill the short distance constraints. And it is clear that the behaviour of the correlator in the mixed formalism couldn't be worse. Similar calculation leads to these relations for coupling constants

$$
\begin{aligned}
c_1 &= -4c_3\\
c_2 &= -4c_3+c_5\\
c_6 &= c_5-\frac{N_C m}{64\sqrt{2}\pi^2F_V}\\
d_1 &= -8d_2-\frac{N_C m^2}{64\pi^2F_V^2}+\frac{F^2}{4F_V^2}+\frac{\sigma_V}{2m}\\
d_3 &= -\frac{N_C m^2}{64\pi^2F_V^2}+\frac{F^2}{4F_V^2}+\frac{\sigma_V}{2m}.
\end{aligned}
$$

This is the analogue of constraints in tensor case. Moreover we obtain the condition $f_V = 0$.

It is possible to preserve the short distance constraints in the mixed formalism. Then the result simplifies to

$$
(\Pi_{VVP})^{abc}_{\mu\nu}(p,q)=\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta d^{abc}\frac{B_0^2F_0^2}{2}\frac{p^2+q^2+r^2-\frac{N_C m^4}{4\pi^2F_0^2}}{(p^2-m^2)(q^2-m^2)r^2}
$$

what is identical with the expression from antisymmetric tensor case.

Chapter 4

Conclusion

In this bachelor thesis we have studied the relationship of the vector field and the antisymmetric tensor field formalisms for the description of massive spin one particles in the context of chiral perturbation theory. We have concentrated on the calculation of VVP correlator using the $\mathcal{O}(p^2)$ chiral Lagrangian, WZW term and the resonance Lagrangian of various types. First we have used the vector field Lagrangian $\mathcal{L}_V$ obtaining the result which doesn't satisfy high energy conditions. Unlike it the antisymmetric tensor field formalism with the Lagrangian $\mathcal{L}_T$ has good high energy behaviour. Next starting with the vector Lagrangian we have derived the effective tensor Lagrangian which has only $\mathcal{L}_T^{eff(\leq 6)}$ contribution in our case (generally it is an infinite series). The expression for VVP correlator was identical with one calculated in the vector formalism. Then we have done the same starting with $\mathcal{L}_T$ for the effective vector Lagrangian $\mathcal{L}_V^{eff(\leq 6)}$ but the equivalence with the antisymmetric tensor formalism up to $\mathcal{O}(p^6)$ hasn't been established. To restore the complete result we had to add terms up to $\mathcal{O}(p^{10})$. Finally we have used mixed formulation based on Lagrangian $\mathcal{L}_{VT}$ which connects both descriptions. Then we have obtained more general formula satisfying high energy conditions. Moreover we can simply get the vector or the antisymmetric tensor result just switching off some coefficients.

Chapter 5

Appendix

Chiral building blocks

In the concrete calculation we have to expand the general chiral building blocks in terms of vector and pseudoscalar sources v^μ and p, pseudoscalar field ϕ and resonance V^μ ($R^{\mu\nu}$). Each building block has generally infinite expansion but for VVP correlator we can restrict ourselves to few terms.

$$
\begin{aligned}
u_\mu &\sim -\frac{\sqrt{2}\partial_\mu\phi}{F}, \\
\chi_+ &\sim \frac{2\sqrt{2}B_0}{F}\left\{p,\phi\right\}, \\
\chi_- &\sim 4iB_0 p, \\
f_+^{\mu\nu} &\sim 2\left(\partial_\mu v_\nu - \partial_\nu v_\mu\right), \\
f_-^{\mu\nu} &\sim \frac{\sqrt{2}i}{F}\left[\phi, \partial_\mu v_\nu - \partial_\nu v_\mu\right], \\
D_\alpha A^{\cdots} &\sim \partial_\alpha A^{\cdots}.
\end{aligned}
$$

SU(3) Traces

Useful traces of T^a matrices are

$$
\begin{aligned}
\mathrm{Tr}[T^a T^b] &= \delta^{ab}, \\
\mathrm{Tr}[\left\{T^a, T^b\right\} T^c] &= \sqrt{2}d^{abc}.
\end{aligned}
$$

Factor in Feynman rules

The generating functional can be written in the form

$$Z[v,a,p,s] = e^{iW[v,a,p,s]} = \left\langle 0|T \exp\left\{ i\int j_V + j_A + j_P - j_S \right\} |0\right\rangle$$

where $W[v,a,s,p]$ is the generating functional of connected Green functions. By definition we give for Green function

$$\begin{aligned}&\langle 0|T(j_V(x_{V_1})...j_V(x_{A_1})...j_S(x_{P_1})...j_P(x_{S_1})...)|0\rangle \\ &= (-i)^{\#v+\#p+\#a-\#s}\frac{\delta}{\delta v}...\frac{\delta}{\delta p}...\frac{\delta}{\delta a}...\frac{\delta}{\delta s}...\left(iW[v,a,s,p]\right)\end{aligned}$$

So for each vertex we have the sign rule

$$\text{sign} = \frac{i}{i^{\#v+\#p}}.$$

Factor i in numerator comes from iW. We can leave the factor in denominator just multiplying the expression for VVP correlator by

$$\text{sign of correlator} = \frac{1}{i^{\#v+\#p}} = i.$$

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