Charles University in Prague

Faculty of Social Sciences
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MASTER THESIS

Monetary policy regime through the lense of New Keynesian DSGE model: case of Mongolia

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Declaration of Authorship

The author hereby declares that he compiled this thesis independently, using only the listeds resources and literature, and the thesis has not been used to obtain a different or the same degree.

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Prague, May 15, 2014

Signature

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I am also very appreciative to my family for their support during my university studies.

Abstract

This paper identifies an optimal monetary policy rule using a calibrated small open economy DSGE model for Mongolian economy. The main result of this study is that domestic inflation-based Taylor rule is the best monetary policy regime for the Central bank of Mongolia (BoM) in terms of welfare loss. Therefore, the result of welfare analysis suggests that BoM should consider not only CPI inflation but also output gap in order to improve household welfare in economy. On the other hand currency board with fixed exchange rate regime could be very harmful to the Mongolian economy because it makes domestic economy more unstable in comparison to the other regimes.

JEL Classification	F41, E52, E58
Keywords	Open economy, Monetary policy, New Kyenesian model,

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Abstrakt

Tento dokument prezentuje optimální měnovou politiku pro mongolskou ekonomiku s využitím modelu DSGE kalibrovaného pro málo otevřené ekonomiky. Hlavním výsledkem studie je, že nejlepším režimem měnové politiky, ve vztahu k poklesu blahobytu, je pro centrální banku Mongolska (BoM) domácí inflace podle Taylorova pravidla. Výsledek analýzy blahobytu naznačuje, že by BoM měla uvážit nejen CPI inflaci, ale i mezera výstupu ve výstupech s cílem zlepšit životní podmínky domácností v ekonomice. Na druhé straně, pevný kurzový režim nastolený měnovou radou by mohl být pro mongolskou ekonomiku velmi škodlivý vzhledem k tomu, že se domácí ekonomika stává méně stabilní ve srovnání s ostatními režimy.

Klasifikace	F41, E52, E58
Klíčová slova	Otevřená ekonomika, Měnová politika, New Kyenesian modelu,

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Acronyms

BoM	The Bank of Mongolia
ROW	Rest of world
IT	Inflation Targeting
RBC	Real Business Cycle
DSGE	Dynamic Stochastic General Equilibrium
CITR	CPI inflation-based Taylor rule
DITR	Domestic inflation-based Taylor rule
PEG	Currency board with fixed exchange rate

Master Thesis Proposal

Author:	Bc. Bilguun Sukhbaatar
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Proposed Topic:

Monetary policy regime through the lense of New Keynesian DSGE model : case of Mongolia

Topic Characteristics:

In the recent years, Mongolia's abundant natural resources have been encouraging and attracting many foreign investors into Mongolia. Due to expansion and increasing foreign investment in mining sector, Mongolia will be forecasted to rapidly increase economy in forthcoming years and real GDP growth will be expected to be an average over 15% a year for the following years. Even though mining sector growth and foreign investment create opportunities of high economic growth rate, they would be raised the problems of economic overheating and sharp appreciation of domestic currency. Hence, the Bank of Mongolia (BoM) is going to face new challenges what kind of monetary policy framework is appropriate for this new situation.

According to the "Monetary policy guidelines for 2013", the Central bank intends to maintain inflation stable by implementing monetary policy instruments. Owing to this, BoM has to keep CPI-inflation below 8 percent at the end of 2013 and in the range of 5-7 percent during 2014-2015. Furthermore, Central bank shall continue the flexible exchange rate policy in line with macroeconomic fundamentals. Hence, BoM targets CPI inflation. However the alternative regimes should be considered.

The main purpose of the thesis is to compare CPI targeting monetary policy with other monetary policy regimes and identify the most suitable monetary policy regime for the BoM when economy is in the cyclical expansion. In order to compare monetary policy regimes, I will work with small scaled open economy New Keynesian Dynamic Stochastic General Equilibrium (DSGE) model based on the paper by Gali and Monacelli (2005). The focus of thesis will be to explain the dynamic properties of Mongolian business cycles.

For the case of Mongolia, there are two small-scale, calibrated small open economy DSGE models and one large-scale, small open economy Bayesian estimated DSGE model which have been developed in the past. The first small-scale model was developed by Altantsetseg and Bayarmaa (2011) based on paper by Soto (2008). It is small open economy inflation targeting DSGE model. The second small-scale model was developed by Batsukh and Avralt-Od based on papers of Berg et al. (2009, 2010, 2011). It is small open economy DSGE model with natural resource sector. This model focuses on the Dutch disease.

The large-scale Bayesian estimated DSGE model was developed by Richard Dutu (2012) from World Bank based on the papers by Christiano, Eichenbaum and Evans (2005), and Adolfson (2007) as choosing benchmark models. The author aimed to conduct risk analysis regarding the challenges and opportunities of several alternative growth paths and forecast public sector contingent liabilities associated with the

Mongolian macroeconomic and investment scenarios. Hence, to date there has been no study carried-out on the optimal monetary policy in case of Mongolia.
I use small-scaled New Keynesian small open economy DSGE model where I consider alternative monetary policy regimes and rules in order to identify the optimal Monetary policy for Mongolia. The results of this thesis might bring more light in to the debate, which is currently going on in Mongolia, what is the most efficient monetary policy regime. The results can be very helpful for policy makers and monetary authorities to make decision on monetary policy which would be appropriate and beneficial for Mongolian economy.

Hypotheses:

1. Hypothesis #1: CPI inflation targeting with floating exchange rate regime is not the optimal monetary policy regime for Mongolia.
2. Hypothesis #2: Taylor rule in which the monetary authority reacts to domestic inflation is higher welfare than other alternative rules in Mongolian.

Methodology:

In order to test my hypothesis and accomplish the aim of the thesis, I will study as follows:

1. I will review literature on theoretical and empirical studies related to the alternative monetary policy regimes.
2. I will derive optimal conditions of small open economy version of the model with Calvo-type staggered price-setting.
3. I will log-linearize nonlinear optimal conditions of the model around the steady state to obtain equilibrium dynamics under the alternative monetary policy rules.
4. I will define steady state and combine the historical data of Mongolian economy to calibrate DSGE model.
5. I will use Dynare package in Matlab software to solve and simulate dynamic model numerically.

Outline:

1) Introduction
2) Literature review
    i) Literature review on International
    ii) Literature review on Mongolia
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4) Empirical results
    i) Data
    ii) Parameter estimates
5) Optimal monetary policy
6) Simple monetary policy rules for the Mongolian economy
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    ii) Comparison analysis of alternative rules
7) Discussion

Conclusion

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Author **Supervisor**

1 Introduction

Since 2007, the Central Bank of Mongolia (BoM) has shifted from money aggregate rule to Consumer Price Index (CPI) inflation targeting rule. According to the “Monetary policy guidelines for 2013”, the Central bank intends to maintain the stable inflation by implementing monetary policy instruments. As a consequence of, BoM had to keep CPI inflation below 8 percent at the end of 2013. However, it appeared to be challenging task and average inflation rate was considerably higher than targeted level in the past years[1]. The main purpose of this thesis is to respond to the policy demand to designing optimal monetary policy suitable for Mongolian economy.

We base our analysis on comparism of CPI targeting monetary policy regime with other alternative monetary policy regimes and identifying of the most suitable monetary policy regime for the BoM. Also, the focus of thesis is to explain the impulse response of different shocks and then examine the welfare loss of alternative monetary rules. To do that, we lay out the small open economy New Keynesian Dynamic Stochastic General Equilibrium (DSGE) model introduced by Gali and Monacelli (2002, 2005). Mongolia is a relatively small open economy as its gross domestic product (GDP) share is 0.01 percent in the World economy in 2012[2]. The main trading partner of Mongolia is China. China consumes 85 percent of Mongolian total export and provides 43 percent of Mongolian total importing goods[3]. For that reason, it is reasonable to assume that firstly China can represent the rest of world for Mongolian economy. Secondly, Mongolian economy has no impact on the rest of world but shocks from the rest of world are considerable impacts to the small open economy.

In general, the results of previous theoretical and empirical papers suggested that strict domestic inflation targeting monetary policy creates the optimal policy which makes the flexible price allocation and maximizes the household welfare in

[1] Mongolian CPI inflation rate was 12.3 percent at the end of 2013.
[2] http://databank.worldbank.org/data/download/GDP.pdf
[3] http://atlas.media.mit.edu/profile/country/mng/

economy (Clarida, Galí and Gertler (2001); Gali and Monacelli (2002); Gali and Monacelli (2005)). Although, Svensson (2000) showed that the strict CPI-inflation targeting implies a strong use of direct exchange rate channel for stabilizing CPI inflation at short term. Moreover, the flexible CPI inflation targeting performs as an attractive alternative and it stabilizes CPI-inflation at long term perspective. However, to the best of my knowledge, there is just one study conducted on Mongolian case on this field. Batsukh, Avralt-Od and Tuvshinjargal (2014) introduced the small open economy model based on the paper Roger, Rest repo & Garcia (2009). The main purpose of the paper was to examine the performance of four different monetary policy rules under the situation of demand, supply and monetary policy shocks. The main result of the paper is that inflation targeting rule with exchange rate band was optimal policy rule for Mongolian economy in terms of welfare loss.

The present thesis contributes to the literature of monetary economic policy in Mongolia. Specifically, it introduces the usage of workable New Keynesian small open economy DSGE model that assesses alternative monetary policy regimes under welfare evaluation. To achieve the purpose, firstly we derive the solutions of the model and welfare loss function. Secondly, we calibrate the parameters based on literature and Mongolian data. Finally, we simulate the New Keynesian small open economy model using Dynare Toolbox, Matlab software.

The main result of this study is that DITR is the best monetary policy regime for BoM in terms of welfare loss. Therefore, the result of welfare analysis suggests that BoM should consider not only CPI inflation but also output gap in order to improve household welfare in economy. However, the hard PEG regime could be very harmful to the Mongolian economy because it makes domestic economy more unstable in comparison to the other regimes.

The thesis is structured as follows: Section 2 discusses literature review on New Keynesian DSGE model and monetary policy analysis in sticky price environment. Section 3 and 4 introduce New Keynesian small open economy model for Mongolian economy. Altenrative monetary policy regimes are introduced in section 5. Section 6 dicusses the calibration of the benchmark model. In section 7, we examine the analyses of impulse response, “cob web” graph, and welfare loss for benchmark models under alternative models. Section 8 concludes.

2 Literature review

2.1 Literature review on international evidence

“... [New Keynesian] models integrate Keynesian elements (imperfect competition, and nominal rigidities) into a dynamic general equilibrium framework that until recently was largely associated with the Real Business Cycle (RBC) paradigm. They can be used (and are being used) to analyze the connection between money, inflation, and the business cycle, and to assess the desirability of alternative monetary policies” [4]

Jorda Gali

In an economy with nominal rigidity, monetary policy can affect the real and nominal economic variables which results in influencing welfare of the economy. Thus, a large number of studies have examined characteristics of monetary policy in sticky-price environment. Lucas (1976) critique was an important basis for developing the dynamic stochastic general equilibrium (DSGE) model which summarized as: *"Given that the structure of an econometric model consists of optimal decision rules of economic agents and that optimal decision rules vary systematically with changes in the structure of series relevant to the decision maker, it follows that any change in policy will systematically alter the structure of econometric models"*[5]. The Lucas critique suggests that the macroeconomic model should be based on the microeconomic foundations. All decisions made by consumers and producers in the model must derive from a well-defined maximization program by which agents equalize the marginal return to the marginal cost of all alternatives. Several years later, Kydland and Prescott (1982) published the paper that reflects the Lucas ideas. A model in this paper was called Real Business Cycle (RBC) model and it became main reference framework for the analysis of the DSGE models.

The baseline New Keynesian (DSGE) model was technically based on RBC model by incorporating two assumptions into RBC model. Firstly, monopolistic competitive goods market assumption was introduced by Dixit and Stiglitz (1977).

4 Gali (2002), p.1
5 Lucas (1976), p.41.

They assumed that a large number of firms work in the market in which each firm produces a distinctive good and has enough pricing power in the market for its particular good. In addition, firms can charge markup over their marginal cost of production. Secondly, staggered price and wage assumption was introduced by Fischer (1977), Taylor (1980) and Calvo (1983) in which the assumption allows precious dynamic effects of monetary policy to the macroeconomic models.

Early examples of micro founded monetary models with monopolistic competition and sticky prices can be found in Akerlof and Yellen (1985), Mankiw (1985), and Blanchard and Kiyotaki (1987). Akerlof and Yellen (1985) studies in which microeconomic foundation for price stickiness developed based on the ideas respective near-rationality. They attempted to explain why changes are not neutral in the supply of nominal money in short run. The model revealed that aggregate demand shocks can cause or trigger significant changes in the real variables such as output and employment if agents adjust wages and prices. Whilst, Mankiw (1985) formed microeconomic foundation for price stickiness based on the idea of small menu cost. He noted that small menu costs can lead to large welfare losses in the economy. Then Blanchard et al. (1987) examined the combination of effect of menu costs and monopolistic competition based on previous two studies. His result also showed that small costs of changing prices may lead to substantial variations in output and welfare. Furthermore, Yun (1996) initially introduced Calvo price setting into stochastic and optimizing-agent model where he studied the basic New Keynesian (DSGE) model by introducing monopolistic competition and nominal price rigidity in RBC model. In this model money supply rule is allowed to be an endogenous. The main result of the paper was that extended models by nominal price rigidity can explain the observed relationship between inflation and output much better than flexible price models.

Early of 2000s, a large number of New Keynesian models developed the framework to analyze the properties of alternative monetary policy regimes in case of open economy. Many significant contributions to the literature on that field have been done by Svensson (2000), Clarida, Galí and Gertler (2001), McCallum and Nelson (2000), Corsetti and Pesenti (2001, 2005), Benigno and Benigno (2003), and Gali and Monacelli (2002, 2005) among others.

Svensson (2000) introduced the small open-economy model with forward-looking aggregate supply and demand. The main purpose of the paper was to extend the closed economy analysis of inflation targeting to small open economy and to examine the performance of inflation targeting monetary policy rules under the situation of exchange rate shock and the shocks influenced from the abroad. Author noticed that the exchange rate provides additional channels for the transmission of the monetary policy in open economy and it influences the cost of domestic goods and domestic inflation. Therefore, Svensson observed that shocks from foreign inflation, foreign interest rates and foreign investors are transmitted through the exchange rate. The result of the paper was that the strict CPI-inflation targeting implies a vigorous use of the direct exchange rate channel for stabilizing CPI inflation at short perspective. Furthermore, flexible CPI inflation targeting appears as an attractive alternative and it stabilizes CPI-inflation at longer perspective.

Clarida et al. (2001) extended their closed economy model to the open economy. Open economy model provides new insights on the usefulness of alternative monetary policy rules. Therefore, this model proposed a number of new issues related to the choice of exchange rate regime, the potential benefits from monetary policy coordination, the optimal response to foreign shocks and CPI inflation versus domestic inflation targeting. In their work, the optimal monetary policy can be obtained by Taylor rule in small open economy. Therefore, they concluded that the optimal policy problem for the small open economy is isomorphic in comparison to the closed economy.

Corsetti and Pesenti (2001) developed a baseline model to analyze the transmission of policy shocks in interdependent open economies. In order evaluate the international policy they formed two country, micro founded general equilibrium model with imperfect competition and nominal rigidities. In other words, they attempted to explore the interaction between internal and external sources of economic distortions in an open economy. As their result, the effect of expansionary policy on the welfare depends on the economic distortion that is related to openness. This result is different from previous results of the literature, for example, Obstfeld and Rogoff (1995). Obstfeld and Rogoff (1995) explained that unexpected small monetary expansion increases the consumption and welfare in the world economy. It does not depend on where the shock originates. However, as a result of Corsetti and Pesenti (2001), inflationary shock has more suffering effect on relatively smaller and more open economies. However, larger economies can have some benefit from demand led expansions and that depends on how much policymakers attempt to decrease the output gap.

Benigno and Benigno (2003) investigateded the conditions under which implementing flexible-price allocation would be monetary policy goal (optimal policy) in open economy model for two-countries. They developed the stochastic general equilibrium model with nominal rigidities and complete markets and they assumed that prices set in the producer's currency. Comparing to the previous open-economy literature focusing on analysis of optimal policy in the framework of dynamic general equilibrium, their key feature was that considering more general constant elasticity of substitution (CES) preference specification in both domestic and foreign consumption goods. Thus, in their model, the intratemporal elasticity of substitution between home and foreign produced goods may be different from the unitary value that has typically chosen in the literature. As a result, they found that the degrees of monopolistic distortion are equal across countries, the flexible-price allocation is an efficient. Otherwise, the intratemporal elasticity of substitution is required to take special values. In case of non-cooperative analysis, their results suggest that the price stability would not appear as an equilibrium. Hence, their conclusion was that international cooperation may be more beneficial.

Gali and Monacelli (2002) set up two-country small open economy version of the Calvo sticky-price. The authors derived a welfare based loss function in small open economy model in order to analyze the properties of alternative monetary policy rules such as domestic inflation targeting, CPI inflation targeting, and an exchange rate PEG. They showed that a key difference of alternative monetary regimes appears in the relative amount of exchange rate volatility which is caused by different regimes. Therefore, the authors found that strict domestic inflation targeting establishes the optimal policy that makes the flexible price allocation and maximizes the household's welfare.

In 2005, Gali and Monacelli extended their small open economy model to the continuum of economies making up the world economy in which the small open economy is just negligible part of the rest of the world (ROW). They analysed the performance of three alternative monetary policy rules based on the welfare analysis: a domestic inflation-based Taylor rule, a CPI-based Taylor rule, and an exchange rate peg. They found that the monetary policy of domestic inflation-based Taylor rule stabilizes both domestic prices and the output gap. However, that policy causes considerably larger volatility on nominal exchange rate and the terms of trade (TOT) relative to the alternative monetary policy rules. Furthermore, Gali and Monacelli showed that the monetary policy rule of exchange rate peg makes higher welfare losses than all other free floating regimes. Although, domestic inflation-based Taylor rule performs lower welfare losses than the CPI inflation-based Taylor rule.

2.2 The DSGE literature on Mongolia

For the case of Mongolia, there are, to my knowledge, only three small-scale, calibrated small open economy DSGE models and one large-scale, small open economy Bayesian estimated DSGE model which have been developed in the past. The first small-scale model was developed by Altantsetseg and Bayarmaa (2011) based on paper by Soto (2008). It is small open economy inflation targeting DSGE model. The second small-scale model was developed by Batsukh and Avralt-Od (2012) based on papers of Berg et al. (2009, 2010, 2011). It is small open economy DSGE model with natural resource sector. This model focused on the Dutch disease. The last model was introduced by Batsukh, Avralt-Od and Tuvshinjargal (2014) based on the paper Roger, Rest repo & Garcia (2009).

Batsukh and Avralt-Od (2011) examined medium term outlook of Mongolian economy using DSGE model. Authors intended to identify that whether it is optimal for the central bank to react to movements in the nominal exchange rate when macroeconomic performance is evaluated by means of inflation and output variability. For this reason, they analyzed different monetary policy rules using calibrated small open economy DSGE model for Mongolia. The authors concluded that less involvement of central bank in foreign exchange market and increasing income in mining sector would lead to higher demand and appreciation of real exchange rate in the economy. Even though real exchange rate appreciation might decrease the production of tradable sector, the government investment would accumulate both social and private capital resulting in stable economic growth. They warned that any attempt to decrease the real exchange rate appreciation may cause crowding out effect on private investment and slow down economic growth in the medium term.

The large-scale Bayesian estimated DSGE model was developed by Richard Dutu (2012) from World Bank based on the papers by Christiano, Eichenbaum and Evans (2005), and Adolfson (2007) as choosing benchmark models. Authors aimed to conduct risk analysis regarding the challenges and opportunities of several alternative growth paths and forecast public sector contingent liabilities associated with the Mongolian macroeconomic and investment scenarios. The study confirmed that rising commodity exports have strong and enduring effects on economic growth, mostly via a rise in private investment. However, the effect on employment is smaller and shorter than an increase in labour productivity. Considering monetary policy, they found that Central bank is quite independent in the sense that it responds quickly to CPI deviations from its target. Although, the authors recognized that it does not respond as strongly as levels of inflation require. Perhaps related to that point, they mentioned that an unexpected increase in the interest rate has long-lasting and quite damaging effects on the economy, though its effect would peak quicker than in more advanced economies.

Batsukh et al. (2014) considered the optimal monetary policy for Mongolian economy using a small open economy DSGE model which is based on the paper by Roger et al. (2009). The main purpose of the paper was to examine the performance of four different monetary policy rules under the situation of demand, supply and monetary policy shocks. Authors noticed that the demand and monetary policy shocks lead to higher volatility of foreign debt and lower volatility of inflation and output. Under the policy rules of inflation targeting (IT) with exchange rate band and exchange rate based IT, supply shock created highest volatility of inflation, interest rate and foreign debt. Therefore, main result of paper was that inflation targeting with exchange rate band was optimal policy rule for Mongolia economy in terms of welfare loss.

3 The Small Open Economy Model

This chapter following small open economy model developed by Gali and Monacelli (2005). Economy has three agents: utility maximizing households, profit maximizing firms and welfare maximizing monetary authority. Agents are modelled by explicit preferences with intertemporal constraints.

3.1 Households

The representative household seeks to maximize her expected discounted lifetime utility function of the form

$$E_0 \sum_{t=0}^{\infty} \beta^{\,t} \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right) \tag{3.1}$$

where E_t expresses the expectation conditional on the information set at period t and $\beta \in (0,1)$ is the subjective discount factor. $N_t = \int_0^1 N_t(j)dj$ is hours of work or labour supply to the domestic firms, and C_t is a composite consumption index; σ is the inverse elasticity of intertemporal substitution between consumption and φ is the inverse elasticity of labour supply to real wage. In the small open economy model, the composite consumption index C_t is determined by both domestic and foreign goods as a constant elasticity of substitution (CES) form (Dixit & Stiglitz, 1977) given by:

$$C_t \equiv \left[(1-\alpha)^{\frac{1}{\eta}} \left(C_{H,t} \right)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} \left(C_{F,t} \right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{3.2}$$

where parameter $\eta > 0$ is an elasticity of substitution between domestic and foriegn goods, parameter $\alpha \in [0,1]$ measures the degree of openness of the small open economy. If α is close to one which means economy is more open. While if it is close to zero, we will face the almost closed economy situation. The trade barriers imposed by governments, geographical resrtrications and country’s infrastructure level of transportations are assumed to be reflected in the degree of openess. The index $C_{H,t}$ is a consumption of domestic produced goods and $C_{F,t}$ is a domestic consumption of foreign produced goods. Both indices are defined by CES function:

$$C_{H,t} \equiv \left(\int_0^1 C_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}}, \qquad C_{F,t} \equiv \left(\int_0^1 \left(C_{i,t}\right)^{\frac{\gamma-1}{\gamma}} di\right)^{\frac{\gamma}{\gamma-1}}$$

Where $j \in [0,1]$ denotes the good variety. Parameter γ measures the substitutability between goods produced in different foreign countries. $C_{i,t}$ is an index of the quantity of goods imported from foreign country i and consumed by domestic households. The index $C_{i,t}$ is formulated by CES function:

$$C_{i,t} \equiv \left(\int_0^1 C_{i,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

Where parameter $\varepsilon > 1$ is the elasticity between varieties types of goods produced within any given country.

The utility maximization of Eq. (3.1) is subject to a intertemporal budget constraints of the following form:

$$\int_0^1 P_{H,t}(j) C_{H,t}(j) dj + \int_0^1 \int_0^1 P_{i,t}(j) C_{i,t}(j) dj di + E_t\{Q_{t,t+1} D_{t+1}\} \leq D_t + W_t N_t \quad (3.3)$$

where $P_{H,t}(j)$ is the domestic price of commoditiy j for home economy and $P_{i,t}(j)$ is the price of commodity j imported from country i. $C_{i,t}(j)$ is the consumption of commodity j imported from country i. D_{t+1} is the nominal payoff in period $t+1$ of portfolio held at the end of period t and $Q_{t,t+1}$ is the stochastic discount factor for one-period ahead nominal payoffs relevant to the domestic household. W_t is the nominal wage of labor. All variables are expressed in units of domestic currency.

As before utility maximization problem is solved, households have to solve the optimization problem that requires the optimal allocation of expenditures across all types of domestic and foreign countries goods. First, households need to choose how much they buy each of goods given level of consumption expenditures on domestic goods. The optimal allocation of any given expenditure within each class of domestic goods yields the following demand function[6]:

$$C_{H,t}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} \cdot C_{H,t} \quad (3.4)$$

[6] See derivation of equations from Appendix A, Domestic goods demand

Then households have to decide how much they to buy each of goods given level of consumption expenditures on imported goods. The demand for ith country's jth good is derived by following equation[7]:

$$C_{i,t}(j) = \left(\frac{P_{i,t}(j)}{P_{i,t}}\right)^{-\varepsilon} \cdot C_{i,t} \tag{3.5}$$

where (3.4) expresses optimal consumption of j-th domestic good and Eq. (3.5) depicts the optimal consumption of good j imported from i-th foreign country which expressed in domestic currency. Aggreagate domestic price index and a price index for goods imported from country i for all $i \in [0,1]$ are given by following formulation

$$P_{H,t} \equiv \left(\int_0^1 P_{H,t}(j)^{1-\varepsilon} dj\right)^{\frac{1}{1-\varepsilon}} \; ; \qquad P_{i,t} \equiv \left(\int_0^1 P_{i,t}(j)^{1-\varepsilon} dj\right)^{\frac{1}{1-\varepsilon}} \tag{3.6}$$

Hence, by combining the optimality conditions in (3.4) and (3.5) with aggeragate price indices in (3.6), the domestic and foreign consumption bundles can be expressed:

$$\int_0^1 P_{H,t}(j) C_{H,t}(j) dj = P_{H,t} C_{H,t}; \qquad \int_0^1 P_{i,t}(j) C_{i,t}(j) dj = P_{i,t} C_{i,t} \tag{3.7}$$

Furthermore, the optimal allocation of expenditures on imported goods by country of origin given by[8]:

$$C_{i,t} = \left(\frac{P_{i,t}}{P_{F,t}}\right)^{-\gamma} C_{F,t}; \qquad P_{F,t} = \left(\int_0^1 P_{i,t}^{1-\gamma} di\right)^{\frac{1}{1-\gamma}} \tag{3.8}$$

where $P_{F,t}$ is the price index for imported goods (expressed in domestic currency). Total expenditures on imported goods can be written as $\int_0^1 P_{i,t} C_{i,t} di = P_{F,t} C_{F,t}$ by using the (3.8) and definition of price index for imported goods.

Finally, the optimal allocation of expenditures between domestic and imported goods implies the demand functions[9]:

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[7] See derivation of Eq. from Appendix A, Section 8.1, Demand for i th country's jth good
[8] See derivation of Eq. from Appendix A, Section 8.1, Demand for imported goods by country origin
[9] See derivation of Eq. from Appendix A, Section 8.1, Eq. (A.15) and Eq. (A.16)

$$C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t\,; \qquad C_{F,t} = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t \tag{3.9}$$

and also domestic Consumer Price Index (CPI)

$$P_t \equiv \left[(1-\alpha)\left(P_{H,t}\right)^{1-\eta} + \alpha\left(P_{F,t}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{3.10}$$

where P_t is the aggregate Consumer Price Index (CPI) in the domestic country. Consequently, total consumption expenditures for the domestic households are given by $P_{H,t}C_{H,t} + P_{F,t}C_{F,t} = P_tC_t$[10]. Let's using those relationships

$$\int_0^1 P_{H,t}(j)C_{H,t}(j)dj + \int_0^1\int_0^1 P_{i,t}(j)C_{i,t}(j)djdi = P_tC_t$$

Hence, the period budget constraint can be rewritten as:

$$P_tC_t + E_t\left\{Q_{t,t+1}D_{t+1}\right\} \le D_t + W_tN_t \tag{3.11}$$

Finally, the household's optimization problem can be solved by above budget constraint with the utility function given in the (3.1):

$$max \rightarrow E_0\sum_{t=0}^{\infty}\beta^t\left\{\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}\right\}$$

$$\text{s.t.} \qquad P_tC_t + E_t\left\{Q_{t,t+1}D_{t+1}\right\} \le D_t + W_tN_t$$

The household has to solve the following optimization problem:

$$\underset{\{C_t,N_t,D_t\}}{\mathcal{L}} \equiv E_0\sum_{t=0}^{\infty}\beta^t\left\{\left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}\right] + \lambda_t\left[D_t + W_tN_t - P_tC_t - E_t\left\{Q_{t,t+1}D_{t+1}\right\}\right]\right\}$$

The household's utility maximisation problem yields following set of First Order Conditions (FOCs).

$$\frac{\partial\mathcal{L}}{\partial C_t}: \qquad \lambda_t = \frac{C_t^{-\sigma}}{P_t} \tag{3.12}$$

$$\frac{\partial\mathcal{L}}{\partial N_t}: \qquad \lambda_t = \frac{N_t^{\varphi}}{W_t} \tag{3.13}$$

$$\frac{\partial\mathcal{L}}{\partial D_{t+1}}: \qquad E_t\left\{Q_{t,t+1}\right\} = \beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\right] \tag{3.14}$$

[10] See derivation of equations from Appendix A, Section 8.1, Eq. (A.17)

The intratemporal optimality condition or labor supply of household can be derived by combining (3.12) with (3.13):

$$C_t^{\sigma} N_t^{\varphi} = {}^{W_t}/_{P_t} \tag{3.15}$$

The intertemporal optimality condition or Euler equation can be derived by inserting (3.12) into (3.14):

$$Q_t = \beta \cdot E_t \left\{ \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \left(\frac{P_{t+1}}{P_t}\right)^{-1} \right\} \tag{3.16}$$

where $Q_t = E_t\{Q_{t,t+1}\}$ is the gross return on a riskless one-period discount bond paying off one unit of domestic currency in $t+1$ and $\log(Q_t) = -\log(1+i_t)$. The equation (3.11) and (3.12) can be respectively written in log-linearized form as:

$$w_t - p_t = \sigma c_t + \varphi n_t \tag{3.17}$$

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho) \tag{3.18}$$

where lowercase letters represent log variables and it approximates the percentage change. $i_t \approx -\log(Q_t)$ is the short term nominal interest rate, $\rho = -\log(\beta)$ is the time discount rate, and $\pi_t = p_t - p_{t-1}$ is CPI inflation. The Euler equation (3.16) implies that higher expected consumption raises current consumption in order to smooth consumption level over period. Current consumption also depends on real interest rate $r_t = i_t - E_t\{\pi_{t+1}\}$ which is defined by the Fisher equation. If real interest rate is high, current consumption is low in consequence of consumers tend to earn high interest gains from tomorrow, which reacts the intertemporal substitution.

3.2 Firms

3.2.1 Technology

Let's assume that a typical firm in the home economy produces a differentiated good according to the following production function incorporating linear technology:

$$Y_t(j) = A_t \cdot N_t(j) \tag{3.19}$$

where $Y_t(j)$ denote the output of a general good in home country and $j \in [0,1]$ is a firm-specific index. A_t is the level of technology and assumed to be common to all domestic firms. $N_t(j)$ is the labor force used by the j-th domestic firm. The logarithm form of technology process is given by $a_t = log(A_t)$ which follows a first-order moving average process of the form

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a \tag{3.20}$$

Aggregate ouptup Y_t is defined by Dixit-Stiglitz CES aggregator:

$$Y_t \equiv \left(\int_0^1 Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{3.21}$$

the representation of index for aggregate domestic output is analogous to the consumption.

With firm's technology, nominal total and marginal cost can be defined as:

$$TC_t^n(j) = \frac{(1-\tau)W_t Y_t(j)}{A_t}; \qquad MC_t^n(j) = \frac{(1-\tau)W_t}{A_t} \tag{3.22}$$

where τ is a labor subsidy of government that is used to offset the distortion due to monopolistic competition. Therefore, all firms nominal marginal cost will be common across firms $MC_t^n(j) = MC_t^n$.

Hence, the real marginal cost will be common across domestic firms and given by

$$MC_t^r = \frac{(1-\tau)}{A_t} \cdot \frac{W_t}{P_{H,t}} \tag{3.23}$$

where $\frac{W_t}{P_{H,t}}$ is the real wage in domestic economy. The real marginal cost can be written in log-linearized form as:

$$mc_t^r = -\nu + w_t - p_{H,t} - a_t \tag{3.24}$$

The employment subsidy is captured in the term $\nu = -\log(1-\tau)$.

3.2.2 Price-setting

Let's assume that firms set prices in a staggered fashion, as in Calvo (1983). Calvo assumes that each domestic firm reset its price with given probability $(1 + \theta_H)$

in each period. So each price adjustment opportunity occurs randomly and independently of the time that has moved since its last price adjustment. Moreover, the number of domestic firms is assumed large and identical, and they can set apart from their differentiated products and the timing of their price adjustments. Result of that $(1+\theta_H)$ represents the fraction of firms adjusting their prices in each period and θ_H expresses the share of domestic firms holding their prices unchanged. The stick price of last period is expressed by $P_{H,t}(j) = P_{H,t-1}$.

Let $\bar{P}_{H,t}(j)$ denote the price set by a firm j adjusting its price in time t. Under the price setting structure $\bar{P}_{H,t+k}(j) = \bar{P}_{H,t}(j)$ with probability θ_H^k for $= 1,2, \ldots$. Since all domestic firms who reset prices in any given period will choose the same price, individual index j can be dropped. Now, aggreagate domestic price level can be defined as[11]:

$$P_{H,t} = \left[\theta_H \cdot P_{H,t-1}^{1-\varepsilon} + (1-\theta_H) \cdot \bar{P}_{H,t}^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{3.25}$$

When setting a new price in period t domestic firm j seeks to maximize the current value of its dividend streams, conditioned on that price being effective:

$$\underset{\bar{P}_{H,t}}{max} \rightarrow \sum_{k=0}^{\infty} \theta_H^k E_t\left\{Q_{t,t+k}\left[\bar{P}_{H,t} Y_{t+k|t}(j) - TC_{j,t+k|t}^n\left(Y_{j,t+k|t}(j)\right)\right]\right\}$$

where $TC_{j,t+k|t}^n$ which depends on $Y_{j,t+k|t}(j)$ is the nominal total cost in period $t+k$ for a domestic firm that last reset its price in period t.

Subject to the sequence of demand constraints

$$Y_{t+k}(j) = C_{H,t+k}(j) + \int_0^1 C_{H,t+k}^i(j)dj$$

where $C_{H,t+k}(j)$ is domestic demand for domestic j-th good and $\int_0^1 C_{H,t+k}^i(j)dj$ is the rest of world's demand for domestic j-th good. After solved[12] this problem the optimal decision rule for $\bar{P}_{H,t}$ is given as:

$$\bar{p}_{H,t} = \mu + (1-\theta_H\beta)\sum_{k=0}^{\infty}(\beta\theta_H)^k E_t\left\{p_{H,t+k} + mc_{t+k}^r\right\} \tag{3.26}$$

[11] See derivation from Appendix A, Section 8.2, The dynamic of the domestic price index
[12] See derivation from Appendix A, Section 8.3, Optimal price setting in the Calvo model

where $\bar{p}_{H,t}$ is the log of newly optimized domestic price, and $\mu = \log\frac{\varepsilon}{\varepsilon-1}$ is the log of the gross markup in the steady state. This implies that firms set their prices to be the sum of the discounted value of nominal marginal costs.

3.3 Terms of Trade, Inflation, and Exchange Rate

3.3.1 Terms of Trade and inflation

Terms of trade (TOT) are the price of the home country's export divided by the price of a foreign country's import. It represents the unit price of imported goods in terms of home good. Thus, the *bilateral terms of trade* between the domestic economy and country *i* are defined as $S_{i,t} = \frac{P_{i,t}}{P_{H,t}}$. So, we can define the *effective terms of trade* as

$$S_t \equiv \frac{P_{F,t}}{P_{H,t}} = \left(\int_0^1 S_{i,t}^{1-\gamma} di\right)^{\frac{1}{1-\gamma}} \tag{3.27}$$

The effective TOT indicates the competitiveness level for the domestic economy. Increase of S_t implies higher competitiveness for the domestic economy, which follows from either raise of imported goods prices $P_{F,t}$ or decline of domestic prices $P_{H,t}$. The equation (3.27) can be approximated (up to first order) around a symmetric steady state satisfying $S_{i,t} = 1$ for all $i \in [0,1]$ by

$$s_t = \int_0^1 s_{i,t} di \tag{3.28}$$

where $s_t = \log(S_t) = p_{F,t} - p_{H,t}$.

Likewise, log-linearized form of the CPI inflation (3.10) around a symmetric steady state yields

$$p_t \equiv (1-\alpha)p_{H,t} + \alpha p_{F,t} \tag{3.29}$$

Let's combine the log-linear domestic price index (3.29) with log-linear effective TOT (3.28) to see the relations between TOT and aggregate price level

$$p_t = p_{H,t} + \alpha s_t \tag{3.30}$$

The first order difference of (3.30) yields the relation of CPI inflation and TOT

$$\pi_t = \pi_{H,t} + \alpha \Delta s_t \tag{3.31}$$

The equation (3.31) implies that the change of TOT is proportional to the difference of overall inflation and domestic inflation. If openness parameter α is high, the change of TOT under shocks will be small.

3.3.2 Exchange rate

Define $\mathcal{E}_{i,t}$ as the *bilateral nominal exchange rate*, i.e. the price of country i's currency in terms of the domestic currency. As an example, the bilateral nominal exchange rate between Mongolia and China could be $\mathcal{E}_{CNY,t} = 250\frac{MNT}{CNY}$. Similarly, define $P_{i,t}^{i}(j)$ the price of country i's good j in terms of producer's currency. For instance, the price of a Lenovo laptop (i) in terms of CNY (i). Now, let's assume that the *law of one price* (LOP) holds for individual goods at all times for both for import and export prices. Thus, LOP is defined for all goods $j \in [0,1]$ in every country $i \in [0,1]$ as

$$P_{i,t}(j) = \mathcal{E}_{i,t} P_{i,t}^{i}(j) \tag{3.32}$$

Suppose that the price of the Lenovo laptop (j) in terms of China currency is $P_{i,t}^{i}(j) = 5{,}000\ CNY$. The LOP implies that the Mongolian price on Lenovo laptop in terms of Mongolian currency is $P_{i,t}(j) = \mathcal{E}_{i,t} P_{i,t}^{i}(j) = 250\frac{MNT}{CNY} \cdot 5{,}000\ CNY = 1.25\ mln\ MNT$. By inserting the LOP assumption in the definition of $P_{i,t}$ yields aggregation across all goods as

$$P_{i,t} = \mathcal{E}_{i,t} P_{i,t}^{i} \tag{3.33}$$

where $P_{i,t}^{i} \equiv \left(\int_0^1 P_{i,t}(j)^{1-\varepsilon} dj\right)^{\frac{1}{1-\varepsilon}}$ is country i's domestic price index.

In turn, by substituting into the definition of $P_{F,t}$ and log-linearizing around the symmetric steady state, we get

$$p_{F,t} = \int_0^1 \left(e_{i,t} + p_{i,t}^{i}\right) di = e_t + p_t^* \tag{3.34}$$

where the (log) effective nominal exchange rate is denoted as $e_t \equiv \int_0^1 e_{i,t} di$ is and the (log) world price index is denoted as $p_t^* \equiv \int_0^1 p_{i,t}^{i} di$.

Combining (3.34) with the definition of the TOT we obtain the following expression:

$$s_t = e_t + p_t^* - p_{H,t} \tag{3.35}$$

The equation (3.35) expresses the TOT as a linear combination of the effective nominal exchange rate, the world price index and the domestic price index.

Let's define the *bilaterial real exchange rate* as a ratio of the home and foreign county i's CPI, both expressed in domestic currency

$$RER_{i,t} = \frac{\mathcal{E}_{i,t} P_t^i}{P_t} \tag{3.36}$$

In log terms (3.36) yields

$$q_{i,t} = e_{i,t} + p_t^i - p_t \tag{3.37}$$

Let $q_t \equiv \int_0^1 q_{i,t}\, di$ be (log) *effective real exchange rate*. Then it follows that

$$q_t = \int_0^1 (e_{i,t} + p_t^i - p_t) di = (1-\alpha) s_t \tag{3.38}$$

Notice that the last equality only holds up to a first order approximation when $\eta \neq 1$.

3.4 International Risk Sharing

Under the assumption of complete international securities markets and perfect capital mobility, the The Euler equation (3.16) must also hold for the any representative household who live in the foreign country i for all $i \in [0,1]$:

$$Q_{t,t+1} = \beta \left\{ \left(\frac{C_{t+1}^i}{C_t^i} \right)^{-\sigma} \left(\frac{\mathcal{E}_{i,t+1}}{\mathcal{E}_{i,t}} \right)^{-1} \left(\frac{P_{i,t+1}}{P_{i,t}} \right)^{-1} \right\} \tag{3.39}$$

Devide domestic intertemporal optimality condition by the foreign country i's intertemporal optimality condition:

$$1 = \frac{E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{P_{t+1}}{P_t} \right)^{-1} \right\}}{E_t \left\{ \left(\frac{C_{t+1}^i}{C_t^i} \right)^{-\sigma} \left(\frac{\mathcal{E}_{i,t+1}^i P_{i,t+1}^i}{\mathcal{E}_{i,t}^i P_{i,t}^i} \right)^{-1} \right\}} \tag{3.40}$$

Combining (3.39) with the real exchange rate definition, it follows that

$$C_t = \vartheta_i C_t^i RER_{i,t}^{1/\sigma} \tag{3.41}$$

where $\vartheta_i \equiv E_t \frac{C_{t+1}}{C_{t+1}^i \cdot RER_{i,t+1}^{1/\sigma}}$ is a constant which will generally depend on initial conditions regarding relative net asset positions. Without loss of generality, assume symmetric initial conditions, i.e. zero net foreign asset holdings and ex-ante identical environment. This implies gives $\vartheta_i = \vartheta = 1$ for any $j \in [0,1]$. Taking logs on both sides of (3.41):

$$c_t = c_t^i + \frac{1}{\sigma} q_{i,t} \tag{3.42}$$

The equation (3.42) expresses consumption in household level. Thus, consumption of whole domestic economy is derived by integrating (3.42) over all i and using (3.39) yields

$$c_t = c_t^* + \frac{1}{\sigma} q_t = c_t^* + \frac{(1-\alpha)}{\sigma} s_t \tag{3.43}$$

where the (log) index of world consumption is denoted as $c_t^* \equiv \int_0^1 c_t^i di$ is.

3.5 Uncovered Interest Parity

Let's assume that households can invest in both domestic and foreign bonds D_t and D_t^i. Hence, the budget constraint (3.11) can be rewritten as:

$$P_t C_t + E_t\{Q_{t,t+1} D_{t+1} + Q_{t,t+1}^* \mathcal{E}_{i,t+1} D_{t+1}^i\} \leq D_t + \mathcal{E}_{i,t} D_t^i + W_t N_t \tag{3.44}$$

Therefore, the household's new utility maximisation problem yields following optimality condition with respect to foreign asset:

$$1 = \beta E_t \left\{ \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \left(\frac{P_{t+1}}{P_t}\right)^{-1} {Q_{t,t+1}^i}^{-1} \left(\frac{\mathcal{E}_{i,t+1}}{\mathcal{E}_{i,t}}\right)^{-1} \right\} \tag{3.45}$$

Divide the optimality condition (3.16) by (3.44), we get:

$$\frac{Q_t^i}{Q_t} = E_t \left\{ \frac{\mathcal{E}_{i,t+1}}{\mathcal{E}_{i,t}} \right\} \tag{3.46}$$

Log-linearizing (3.45) around a perfect foresight steady state, and aggregating over i, yields the following expression:

$$i_t = i_t^* + E_t\{\Delta e_{t+1}\} \tag{3.47}$$

The equation (3.47) states that uncovered interest parity (UIP) hold in the economy. The (3.47) implies nominal interest rate at home is equal to the world nominal interest plus expected rate of depreciation of the home currency.

Now, combining the definition of the (log) terms of trade with (3.47) yields the following stochastic difference equation:

$$s_t = (i_t^* - E_t\{\pi_{t+1}^*\}) - \left(i_t - E_t\{\pi_{H,t+1}\}\right) + E_t\{s_{t+1}\} \tag{3.48}$$

Given that the terms of trade are pinned down uniquely in the perfect foresight steady state, and the assumption of stationary in the model's driving forces and a convenient normalization implies that $\log_{T\to\infty} E_t(s_T) = 0$. Hence, (3.48) can be solved forward to obtain

$$s_t = E_t\left\{\sum_{k=0}^{\infty}\left[(i_t^* - \pi_{t+k+1}^*) - \left(i_t - \pi_{H,t+k+1}\right)\right]\right\} \tag{3.49}$$

Equation (3.49) expresses the terms of trade are a function of current and anticipate dreal interest rate differentials.

4 EQUILIBRIUM

4.1 The Demand Side

4.1.1 Consumption and output

Goods market clearing in the representative small open economy requires

$$Y_t(j) = C_{H,t}(j) + \int_0^1 C_{H,t}^i(j)di \tag{4.1}$$

where the supply of domestically produced good j is denoted $Y_t(j)$. The domestic demand is denoted $C_{H,t}(j)$ and country i's demand for good j produced in home economy is denoted $C_{H,t}^i(j)$. The assumption of symmetric preferences across countries yields the demand for domestically produced good j in country i as a function of total consumption

$$C_{H,t}^i(j) = \left[\frac{P_{H,t}(j)}{P_{H,t}}\right]^{-\varepsilon} \left[\frac{P_{H,t}}{\mathcal{E}_{i,t}P_{F,t}^i}\right]^{-\gamma} \left[\frac{P_{F,t}^i}{P_t^i}\right]^{-\eta} C_t^i \tag{4.2}$$

Inserting (3.51) into the goods market clearing (3.50) yields

$$Y_t(j) = \left[\frac{P_{H,t}(j)}{P_{H,t}}\right]^{-\varepsilon} \left\{(1-\alpha)\left[\frac{P_{H,t}}{P_t}\right]^{-\eta} C_t + \alpha \int_0^1 \left[\frac{P_{H,t}}{\mathcal{E}_{i,t}P_{F,t}^i}\right]^{-\gamma} \left[\frac{P_{F,t}^i}{P_t^i}\right]^{-\eta} C_t^i di\right\} \tag{4.3}$$

Then, plugging (4.2) into the definition of aggregate domestic output (3.21), we can write it as:

$$Y_t = \left[\frac{P_{H,t}}{P_t}\right]^{-\eta} \left[(1-\alpha)C_t + \alpha \int_0^1 \left(\frac{\mathcal{E}_{i,t}P_{F,t}^i}{P_{H,t}}\right)^{\gamma-\eta} RER_{i,t}^{\eta} C_t^i di\right] \tag{4.4}$$

Let's define effective TOT for country i as $S_t^i = \frac{\mathcal{E}_{i,t}P_{F,t}^i}{P_{i,t}}$. Then insert the previous definition with bilateral TOT between the domestic economy and country i, and for $C_t = C_t^i RER_{i,t}^{1/\sigma}$ from (3.41) into the (4.4) yields

$$Y_t = \left[\frac{P_{H,t}}{P_t}\right]^{-\eta} \left[(1-\alpha)C_t + \alpha \int_0^1 \left(S_t^i S_{i,t}\right)^{\gamma-\eta} RER_{i,t}^{\eta} C_t^i di\right] \tag{4.5}$$

We can derive the first order log-linear approximation to (4.5) around symmetric steady state as

$$y_t = c_t + \frac{\alpha w}{\sigma} s_t \tag{4.6}$$

where $w \equiv \sigma\gamma + (1-\alpha)(\sigma\eta - 1) > 0$. For all other countries, a condition analogous to (4.6) will hold. Hence, for any country i condition can be written as $y_t^i = c_t^i + \frac{\alpha w}{\sigma} s_t^i$. By aggregating over all countries, a world market clearing condition can be derived as

$$y_t^* = \int_0^1 y_t^i di = \int_0^1 \left(c_t^i + \frac{\alpha w}{\sigma} s_t^i\right) di = \int_0^1 c_t^i di + \frac{\alpha w}{\sigma}\int_0^1 s_t^i di = c_t^* \tag{4.7}$$

where y_t^* and c_t^* are indexes for world output and consumption in log terms. The result follows from the fact that $\int_0^1 s_t^i di = 0$.

Inserting (3.43) and (4.6) into (4.5) yields

$$y_t = y_t^* + \frac{1}{\sigma_\alpha} s_t \tag{4.8}$$

where $\sigma_\alpha = \frac{\sigma}{1+\alpha(w-1)} > 0$.

Finally, combinig (3.55) with Euler equation (3.18) gives[13]

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma_\alpha}\left(i_t - E_t\{\pi_{H,t+1}\} - \rho\right) + \alpha\Theta E_t\{\Delta y_{t+1}^*\} \tag{4.9}$$

where $\Theta \equiv w - 1 = \sigma\gamma + (1-\alpha)(\sigma\eta - 1) - 1 > 0$.

4.1.2 The trade balance

Let denote net exports in terms of domestic output as $nx_t \equiv \left(\frac{1}{Y}\right)\left(Y_t - \frac{P_t}{P_{H,t}} C_t\right)$. The net eport expressed as a fraction of steady state output Y. The first order approximation yields:

$$nx_t = y_t - c_t - \alpha s_t \tag{4.10}$$

Combining (3.59) with (3.55) gives

$$nx_t = \alpha\left(\frac{w}{\sigma} - 1\right) s_t \tag{4.11}$$

the sign of the relationship between the TOT and net export depend on the relative size of σ, γ and η.

[13] See derivation from Appendix A, Section 8.4, Dynamic IS equation

4.2 The Supply Side

4.2.1 Aggregate output and Employment

Labor market clearing in the representative small open economy requires

$$N_t = \int_0^1 N_t(j)\, dj \tag{4.12}$$

where N_t is aggregate employement.

Combining aggregate employement (4.12) with firm's output (3.19) yields

$$N_t = \frac{Y_t}{A_t}\int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} dj \tag{4.13}$$

where first-order approximation of $\int_0^1 \left(P_{H,t}(j)/P_{H,t}\right)^{-\varepsilon} dj$ is equal to zero[14]. Thus, the first-order approximation of (4.13) around the perfect foresight steady state gives the following relationship between aggregate output and employment:

$$y_t = a_t + n_t \tag{4.14}$$

4.2.2 Marginal cost and inflation dynamics

The log-linearized optimal price-setting condition (3.26) can be formulated in the terms of marginal cost and inflation as

$$\bar{p}_{H,t} - p_{H,t-1} = \mu + (1-\theta_H\beta)\sum_{k=0}^{\infty}(\theta_H\beta)^k E_t\{mc_{t+k}^r\} + \sum_{k=0}^{\infty}(\theta_H\beta)^k E_t\{\pi_{H,t+k}\} \tag{4.15}$$

Combining (3.64) with log-linearized difference equation describing the evolution of domestic prices (A.21) yields

$$\pi_{H,t} = \beta E_t\{\pi_{H,t+1}\} + \lambda_H \widehat{mc}_t^r \tag{4.16}$$

where $\lambda_H \equiv \frac{(1-\theta_H)(1-\theta_H\beta)}{\theta_H}$ and $\widehat{mc}_t^r = mc_t^r - mc^r$. The equation (4.16) shows that current domestic inflation depends on the sum of expected domestic inflation and real marginal costs.

By inserting (3.17) and (3.30) into (3.24) the real marginal cost can be redefined as

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[14] See derivation from Appendix A, Section 8.7 Welfare loss function, Equation (A.48)

$$mc_t^r = -\nu + \sigma c_t + \varphi n_t + s_t - (1+\varphi)a_t \tag{4.17}$$

Next, using world market equilbirum condition with (3.43) and (4.14), the (4.17) yields

$$mc_t^r = -\nu + \sigma y_t^* + \varphi y_t + s_t - (1+\varphi)a_t \tag{4.18}$$

Thus, from the result it can be seen that marginal cost is increasing in the TOT and world output.

Finally, using (4.8) to substitute for s_t, the expression (4.18) can be rewritten

$$mc_t^r = -\nu + (\varphi + \sigma_\alpha)y_t + (\sigma - \sigma_\alpha)y_t^* - (1+\varphi)a_t \tag{4.19}$$

Hence, the real marginal cost is rising in domestic output and TOT, while the rise of technology decreases it.

4.3 Equilibrium Dynamics: Canonical Representation

4.3.1 Equilibrium Dynamic for the Small open Economy

The linearized equilibrium dynamics for the small open economy have a representation in terms of output gap and domestic inflation. To derive output gap version of dynamic IS curve, first we need to define domestic output gap $\widetilde{y_t}$:

$$\widetilde{y_t} \equiv y_t - y_t^n \tag{4.20}$$

where y_t^n is domestic natural level of output. It can be found after imposing $mc_t = -\mu$ for all t and solving for domestic output in equation (4.19):

$$y_t^n = \Gamma_0 + \Gamma_\mathrm{a} \cdot a_t + \Gamma_* \cdot y_t^* \tag{4.21}$$

where $\Gamma_0 = \frac{\nu-\mu}{\varphi+\sigma_\alpha}$; $\Gamma_a = \frac{(1+\varphi)}{\varphi+\sigma_\alpha}$; $\Gamma_* = \frac{\alpha\Theta\sigma_\alpha}{\varphi+\sigma_\alpha}$ and $\Gamma_a > 0$.

Using equation (4.21), the domestic real marginal cost can be reexpressed as

$$\widehat{mc}_t = (\sigma_\alpha + \varphi)x_t \tag{4.22}$$

Let's insert the expression (4.22) into (4.16) to derive a New Keynesian Phillips curve (NKPC) for the small open economy in terms of the output gap:

$$\pi_{H,t} = \beta E_t\{\pi_{H,t+1}\} + k_\alpha \widetilde{y}_t \tag{4.23}$$

where $k_\alpha = \lambda(\varphi + \sigma_\alpha)$ and the degree of openness α affects the dynamic of domestic inflation through its influence on the slope of NKPC.

Let's insert Fisher equation $r_t \equiv i_t - E_t\{\pi_{H,t+1}\}$ into dynamic IS equation (4.9) to define natural level output (flexible price level output):

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma_\alpha}(r_t^n - \rho) - \alpha\Theta E_t\{\Delta y_{t+1}^*\} \tag{4.24}$$

From (4.24) the natural output level is expressed as

$$y_t^n = E_t\{y_{t+1}^n\} - \frac{1}{\sigma_\alpha}(r_t^n - \rho) + \alpha\Theta E_t\{\Delta y_{t+1}^*\} \tag{4.25}$$

Now, by combining the equation (4.20) and (4.25) to derive dynamic IS equation for the open economy in terms of the output gap:

$$\widetilde{y}_t = E_t\{\tilde{y}_{t+1}\} - \frac{1}{\sigma_\alpha}(i_t - E_t\{\pi_{H,t+1}\} - r_t^n) \tag{4.26}$$

where $r_t^n \equiv \rho - \sigma_\alpha \Gamma_\alpha(1 - \rho_a)a_t + \alpha\sigma\_\alpha(\Theta + \Psi)E_t\{\Delta y_{t+1}^*\}$ is the small open economy's natural rate of interest.

4.3.2 Equilibrium Dynamic for the rest of the world

Let's assume that the size of small open economy is negligible relative to the rest of the world (ROW) economy. This assumption allows to deal the ROW economy as a closed economy. In the ROW, the representative households face a similar problem as domestic households. Hence, the log-linear labor supply of household (3.17) and the log-linearized Euler equation (3.18) can be rewritten as

$$w_t^* - p_t^* = \sigma c_t^* + \varphi n_t^* \tag{4.27}$$

$$c_t^* = E_t\{c_{t+1}^*\} - \frac{1}{\sigma}(i_t^* - E_t\{\pi_{t+1}^*\} - \rho) \tag{4.28}$$

By combining the world market clearing condition (4.7) with the log-linearized Euler equation of ROW (4.28), we can get the world output:

$$y_t^* = E_t\{y_{t+1}^*\} - \frac{1}{\sigma}(i_t^* - E_t\{\pi_{t+1}^*\} - \rho) \tag{4.29}$$

Therefore, the small economy's log-linear real marginal cost (3.24) and aggregate production (4.14) are analogue to the world economy:

$$mc_t^* = -\nu^* + w_t^* - p_t^* - a_t^* \tag{4.30}$$

$$y_t^* = a_t^* + n_t^* \tag{4.31}$$

where ν^* is the optimal subsidy that can exactly offset monopolistic distortion in the world economy and equals to the $-\log(1 - 1/\varepsilon)$ and a_t^* is world productivity shock which is described as $a_{t+1}^* = \rho_a^* a_t^* + \varepsilon_t^*$.

By inserting (4.27) and (4.31) into (4.30) the log linearized real marginal cost for ROW can be redefined as

$$mc_t^* = -\nu^* + (\sigma + \varphi)y_t^* - (1 + \varphi)a_t^* \tag{4.32}$$

Under the flexible price, the ROW economy's real marginal cost is constant over time and given by $mc^* =-\mu$. Hence, the natural level of world output can be defined from (4.32) as

$$\overline{y_t^*} = \left(\frac{\nu^* - \mu}{\sigma + \varphi}\right) - \left(\frac{1 + \varphi}{\sigma + \varphi}\right) a_t^* \tag{4.33}$$

Let's redefine natural level of world output from (4.29) in the terms of flexible price level

$$\bar{y}_t^* = E_t\{\bar{y}_{t+1}^*\} - \frac{1}{\sigma}\left(\overline{r_t^*} - \rho\right) \tag{4.34}$$

where $\overline{r_t^*} = \rho - \sigma(1 - \rho_a^*)$ is the natural level real interest of the world economy. Hence, the world economy's dynamic IS equation in terms of the output gap can be obtained by subtracting equation (4.34) from equation (4.29):

$$\tilde{y}_t^* = E_t\{\tilde{y}_{t+1}^*\} - \frac{1}{\sigma}\left(i_t^* - E_t\{\pi_{t+1}^*\} - \overline{r_t^*}\right) \tag{4.35}$$

Furthermore, the world economy's Phillips curve in terms of marginal cost is analogue to small open economy version (4.16) and expressed as

$$\pi_t^* = \beta E_t\{\pi_{t+1}^*\} + \lambda_F \widehat{mc}_t^* \tag{4.36}$$

where $\lambda_F \equiv (1-\theta_F)(1-\theta_F\,\beta)/\theta_F$ and $\widehat{mc}_t^* = mc_t^* - mc^*$.

Now, let's reexpress (4.32) in the terms of flexible price level or natural level real marginal cost:

$$mc^* = -\nu^* + (\sigma+\varphi)\bar{y}_t^* - (1+\varphi)a_t^* \tag{4.37}$$

The world economy's New Keynesian Phillips equation in terms of output gap can be achieved by subtracting the equation (4.37) from the equation (4.34) and inserting it into the equation (4.36):

$$\pi_t^* = \beta E_t\{\pi_{t+1}^*\} + \lambda_F(\sigma+\varphi)\tilde{y}_t^* \tag{4.38}$$

5 Monetary Policy

5.1 Monetary Policy rules

The Central Bank of Mongolia has shifted from money aggregate rule to CPI inflation targeting rule since 2007. However, average inflation rate was considerably higher than targeted level in the past years. For that reason, examining alternative monetary policy rules is important. Hence, let's assume that BoM follows the CPI inflation-based Taylor rule (CITR) of the form:

$$i_t = \rho + \phi_\pi^{CPI} \cdot \pi_t \tag{5.1}$$

where ϕ_π^{CPI} is the relative weight on the CPI inflation. The equation (5.1) implies that BoM changes the nominal interest rate (policy interest rate) only if the CPI inflation deviates from its targeting level.

In order to compare the CITR performance to the other alternative monetary rules, let specify three different simple monetary rules: the domestic inflation-based Taylor rule (DITR), currency board with fixed exchange rate (PEG), and the simple Taylor rule. Under DITR monetary policy regime, BoM objects at stabilization of domestic prices which expressed as

$$i_t = \rho + \phi_\pi^{DPI} \cdot \pi_{H,t} \tag{5.2}$$

where ϕ_π^{DPI} is the relative weight on the domestic price inflation.

The currency board implies that BoM is required to keep a fixed nominal exchange rate over the time. Thus, the main goal of monetary authority is be subordinated to the exchange rate target formulated as

$$e_t = 0 \tag{5.3}$$

where e_t is effective nominal exchange rate. The analogue monetary policy rule can be defined as $i_t = i_t^*$.

The simple Taylor rule (flexible inflation targeting) is specified as

$$i_t = \rho + \phi_\pi^{CPI} \cdot \pi_t + \phi_y \widetilde{y}_t \tag{5.4}$$

where ϕ_y is the relative weight on the output gap. The Taylor rule tells that central bank concerns both on the output gap and the deviation of an inflation from the target. However, parameter ϕ_π^{CPI} should be higher than ϕ_y because the main goal of BoM is the still price stability.

5.2 The Welfare Loss Function

The analysis of welfare evaluation to the alternative monetary policy rules has become an important field of study since firstly introduced by Taylor (1999). The main idea of welfare evaluation concerns the importance for policy makers to have a set of tools that allow them to compare alternative policy rules.

Under the special parameters configuration $\sigma = \eta = \gamma = 1$, the employment subsidy τ_H that exactly offset the market distortion because of monopolistic power of the firm can be derived analytically. Therefore, strict domestic inflation targeting has been showed to be optimal by Galí and Monacilli (2005)[15]. Also, they have derived a second-order approximation to the utility losses of the domestic representative consumer resulting from the optimal allocation level which is expressed as a fraction of steady state consumption as[16]

$$\boldsymbol{W} = -\frac{(1-\alpha)}{2}\left[\frac{\varepsilon}{\lambda}\sum_{t=0}^{\infty}\beta_H^t\,\pi_{H,t}^2 + (1+\varphi)\sum_{t=0}^{\infty}\beta_H^t\,\tilde{y}_t{}^2\right] \tag{5.5}$$

The expected period welfare losses of any policy that deviates from strict inflation targeting can be written as

$$\boldsymbol{L} = -\frac{(1-\alpha)}{2}\left[\frac{\varepsilon}{\lambda}var(\pi_{H,t}) + (1+\varphi)var(\tilde{y}_t)\right] \tag{5.6}$$

by using this welfare loss function, alternative monetary policies are evaluated and compared.

[15] See derivation from Appendix A, Section 8.5 and 8.6, Optimal allocation and flexible price equilibrium

[16] See derivation from Appendix A, Section 8.7 Welfare loss function

6 Calibration

In this section, the parameters values of the model are determined. The parameters are calibrated based on the existence literature related to Mongolia and China. I follow Richard (2012), Batsukh and Avralt-Od (2014) for Mongolian calibrations, Zhang (2009) and others for Chinese parameters as well as Scott, Jorge and Carlos (2009) for developing countries calibrations.

Table 1: Calibrated values for a two country

Mongolian		Chinese	
Label	Value	Label	Value
σ	1	θ_F	0.84
φ	3	ρ_a^*	0.7107
η	1	σ_a^*	0.551
γ	1	ϕ_π^*	1.34
ε	6	ϕ_y^*	0.00
θ_H	0.84		
ϕ_π^{CPI}	2.5		
ϕ_π^{DPI}	2.5		
ϕ_π^{TR}	1.62		
ϕ_y	0.18		
ρ_a	0.89		
σ_a	0.0153		

The discount factor of household is taken from Scott et al (2009) and it is set at 0.988 which implies an annual real interest rate of 4.8% in steady state for developing economies. The elasticity of labor supply in CES utility function, φ, is chosen to be 3 which is the same as Gali and Monacelli (2005) and meaning that 1% increase of real wage leads to 3 percent increase of labor supply.

Let’s set $\sigma = \eta = \gamma = 1$ following Galí and Monacelli (2005) to evaluate welfare loss of alternative policy rules. The inverse elasticity of intertemporal substitution of consumption σ set at 1 which expresses utility function of household is log utility. The elasticity of substitution between domestic and foreign produced goods for consumption, η, is 1. This elasticity describes the change of imported goods consumption in response to change in the price of domestically produced good. The preferred value of the parameter implies that the demand of imported goods increase by exactly 1 percent when the price of domestic goods increase by 1 percent.

Following Scott *et al.* (2009) the price elasticity of demand for domestically produced goods is 6, which yields a markup of domestic firms 20 percent in steady state. The degree of openness, α, is taken from Richard (2012). He computed the share of imported consumption goods over total consumption 0.55 which expresses the degree of openness. Therefore, I follow the Richard (2012) set the value 0.84 to calibrate degree of price stickiness of Mongolia. It means that Mongolian economy has more flexible price stickiness than developed countries and prices are settled every 6 months.

I followed Batsukh and Avralt-Od (2014), the relative weight on the domestic price inflation ϕ_{π}^{DPI} and CPI inflation ϕ_{π}^{DPI} are set 2.4 respectively DITR and CITR. Furthermore, the values of parameters of simple Taylor rule are taken from Richard (2012). The relative weight on CPI inflation and output gap are respectively 1.62 and 0.18.

In order to calibrate Chinese price stickiness of China, I followed the Zhang (2009) set the value as 0.84. Then, I followd the parameters of Chineses monetary policy rule Aaron, Riikka and Jenni (2011), the parameters ϕ_{π}^{*} and ϕ_{π}^{*} are respectively 1.34 and 0. They estimated Chinese Taylor rule by General Method of Moment estimator from 1994Q1 to 2008Q4.

Also, I fit the AR(1) processes to (log) labour productivity in Mongolian data from 1996Q1 to 2011Q4 in order to calibrate the domestic productivity. The estimated parameter of persistence of domestic shock is 0.89 after Hodrick-Prescott filtered. The standart deviation of labour productivity shock is estimated of 0.0153. In addition, to calibrate foreign productivity shock, I followed Zheng and Guo (2013) set the parameters as $\rho_{a}^{*} = 0.7107$ and $\sigma_{a}^{*} = 0.551$.

7 Model simulation

7.1 Impulse Response Analysis

The impulse responses analysis can give the useful information about the dynamic behavior of the economy in response to the various shocks and the reaction of the monetary authority. In reality, it takes some time until the monetary authority realizes that the shock has hit the economy. For simplicity let's assume that BOM can identify the shocks on time and react immediately.

Figure 7.1 presents the impulse response to domestic technology shock under CITR, DITR, PEG and Taylor rule regimes. The positive productivity shock decreases real marginal cost causing both the natural level of output and output to increase.

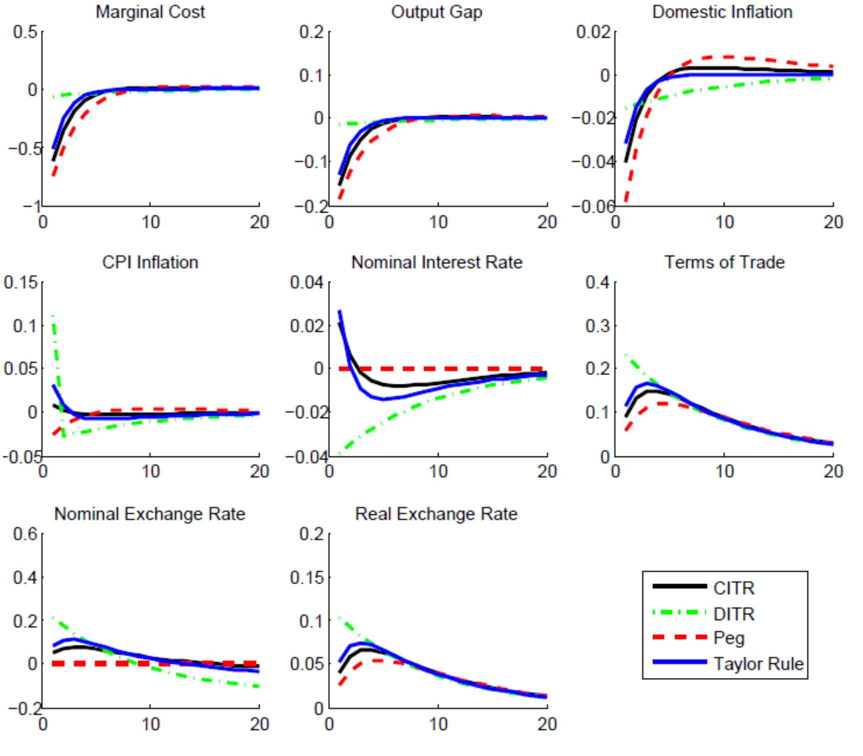

Figure 7.1 Imulse Response to the Domestic Productivity Shock under Alternative Policy Rules

However, increasing output level is less than the natural level of output which makes the output gap to fall. From the figure, the output gap is the most volatile under the PEG regime. Reduced real marginal cost allows to domestic producers lower their prices in order to make their goods more attractive for costumers in the good market. A monetary authority that follows DITR and simple Taylor rule lowers nominal interest rate to stabilize the domestic price and output gap changes respectively.

From the view of the domestic economy, lower domestic price level rises TOT that means the competitiveness of domestic products advances in the rest of the world. Increasing TOT causes CPI price level to inflate and nominal exchange rate to depreciate. However, under the PEG the nominal exchange rate is kept completely stable because the main goal of the monetary authority is to hold nominal exchange rate at constant level. Therefore, the monetary authority that follows CITR and Taylor rule increase nominal interest rate to reduce CPI inflation. Furthermore, the impact of domestic productivity shock on the foreign economy is negligible. It implies that the world interest rate remains unchanged. Hence, the expected appreciation of the domestic currency in the future is induced by UIP in the domestic economy.

First, foreign positive productivity shock causes foreign real marginal cost and foreign output to decrease. Foreign producers lower their prices to attract more costumers in the good market because of reducing real marginal cost. Result of that foreign monetary authority reduces key interest rate to ease deflationary pressure in the economy (Figure 7.2).

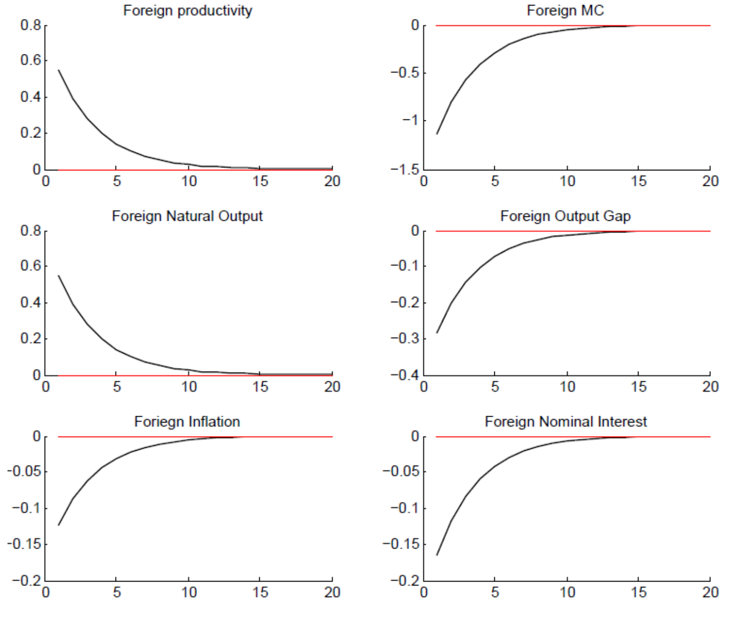

Figure 7.2 Imulse Response to the Foreign Productivity Shock under Foreign Economy

Figure 7.3 displays the impulse response to the foreign productivity shock for the SOE model under alternative monetary policy. The competitiveness (TOT) of domestic products falls in the international market because of decreasing foreign price level. The domestic monetary authorities that follow CITR, PEG, and simple Taylor rule react in the same way by reducing their policy interest rate to counterpart the real exchange appreciation caused by the foreign policy. Subsequently, real exchange rate gradually depreciates until both policy interest rates converge to their steady state.

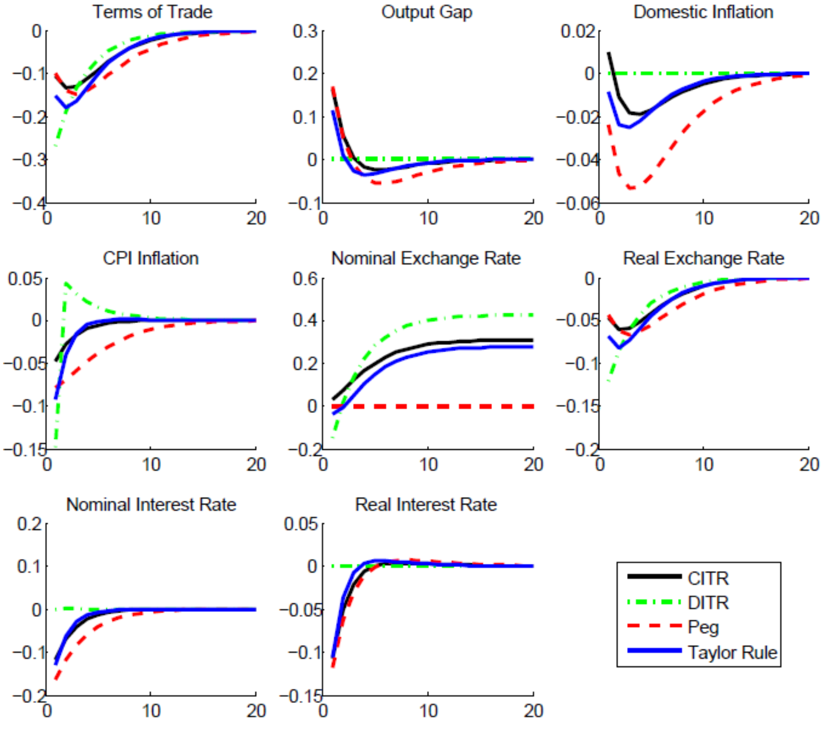

Figure 7.3 Imulse Response to the Foreign Productivity Shock under Alternative Policy Rules

Therefore, lower domestic policy rate allows SOE to increase domestic production in the short run that causes output gap to increase (except DITR). However, domestic economy has no room for further expansion in production because it falls behind the booming foreign economy. Consequently, output gap gradually decrease until it reaches its steady state. After two periods, deflated domestic price level reverts to the initial value because of domestic policy reaction. Under the PEG regime, domestic price is highly deflated. Under the CITR and DITR regimes, reaction of domestic price deflation is relatively similar.

7.2 Performance of Alternative Regimes

The Figure 7.4 presents the "cobweb" graphs that plot standard deviations of output, domestic price inflation, CPI inflation, nominal depreciation rate, and interest rate associated with the orthogonal domestic productivity shock under the alternative monetary policy regimes: CITR, DITR, PEG and simple Taylor rule.

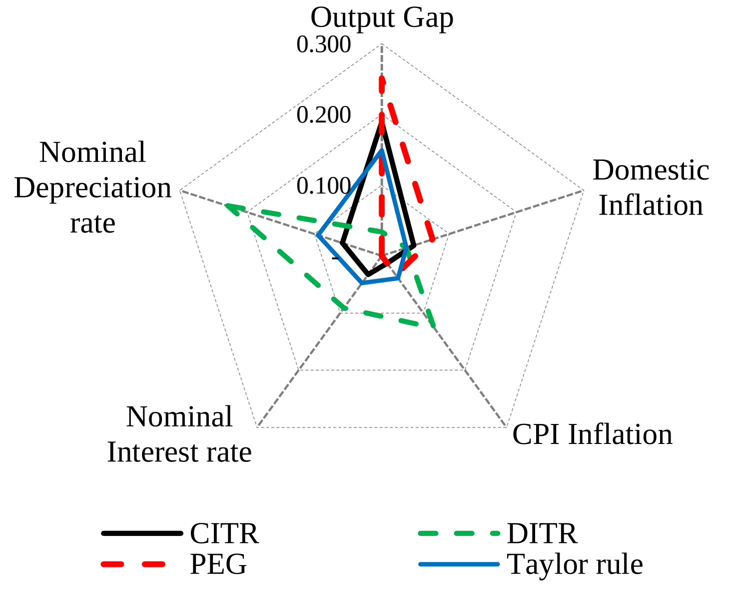

Figure 7.4 Performance of Alternative Policy Rules under Domestic Productivity Shock

Under the domestic productivity shock, the hard PEG regime creates the highest volatility in output gap and domestic inflation relative to the other three Taylor type rules. While it can keep nominal exchange depreciation rate at constant level and makes relatively lower volatility in CPI inflation. Then strict CPI-based Taylor rule performs lower volatility in CPI inflation and nominal currency depreciation rate. However it generates large fluctuation in the output gap relative to the other three Taylor rules. Furthermore, the standard deviations of the output gap and domestic inflation tend to lower under simple Taylor rule relative to the hard PEG and CITR. Eventhough it generates a little bit higher volatility in CPI inflation. In addition, the domestic inflation-based Taylor rule performs very well in domestic inflation and output gap, but it works defectively in volatilities of nominal exchange depreciation rate, CPI inflation and nominal interest rate.

The “cobweb” graph for orthogonal foreign productivity shock under the alternative monetary policy regimes is illustrated in the Figure 7.5. It plots standard deviations of output, domestic price inflation, CPI inflation, nominal depreciation rate, and interest rate associated with the: CITR, DITR, PEG and simple Taylor rule.

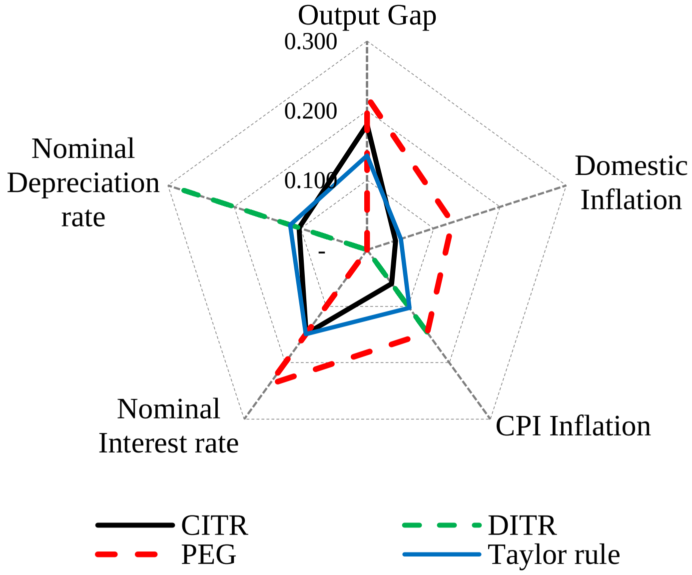

Figure 7.5 Performance of Alternative Policy Rules under Foreign Productivity Shock

The hard PEG regime executes relatively better performance in the previous situation except output gap volatility. However, in Figure 7.5, this result has changed dramatically under the foreign productivity shock. The hard PEG creates very poor performance in the economy except nominal exchange rate when economy faces foreign technology shock. If the Central bank of Mongolia decides to implement the hard PEG regime, it could be very costly to the economy because volatility of the output gap, domestic inflation, CPI inflation and nominal interest rate are much higher than Taylor type rules. While, if BoM implements DITR, the volatility of nominal interest rate, domestic inflation, and the output gap are fully stabilized but volatility of CPI inflation and nominal depreciation rate are much higher than other rules. Therefore, both of the CITR and simple Taylor rule create almost same performance. It is difficult to see which one is better than another. If BoM only considers the CPI inflation, the performance of CITR regime is better than simple Taylor rule regime.

From the “cobweb graph” analysis, it is difficult to determine which monetary policy rule is the best one because every policy rule has its advantages and disadvantages regarding the shock's form and volatility of variables. For That Reason, it is necessary to consider welfare analysis of alternative monetary policy regimes.

7.3 Welfare Analysis of Alternative Regimes

The Table 7.1 reports the standard deviations of main macro variables and welfare losses under the alternative regimes: CITR, DITR, PEG and simple Taylor rule. The performance of alternative monetary policy rules in terms on volatility of variables has already examined in section 7.2. Therefore, welfare losses of alternative monetary rules are considered in this section. The welfare losses result from a decrease in the output gap and domestic inflation volatility deviating from alternative monetary policy regimes and it is expressed as a fraction of steady state consumption.

Table 7-1 The Volatility and Welfare Loss under Alternative Policy Regimes

Variables	CITR	DITR	PEG	Taylor rule
Output Gap	0.27	0.03	0.34	0.20
Domestic Inflation	0.06	0.04	0.14	0.06
CPI Inflation	0.06	0.21	0.14	0.11
Nominal Interest Rate	0.15	0.09	0.23	0.15
Exchange Rate	6.80	9.07	0.00	5.95
CPI price level	2.82	2.55	9.52	3.78
Domestic Price level	2.85	2.53	9.55	3.80
Terms of Trade	0.55	0.68	0.54	0.61
Var(Domestic inflation)	0.004	0.001	0.020	0.004
Var(Output Gap)	0.071	0.001	0.114	0.041
Welfare Loss (W)	**-0.23**	**-0.06**	**-0.94**	**-0.18**

Standart deviations in percent

From the table, the DITR is the best monetary policy regime in terms on welfare loss. Furthermore, the welfare losses of CITR rule and simple Taylor rule exhibit almost similar result. However, simple Taylor rule performs a little bit better than CITR in terms on welfare loss. The result implies that BoM should consider not only IT but also output gap. Additionally, the hard PEG regime creates much higher welfare loss than other regimes.

8 Conclusion

In the present thesis, New Keynesian small open economy model, introduced by Gali and Monacelli (2005), is redesigned for Mongolian economy. The thesis mainly focuses on to compare qualitative and quantitative properties of the model under alternative monetary policy regimes such as CITR, DITR, PEG and simple Taylor rule by studying the analyses of the impulse response functions, “cob-web” graphs and welfare losses. To evaluate alternative monetary policy rules the welfare loss function is derived.

According to the result of welfare analysis, the DITR is the best monetary policy regime for BoM. In addition, the simple Taylor rule is better than CITR in terms on welfare loss. The result suggests that BoM should consider not only CPI inflation but also output gap. Additionally, the hard PEG regime creates much higher welfare loss than other regimes. However, from the result of “cob-web” graph analysis, the hard PEG regime creates ambiguous effect on the economy in regards to the type of shocks. For instance, the hard PEG regime performsrelatively better in the economy under the domestic productivity shock. However, the hard PEG regime doesvery poor performance in the economy under the foreign productivity shock.

Moreover, I used impulse response analyses to examine the dynamic properties of the model. The impulse response analysis presents the dynamic behavior of the small open economy in response to the domestic and foreign productivity shocks under the reaction of four alternative monetary regimes. The model provides reasonably meaningful results for Mongolian economy (Section 7.1).

The future research should consider on improving the data fitting ability of current model to increase forecasting ability of the model. In order to do that, more comprehensive examinations should be carried out.

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Appendix A: Derivation of equations

8.1 Household's demand fucntion

Demand for domestic good

The representative household solves following optimization problem to minimize aggregate domestic goods consumption $C_{H,t}$ for any given level of consumption expenditures within each category of domestic goods.

$$min \to \int_0^1 P_{H,t}(j)C_{H,t}(j)dj$$

$$s.t.\ C_{H,t} \equiv \left(\int_0^1 C_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

The household has to solve the following optimization problem:

$$\mathcal{L} \equiv \int_0^1 P_{H,t}(j)C_{H,t}(j)dj - \lambda_t^H \left[\left(\left(\int_0^1 C_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}} - C_{H,t}\right]\right.$$

The first order condition is

$$\frac{\partial \mathcal{L}}{\partial C_{H,t}(j)} = P_{H,t}(j) - \lambda_t^H \cdot \frac{\varepsilon}{\varepsilon-1} \cdot \left(\int_0^1 C_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}-1} \cdot \frac{\varepsilon-1}{\varepsilon} \cdot C_{H,t}(j)^{-\frac{1}{\varepsilon}} = 0$$

Let's multiply FOC by $C_{H,t}(j)$ and after doing some algebra

$$C_{H,t}(j) \cdot P_{H,t}(j) = \lambda_t^H \cdot \left(\int_0^1 C_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{1}{\varepsilon-1}} \cdot C_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} \tag{A.1}$$

Integrate both side of equation (A.1) by *j* and using definition of aggeragte domestic goods, we can get

$$\int_0^1 P_{H,t}(j)C_{H,t}(j)dj = \lambda_t^H \cdot C_{H,t} \to \lambda_t^H = P_{H,t} \tag{A.2}$$

Insert (A.2) into (A.1), we can get

$$P_{H,t}(j) = P_{H,t} \cdot C_{H,t}^{\frac{1}{\varepsilon}} \cdot C_{H,t}(j)^{-\frac{1}{\varepsilon}} \tag{A.3}$$

So using (A.3), demand function for domestic *j*-th good is given by following expression:

$$C_{H,t}(j) = \left[\frac{P_{H,t}(j)}{P_{H,t}}\right]^{-\varepsilon} C_{H,t} \tag{A.4}$$

After insert (A.4) into the total expenditure level, the first section of epxpression (3.4) will be proved:

$$\int_0^1 P_{H,t}(j)C_{H,t}(j)dj = \int_0^1 P_{H,t}(j)\left[\frac{P_{H,t}(j)}{P_{H,t}}\right]^{-\varepsilon} C_{H,t}dj = P_{H,t}C_{H,t}$$

Demand for *i*-th country's *j*-th good

The representative household needs to solve following optimization problem to minimize i-th country's j-th good consumption $C_{i,t}$ for any given level of consumption expenditures within each i-th country's imported goods.

$$\min \rightarrow \int_0^1 P_{i,t}(j)C_{i,t}(j)dj$$

$$s.t{:}\ C_{i,t} = \left(\int_0^1 C_{i,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

The household has to solve the following optimization problem:

$$\mathcal{L} \equiv \int_0^1 P_{i,t}(j)C_{i,t}(j)dj - \lambda_t^i \left[\left(\int_0^1 C_{i,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}} - C_{i,t}\right]$$

The first order condition is

$$\frac{\partial\mathcal{L}}{\partial C_{i,t}(j)} = P_{i,t}(j) - \lambda_t^i \cdot \left[\frac{\varepsilon}{\varepsilon-1}\cdot\left(\int_0^1 C_{i,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{1}{\varepsilon-1}} \cdot \frac{\varepsilon-1}{\varepsilon}\cdot C_{i,t}(j)^{-\frac{1}{\varepsilon}}\right] = 0$$

Let's multiply FOC by $C_{i,t}(j)$ and after doing some algebra

$$C_{i,t}(j)\cdot P_{i,t}(j) = \lambda_t^i \cdot \left(\int_0^1 C_{i,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{1}{\varepsilon-1}} \cdot C_{i,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} \tag{A.5}$$

Integrate both side of equation (A.2) by *j* and using definition of aggeragte imported goods, we can get

$$\int_0^1 P_{i,t}(j)C_{i,t}(j)dj = \lambda_t^i \cdot C_{i,t} \rightarrow\ \lambda_t^i = P_{i,t} \tag{A.6}$$

Insert (A.6) into (A.5), we can get

$$P_{i,t}(j) = P_{i,t}\cdot C_{i,t}^{\frac{1}{\varepsilon}} \cdot C_{i,t}(j)^{-\frac{1}{\varepsilon}} \tag{A.7}$$

So using (A.7), demand function for i-th country's j-th good is given by following expression:

$$C_{i,t}(j) = \left[\frac{P_{i,t}(j)}{P_{i,t}}\right]^{-\varepsilon} C_{i,t} \tag{A.8}$$

After insert (A.8) into the total expenditure of i-th country's j-th good level, the second section of epxpression (3.4) will be proved:

$$\int_0^1 P_{i,t}(j)C_{i,t}(j)dj = \int_0^1 P_{i,t}(j)\left[\frac{P_{i,t}(j)}{P_{i,t}}\right]^{-\varepsilon} C_{i,t}dj = P_{i,t}C_{i,t}$$

Demand for imported goods by country origin

Now representative household have to decide how much to import from each foreign country, for any given level of consumption expenditures on imported goods.

$$\min \rightarrow \int_0^1 P_{i,t}C_{i,t}di$$

$$s.t{:}\ C_{F,t} = \left(\int_0^1 C_{i,t}^{\frac{\gamma-1}{\gamma}} di\right)^{\frac{\gamma}{\gamma-1}}$$

The household has to solve the following optimization problem:

$$\mathcal{L} \equiv \int_0^1 P_{i,t}C_{i,t}di - \lambda_t^F\left[\left(\int_0^1 C_{i,t}^{\frac{\gamma-1}{\gamma}} di\right)^{\frac{\gamma}{\gamma-1}} - C_{F,t}\right]$$

The first order condition is

$$\frac{\partial\mathcal{L}}{\partial C_{i,t}} = 0 \rightarrow P_{i,t} - \lambda_t^F\left[\frac{\gamma}{\gamma-1}\left(\int_0^1 C_{i,t}^{\frac{\gamma-1}{\gamma}} di\right)^{\frac{1}{\gamma-1}}\frac{\gamma-1}{\gamma}C_{i,t}^{-\frac{1}{\gamma}}\right] = 0$$

Let's multiply FOC by $C_{i,t}$and after doing some algebra

$$C_{i,t}\cdot P_{i,t} = \lambda_t^F\cdot\left(\int_0^1 C_{i,t}{}^{\frac{\gamma-1}{\gamma}} dj\right)^{\frac{1}{\gamma-1}}\cdot C_{i,t}{}^{\frac{\gamma-1}{\gamma}} \tag{A.9}$$

Integrate both side of equation (A.9) by i sand using definition of aggeragte imoirted goods, we can get

$$\int_0^1 P_{i,t}C_{i,t}di = \lambda_t^F\cdot C_{F,t} \rightarrow\ \lambda_t^F = P_{F,t} \tag{A.10}$$

Insert (A.10) into (A.9), we get

$$P_{i,t} = P_{F,t} \cdot C_{F,t}^{\frac{1}{\gamma}} \cdot C_{i,t}^{-\frac{1}{\gamma}} \tag{A.11}$$

So using (A.11), demand function for imported goods by country origin is given by

$$C_{i,t} = \left[\frac{P_{i,t}}{P_{F,t}}\right]^{-\gamma} C_{F,t} \tag{A.12}$$

After insert (A.12) into the total expenditure on imported goods level, the following epxpression will be proved:

$$\int_0^1 P_{i,t}C_{i,t}di = \int_0^1 P_{i,t}\left[\frac{P_{i,t}}{P_{F,t}}\right]^{-\gamma} C_{F,t}di = P_{F,t}^{\gamma}C_{F,t}P_{F,t}^{1-\gamma} = C_{F,t}P_{F,t}$$

The optimal allocation of expenditures between domestic and imported goods

The representative household’s total domestic consumption is defined by sum of domestic and foreign goods. Household face following optimization problem:

$$\min \rightarrow P_{H,t}C_{H,t} + P_{F,t}C_{F,t}$$

$$s.t{:}\ C_t = \left[(1-\alpha)^{\frac{1}{\eta}}\left(C_{H,t}\right)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}\left(C_{F,t}^{\frac{\eta-1}{\eta}}\right)\right]^{\frac{\eta}{\eta-1}}$$

The optimization problem can be solved by Lagrangian method:

$$\mathcal{L} \equiv P_{H,t}C_{H,t} + P_{F,t}C_{F,t} - \lambda_t\left\{\left[(1-\alpha)^{\frac{1}{\eta}}\left(C_{H,t}\right)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}\left(C_{F,t}^{\frac{\eta-1}{\eta}}\right)\right]^{\frac{\eta}{\eta-1}} - C_t\right\}$$

The first order condition is

$$\begin{cases} \dfrac{\partial L}{\partial C_{H,t}}: P_{H,t} - \lambda_t\left\{\dfrac{\eta}{\eta-1}\left[(1-\alpha)^{\frac{1}{\eta}}\left(C_{H,t}\right)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}\left(C_{F,t}^{\frac{\eta-1}{\eta}}\right)\right]^{\frac{1}{\eta-1}}(1-\alpha)^{\frac{1}{\eta}}\dfrac{\eta-1}{\eta}C_{H,t}^{-\frac{1}{\eta}}\right\} \\ \dfrac{\partial L}{\partial C_{F,t}}: P_{F,t} - \lambda_t\left\{\dfrac{\eta}{\eta-1}\left[(1-\alpha)^{\frac{1}{\eta}}\left(C_{H,t}\right)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}\left(C_{F,t}^{\frac{\eta-1}{\eta}}\right)\right]^{\frac{1}{\eta-1}}\alpha^{\frac{1}{\eta}}\dfrac{\eta-1}{\eta}C_{F,t}^{-\frac{1}{\eta}}\right\} \end{cases} \tag{A.13}$$

after doing some algebra

$$\begin{cases} P_{H,t} = \lambda_t C_t^{\frac{1}{\eta}}(1-\alpha)^{\frac{1}{\eta}}C_{H,t}^{-\frac{1}{\eta}} \\ P_{F,t} = \lambda_t C_t^{\frac{1}{\eta}}\alpha^{\frac{1}{\eta}}C_{F,t}^{-\frac{1}{\eta}} \end{cases} \rightarrow \begin{cases} P_{H,t}^{1-\eta} = \lambda_t^{1-\eta}C_t^{\frac{1-\eta}{\eta}}(1-\alpha)^{\frac{1-\eta}{\eta}}C_{H,t}^{-\frac{1-\eta}{\eta}} \\ P_{F,t}^{1-\eta} = \lambda_t^{1-\eta}C_t^{\frac{1-\eta}{\eta}}\alpha^{\frac{1-\eta}{\eta}}C_{F,t}^{-\frac{1-\eta}{\eta}} \end{cases}$$

Let’s add both equations

$$(1-\alpha)P_{H,t}^{1-\eta}+\alpha P_{F,t}^{1-\eta}=\lambda_t^{1-\eta}C_t^{\frac{1-\eta}{\eta}}\left[(1-\alpha)^{\frac{1}{\eta}}C_{H,t}^{\frac{\eta-1}{\eta}}+\alpha^{\frac{1}{\eta}}C_{F,t}^{\frac{\eta-1}{\eta}}\right] \tag{A.14}$$

Using definition of total consumption and the fact $\lambda_t = P_t$, we get CPI from (A.14)

$$P_t=\left[(1-\alpha)P_{H,t}^{1-\eta}+\alpha P_{F,t}^{1-\eta}\right]^{\frac{1}{1-\eta}}$$

Finally, using (A.14) the optimal allocation of expenditures are driven by

$$P_{H,t}=\lambda_t C_t^{\frac{1}{\eta}}(1-\alpha)^{\frac{1}{\eta}}C_{H,t}^{-\frac{1}{\eta}} \rightarrow\ C_{H,t}=\left[\frac{P_{H,t}}{P_t}\right]^{-\eta}(1-\alpha)C_t \tag{A.15}$$

$$P_{F,t}=\lambda_t C_t^{\frac{1}{\eta}}\alpha^{\frac{1}{\eta}}C_{F,t}^{-\frac{1}{\eta}} \rightarrow\ C_{F,t}=\left[\frac{P_{F,t}}{P_t}\right]^{-\eta}\alpha C_t \tag{A.16}$$

Total consumption expenditures for the domestic household

Let's prove that the total domestic consumption is split into domestic and foreign goods which is expressed by $P_tC_t=P_{H,t}C_{H,t}+P_{F,t}C_{F,t}$. Multiply (A.15) and (A.16) by (A.15):

$$\begin{cases} C_{H,t}P_{H,t}=\left[\frac{P_{H,t}}{P_t}\right]^{-\eta}(1-\alpha)C_tP_{H,t}=(1-\alpha)P_t^{\eta}P_{H,t}^{1-\eta}C_t \\ C_{F,t}P_{F,t}=\left[\frac{P_{F,t}}{P_t}\right]^{-\eta}\alpha C_tP_{F,t}=\alpha P_t^{\eta}P_{F,t}^{1-\eta}C_t \end{cases}$$

Let's add last two equations

$$C_{H,t}P_{H,t}+C_{F,t}P_{F,t}=P_t^{\eta}C_t\left[(1-\alpha)P_{H,t}^{1-\eta}+\alpha P_{F,t}^{1-\eta}\right]=P_t^{\eta}C_tP_t^{1-\eta}=P_tC_t \tag{A.17}$$

8.2 The dynamics of the domestic price index

Let A_t is the set of domestic firms who did not reoptimizing their posted price in period *t* and $A_t\subset[0,1]$. From the equation (3.6) domestic aggregate price is given as

$$P_{H,t}=\left[\int_{A(t)}^{1}P_{H,t}(j)^{1-\varepsilon}dj+(1-\theta)\bar{P}_{H,t}^{1-\varepsilon}\right]=\theta P_{H,t}^{1-\varepsilon} \tag{A.18}$$

Hence, we can rewrite domestic price index as

$$P_{H,t}=\left[\theta P_{H,t}^{1-\varepsilon}+(1-\theta)\bar{P}_{H,t}^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{A.19}$$

Let's divide both side of equation (A.19) by $P_{H,t-1}$, we get:

$$\left[\frac{P_{H,t}}{P_{H,t-1}}\right]^{1-\varepsilon} = \theta + (1-\theta)\left[\frac{\bar{P}_{H,t}}{P_{H,t-1}}\right]^{1-\varepsilon} \tag{A.20}$$

If we log-linearize the equation (A.20) around zero inflation:

$$\pi_{H,t} = (1-\theta)\left(\bar{p}_{H,t} - p_{H,t-1}\right) \tag{A.21}$$

The equation (A.21) implies that domestic price inflation results from the domestic firms who re-optimizing in any given period choose a price that differs from the economy's average price.

8.3 Optimal price setting in the Calvo model

$$\max_{\bar{P}_{H,t}} \rightarrow \sum_{k=0}^{\infty} \theta_H^k E_t\left\{Q_{t,t+k}\left[\bar{P}_{H,t} Y_{t+k|t}(j) - TC_{jt+k|t}^n(Y_{jt+k|t}(j))\right]\right\}$$

$$s.t{:}\ Y_{t+k}(j) = C_{H,t+k}(j) + \int_0^1 C_{H,t+k}^i(j)dj$$

where $C_{H,t+k}(j)$ is domestic demand for domestic j-th good from (3.4). From (3.5), foreign demand for home country's j-th good also can be defined like home country:

$$C_{H,t+k}(j) = \left(\frac{\bar{P}_{H,t}(j)}{P_{H,t+k}}\right)^{-\varepsilon} C_{H,t+k}; \qquad C_{H,t+k}^i(j) = \left(\frac{\bar{P}_{H,t}(j)}{P_{H,t+k}}\right)^{-\varepsilon} C_{H,t+k}^i;$$

Hence, we can rewrite constraint as

$$Y_{t+k|t}(j) = \left(\frac{\bar{P}_{H,t}(j)}{P_{H,t+k}}\right)^{-\varepsilon}\left[C_{H,t+k} + \int_0^1 C_{H,t+k}^i\, di\right] \equiv Y_{t+k}^d(\bar{P}_{H,t}) \tag{A.22}$$

where $Y_{t+k}^d(\bar{P}_{H,t})$ is the demand of domestic firm's product k-period after resetting its price. Now we can redefine optimization problem as

$$\max_{\bar{P}_{H,t}} \rightarrow \sum_{k=0}^{\infty} \theta_H^k E_t\left\{Q_{t,t+k}\left[\bar{P}_{H,t}\left(\frac{\bar{P}_{H,t}(j)}{P_{H,t+k}}\right)^{-\varepsilon}\left[C_{H,t+k} + \int_0^1 C_{H,t+k}^i\, di\right] - TC_{jt+k|t}^n(Y_{jt+k|t}(j))\right]\right\}$$

FOC:

$$\frac{\partial \Pi_{t+k|t}}{\partial \bar{P}_{H,t}}:\ \sum_{k=0}^{\infty} \theta_H^k E_t \left\{ Q_{t,t+k} \left[(1-\varepsilon) \left(\frac{\bar{P}_{H,t}(j)}{P_{H,t+k}} \right)^{-\varepsilon} \left[C_{H,t+k} + \int_0^1 C_{H,t+k}^i \, di \right] - \frac{\partial TC_{jt+k|t}^n}{\partial Y_{jt+k|t}(j)} \cdot \frac{\partial Y_{jt+k|t}(j)}{\partial \bar{P}_{H,t}} \right] \right\} = 0$$

$$\rightarrow \sum_{k=0}^{\infty} \theta_H^k E_t \left\{ Q_{t,t+k} \left[(1-\varepsilon) Y_{t+k|t}(j) + \varepsilon \cdot MC_{t+k|t}^n(j) \cdot Y_{t+k|t}(j) \cdot \left(\frac{\bar{P}_{H,t}(j)}{P_{H,t+k}} \right)^{-1} \cdot \frac{1}{P_{H,t+k}} \right] \right\} = 0$$

$$\sum_{k=0}^{\infty} \theta_H^k E_t \left\{ Q_{t,t+k} Y_{t+k|t}(j) \left[\bar{P}_{H,t} + \frac{\varepsilon}{1-\varepsilon} \cdot MC_{t+k|t}^n(j) \right] \right\} = 0 \qquad \text{(A.23)}$$

where $MC_{t+k|t}^n(j)$ is the nominal marginal cost for *j*-th firm. From the fact that all firms marginal cost is equal when hold CRS and Euler equation (3.12), we can rewrite first order condition as

$$\sum_{k=0}^{\infty} (\theta\beta)^k E_t \left\{ C_{t+k}^{-\sigma} Y_{t+k|t}(j) \frac{P_{H,t-1}}{P_{t+k}} \left[\frac{\bar{P}_{H,t}}{P_{H,t-1}} - \frac{\varepsilon}{\varepsilon-1} \cdot \Pi_{t-1,t+k}^H \cdot MC_{t+k}^r \right] \right\} = 0 \qquad \text{(A.24)}$$

Where: $MC_{t+k}^r = \frac{MC_{t+k}^n}{P_{H,t+k}}$ is the real marginal cost and $\Pi_{t-1,t+k}^H = \frac{P_{H,t+k}}{P_{H,t+k-1}} \cdot \frac{P_{H,t+k-1}}{P_{H,t+k-2}} \cdot \ldots \cdot \frac{P_{H,t-3}}{P_{H,t-2}} \cdot \frac{P_{H,t-2}}{P_{H,t-1}} = \frac{P_{H,t+k}}{P_{H,t-1}}$. Log-linearize the equation (A.19) around zero inflation steady state with balanced trade, we can get the optimal price setting (3.23).

8.4 Dynamic IS equation

Combinig (3.55) with Euler equation (3.18) gives

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho) - \frac{\alpha w}{\sigma} E_t\{\Delta s_{t+1}\} \qquad \text{(A.25)}$$

Inserting domestic and CPI inflation relation of (3.31) into the (A.25), we get

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}\left(i_t - E_t\{\pi_{H,t+1}\} - \rho\right) - \left(\frac{\alpha}{\sigma} - \frac{\alpha w}{\sigma}\right) E_t\{\Delta s_{t+1}\} \qquad \text{(A.26)}$$

Combining (A.26) and (3.57), it gives

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma_\alpha}\left(i_t - E_t\{\pi_{H,t+1}\} - \rho\right) + \alpha\Theta E_t\{\Delta y_{t+1}^*\} \qquad \text{(A.27)}$$

8.5 Optimal Allocation

In this section, optimal allocation (optimal monetary policy) for the small open economy will be derived. In order to get analytical solution, I follow the Gali (2005) and make the assumptions regarding to the parameter coefficients of $\sigma = \eta = \gamma = 1$.

First, I need to define efficient allocation from the view point of Social planner. The benevolent social planner is seeking to maximize the representative households' life time utility subject to i) the technological constraint, ii) a consumption/output possibilities set implicit in the international risk-sharing conditions and iii) the market clearing condition. The optimization problem can be formulated as

$$max \rightarrow E_0 \sum_{t=0}^{\infty} U(C_t, N_t)$$

Subject to

i) $Y_t(j) = A_t \cdot N_t(j)$

ii) $C_t = \vartheta_i \cdot C_t^i \cdot RER_{i,t}$

iii) $$Y_t = \left[\frac{P_{H,t}}{P_t}\right]^{-\eta} \left[(1-\alpha)C_t + \alpha \int_0^1 \left(S_t^i S_{i,t}\right)^{\gamma-\eta} C_t RER_{i,t}^{\eta-\frac{1}{\sigma}} di\right]$$

Consider in special case $\sigma = \eta = \gamma = 1$ the market clearing condition (4.3) can be expressed as

$$Y_t = C_t \cdot S_t^{\alpha} \tag{A.28}$$

The domestic consumption relation to the world consumption and terms of trade

$c_t = c_t^* + \left(\frac{1-\alpha}{\sigma}\right) \cdot S_t$ can be formulated in our case $\sigma = 1$ as

$$c_t = c_t^* + (1-\alpha) \cdot s_t \rightarrow \log(C_t) = \log(C_t^*) + (1-\alpha) \cdot \log(S_t) \rightarrow C_t = C_t^* \cdot S_t^{(1-\alpha)}$$

$$S_t = \left(\frac{C_t}{C_t^*}\right)^{\frac{1}{(1-\alpha)}} \tag{A.29}$$

Where $c_t^* = \int_0^{\infty} c_t^* di$ is world consumption index. Lets combine (A.28) with (A.29):

$$Y_t = (C_t)^{\frac{1}{(1-\alpha)}} (C_t^*)^{\frac{\alpha}{(1-\alpha)}} \tag{A.30}$$

From the international market clearing condition $y_t^* = c_t^*$, we know that $Y_t^* = C_t^*$. Now insert international market clearing condition to the (A.30), we will get following result:

$$C_t = (Y_t)^{(1-\alpha)}(Y_t^*)^{\alpha} \tag{A.31}$$

Let combine (A.31) with the technological constraint and the labor market optimality condition:

$$\begin{cases} C_t = (Y_t)^{(1-\alpha)}(Y_t^*)^{\alpha} \\ Y_t(j) = A_t \cdot N_t(j) \\ N_t(j) = N_t, \quad all\ j \in [0,1] \end{cases} \rightarrow \begin{cases} C_t = (Y_t)^{(1-\alpha)}(Y_t^*)^{\alpha} \\ Y_t = A_t \cdot N_t \end{cases}$$

$$C_t = (A_t \cdot N_t)^{(1-\alpha)} \cdot (Y_t^*)^{\alpha} \tag{A.32}$$

Now social planner's optimization problem can be rewritten as following formulation:

$$\max_{N_t, C_t} \rightarrow E_0 \sum_{t=0}^{\infty} U(C_t, N_t)$$

$$\text{s.t.} \qquad C_t = (A_t \cdot N_t)^{(1-\alpha)} \cdot (Y_t^*)^{\alpha}$$

Social planner's problem can be solved by using Lagrangean multiplier method.

$$\mathcal{L} \equiv E_0 \sum_{t=0}^{\infty} \left\{ U(C_t, N_t) - \lambda_t [(A_t \cdot N_t)^{(1-\alpha)} \cdot (Y_t^*)^{\alpha} - C_t] \right\}$$

The associated first order conditions are

$$\begin{cases} \dfrac{\partial \mathcal{L}}{\partial C_t} = 0 \\ \dfrac{\partial \mathcal{L}}{\partial N_t} = 0 \\ \dfrac{\partial \mathcal{L}}{\partial \lambda_t} = 0 \end{cases} \quad \Rightarrow \quad \begin{cases} U(C_t, N_t)_C + \lambda_t = 0 \\ U(C_t, N_t)_N - \lambda_t \dfrac{(1-\alpha)(A_t \cdot N_t)^{(1-\alpha)}}{N_t} = 0 \\ (A_t \cdot N_t)^{(1-\alpha)} \cdot (Y_t^*)^{\alpha} = C_t \end{cases} \quad \Rightarrow$$

$$-\frac{U(C_t, N_t)_N}{U(C_t, N_t)_C} = (1-\alpha)\frac{C_t}{N_t} = (1-\alpha)MPN_t \tag{A.33}$$

When utility function equal to $U_t(C_t, N_t) = \log(C_t) - N_t^{1+\varphi}/1+\varphi$; optimal allocation (A.33) can be expressed as $N_t = (1-\alpha)^{\frac{1}{1+\varphi}}$ which implies optimal employment is constant.

8.6 Flexible Price Equilibrium

Firms problem in flexible price and free competition market

In the situation where optimal allocation hold in economy, firms optimization problem can be expressed as

$$\max_{Y_t,N_t} \rightarrow \ \Pi_t = P_t Y_t - W_t N_t$$

$$\text{s.t.} \qquad Y_t = A_t \cdot N_t$$

The optimization problem can be rewritten as

$$\max_{N_t} \rightarrow \ \Pi_t = P_t A_t N_t - W_t N_t$$

FOC:

$$\frac{\partial \Pi_t}{\partial N_t} = 0 \rightarrow \ P_t A_t - W_t = 0 \rightarrow \ A_t = \frac{W_t}{P_t} \tag{A.34}$$

Also, the marginal productivity of labor MPN_t can be expressed as

$$\frac{\partial \mathrm{Y}_t}{\partial N_t} = MPN_t \rightarrow \ A_t = MPN_t \tag{A.35}$$

From (A.34) and (A.35), the marginal productivity of labor equal to

$$MPN_t = \ A_t = \frac{W_t}{P_t} \tag{A.36}$$

From the optimal allocation condition (A.33) and firm’s optimality condition in flexible price and free competitive market (A.36), the optimal allocation of domestic economy can be derived:

$$-\frac{U(C_t,N_t)_N}{U(C_t,N_t)_C} = (1-\alpha)MPN_t = (1-\alpha)\frac{W_t}{P_t} \tag{A.37}$$

Market distortion because of monopolistic power of firm

First, we need to make two assumptions: firms are monopolistic and prices are flexible in the market. Hence, firms’ optimization problem can be defined as

$$\max_{P^n_{H,t}(j)} \rightarrow \Pi^n_t = P^n_{H,t}(j) Y^n_t(j) - (1-\tau) W^n_t N^n_t(j)$$

s.t.

i) $Y^n_t(j) = A_t \cdot N^n_t(j)$

$$\text{ii)} \quad C_{H,t}^n(j) = \left[\frac{P_{H,t}^n(j)}{P_{H,t}^n}\right]^{-\varepsilon} C_{H,t}^n$$

$$\text{iii)} \quad Y_t^n(j) = C_{H,t}^n(j)$$

Where parameter τ is labor subsidy conducted by authority and superscript n refers natural level equilibrium in economy. The optimization problem can be redefined as

$$\max_{P_{H,t}^n(j)} \to \Pi_t^n = P_{H,t}^n(j)\left[\frac{P_{H,t}^n(j)}{P_{H,t}^n}\right]^{-\varepsilon} C_{H,t}^n - (1-\tau)W_t^n\left[\frac{P_{H,t}^n(j)}{P_{H,t}^n}\right]^{-\varepsilon} C_{H,t}^n \frac{1}{A_t}$$

FOC:

$$\frac{\partial \Pi_t^n}{\partial P_{H,t}^n(j)} = 0 \to (1-\varepsilon)P_{H,t}^n(j)\left[\frac{P_{H,t}^n(j)}{P_{H,t}^n}\right]^{-\varepsilon} C_{H,t}^n$$

$$-(1-\tau)W_t^n(-\varepsilon)\left[\frac{P_{H,t}^n(j)}{P_{H,t}^n}\right]^{-\varepsilon} C_{H,t}^n \frac{1}{P_{H,t}^n(j)A_t} = 0$$

$$(1-\varepsilon)C_{H,t}^n(j) = (1-\tau)W_t^n(-\varepsilon)\frac{C_{H,t}^n(j)}{P_{H,t}^n(j)A_t}$$

$$\frac{\varepsilon-1}{\varepsilon} = (1-\tau)\frac{W_t^n}{P_{H,t}^n(j)A_t} \tag{A.38}$$

The real marginal cost expressed as

$$MC_t^r = \frac{MC_t}{P_{H,t}} = (1-\tau)\frac{W_t}{P_{H,t}}\frac{1}{A_t} \tag{A.39}$$

I can get real marginal cost in natural level by inserting (A.38) into (A.39).

$$(MC_t^r)^n = \frac{\varepsilon-1}{\varepsilon} = (1-\tau)\frac{W_t^n}{P_{H,t}^n(j)}\frac{1}{A_t} \tag{A.40}$$

From the households' intertemporal optimality condition (labor supply) and (3.15)

$$\begin{cases} \dfrac{\varepsilon-1}{\varepsilon} = \dfrac{P_t^n}{P_{H,t}^n(j)}\dfrac{W_t^n}{P_t^n}\dfrac{1}{A_t} \\ \dfrac{W_t^n}{P_t^n} = C_t^n(N_t^n)^{\varphi} \end{cases} \to \frac{\varepsilon-1}{\varepsilon} = (1-\tau)\frac{P_t^n}{P_{H,t}^n(j)}\frac{C_t^n(N_t^n)^{\varphi}}{A_t}$$

From the market equilibrium condition (A.28), relation between terms of trade and price level as well as above expression, we can take employment subsidy expressed in the labor:

$$(1-\tau) = \frac{\varepsilon-1}{\varepsilon}\cdot(N_t^n)^{-(1+\varphi)} \tag{A.41}$$

Now we can take optimal subsidy by inserting social planner's solution (A.37) into the (A.41):

$$\begin{cases} N_t = (1-\alpha)^{\frac{1}{1+\varphi}} \\ (1-\tau) = \dfrac{\varepsilon - 1}{\varepsilon} \cdot (N_t^n)^{-(1+\varphi)} \end{cases} \rightarrow$$

$$\tau = \frac{1}{(1-\alpha)} \cdot \left(\frac{1-\varepsilon\alpha}{\varepsilon}\right) \tag{A.42}$$

Now, let's assume that optimal subsidy is in place which means the optimality of flexible price equilibrium is guaranteed.

8.7 Welfare loss function

In this section, I will follow Gali (2005) to derive a second-order approximation to the utility of the representative household when economy remains in a neighborhood of an efficient steady state. In other words authorities implement an optimal employment subsidy that removes the distortion caused by monopolistic competition. Hence, assume that the subsidy is given by (A.42).

A second-order approximation of utility is derived around a given steady state allocation. For any variable the second-order approximation of relative deviations in terms of log deviations can be expressed as

$$\begin{aligned} \frac{X_t - \bar{X}}{\bar{X}} = \frac{e^{\log(x_t)} - \bar{X}}{\bar{X}} &\approx 0 + \frac{e^{\bar{x}}}{\bar{X}}(x_t - \bar{x}) + \frac{1}{2}\frac{e^{\bar{x}}}{\bar{X}}(x_t - \bar{x})^2 \\ &= \widehat{x_t} + \frac{1}{2}\widehat{x_t}^2 + o(\|a\|^n) \end{aligned} \tag{A.43}$$

Where $\widehat{x_t}$ refers the log deviation from steady state and $o(\|a\|^n)$ represents terms that are of order higher than n-th, in the bound $\|a\|$ on the amplitude of the relevant shocks. The utility function is assumed separable in consumption and hours ($U_{cn} = 0$). In order to lighten the notation, let denotes t period utility function as $U_t \equiv U(C_t, N_t)$ and the steady state utility as $U \equiv U(C, N)$.

The second-order Taylor expansion of U_t around a steady state U yields

$$U_t - U = U_C C\left(\frac{C_t - C}{C}\right) + U_N N\left(\frac{N_t - N}{N}\right)\frac{1}{2}U_{CC}C^2\left(\frac{C_t - C}{C}\right)^2 + \frac{1}{2}U_{NN}N^2\left(\frac{N_t - N}{N}\right)^2 + o(||a||^3)$$

Using the second-order approximation property defined in (A.43), we can rewritten the above expression as

$$U_t - U = U_C C\left(\hat{c_t} + \frac{1}{2}\hat{c_t}^2\right) + U_N N\left(\widehat{n_t} + \frac{1}{2}\widehat{n_t}^2\right)\frac{1}{2}U_{CC}C^2\hat{c_t}^2 + \frac{1}{2}U_{NN}N^2\widehat{n_t}^2 + o(||a||^3)$$

Let divide both side of equation by $U_C C$

$$\frac{U_t - U}{U_C C} = \left(\hat{c_t} + \frac{1}{2}\hat{c_t}^2\right) + \frac{U_N N}{U_C C}\left(\widehat{n_t} + \frac{1}{2}\widehat{n_t}^2\right) + \frac{1}{2}\frac{U_{CC}C}{U_C}\hat{c_t}^2 + \frac{1}{2}\frac{U_{NN}N^2}{U_C C}\widehat{n_t}^2 + o\left(||a||^3\right) \rightarrow$$

$$\frac{U_t - U}{U_C C} = \left[\hat{c_t} + \frac{1}{2}\hat{c_t}^2 + \frac{1}{2}\frac{U_{CC}C}{U_C}\hat{c_t}^2\right] + \frac{U_N N}{U_C C}\left(\widehat{n_t} + \frac{1}{2}\widehat{n_t}^2\right) + \frac{1}{2}\frac{U_{NN}N^2}{U_C C}\widehat{n_t}^2 + o\left(||a||^3\right) \rightarrow$$

$$\frac{U_t - U}{U_C C} = \hat{c_t} + \left[\frac{1}{2} + \frac{1}{2}\frac{U_{CC}}{U_C}C\right]\hat{c_t}^2 + \frac{U_N N}{U_C C}\left[\widehat{n_t} + \left(\frac{1}{2} + \frac{1}{2}\frac{U_{NN}}{U_N}N\right)\widehat{n_t}^2\right] + o\left(||a||^3\right) \tag{A.44}$$

Utility function given by $U_t = C_t^{1-\sigma}/(1-\sigma) - N_t^{1+\varphi}/(1+\varphi)$. Hence we get following expressions:

$$U_C = C^{-\sigma};\ U_N = -N^{\varphi};\ U_{CC} = -\sigma C^{-\sigma-1};$$

$$U_{NN} = -\varphi N^{\varphi-1};\ \frac{U_{CC}}{U_C} = -\frac{\sigma}{C};\ \frac{U_{NN}}{U_N} = \frac{\varphi}{N}$$

Let insert expressions defined above into the (A.44):

$$\frac{U_t - U}{U_C C} = \hat{c_t} + \left[\frac{1-\sigma}{2}\right]\hat{c_t}^2 + \frac{U_N N}{U_C C}\left[\widehat{n_t} + \left(\frac{1+\varphi}{2}\right)\widehat{n_t}^2\right] + o\left(||a||^3\right) \tag{A.45}$$

From the (4.8) $y_t = y_t^* + \left(\frac{1+\alpha(\omega-1)}{\sigma}\right) s_t \rightarrow\ s_t = y_t - y_t^*$ because of $\sigma = \eta = \gamma = 1$. Also, from the expression (3.43) $c_t = c_t^* + \left(\frac{1-\alpha}{\sigma}\right) s_t \rightarrow\ c_t = c_t^* + (1-\alpha)s_t$. If we combine those two equations, we will get the following equation:

$$c_t = (1-\alpha)y_t + \alpha y_t^* \tag{A.46}$$

Insert expression (A.46) into the (A.45)

$$\frac{U_t - U}{U_C C} = (1-\alpha)\widehat{y_t} + \frac{U_N N}{U_C C}\left[\widehat{n_t} + \left(\frac{1+\varphi}{2}\right)\widehat{n_t}^2\right] + o\left(\left||a|\right|^3\right) + t.i.p \tag{A.47}$$

Where *t.i.p* stands for terms independent of policy. I will combine following expressions in order to express $\widehat{n_t}$ in terms of output gap and inflation:

$$\begin{cases} N_t(j) = \dfrac{Y_t(j)}{A_t} \\ N_t = \displaystyle\int_0^1 N_t(j)dj \\ C_{H,t}(j) = \left(\dfrac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t} \\ Y_t(j) = C_{H,t}(j) + \displaystyle\int_0^1 C_{H,t}^i(j)di \end{cases} \rightarrow N_t = \int_0^1 \frac{C_{H,t}(j) + \int_0^1 C_{H,t}^i(j)di}{A_t} dj$$

$$= \int_0^1 \frac{C_{H,t}(j)}{A_t} dj + t.i.p \rightarrow$$

$$N_t = \int_0^1 \frac{\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t}}{A_t} dj + t.i.p = \frac{Y_t}{A_t}\int_0^1 \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} dj + t.i.p$$

Now take logarithm for both side of expression

$$n_t = y_t - a_t + d_t + t.i.p \tag{A.48}$$

Where $d_t = log\left[\int_0^1 \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} dj\right]$. The following lemma shows that d_t is proportional to the cross-sectional distribution of relative price.

Lemma 1

$$d_t = \frac{\varepsilon}{2} var_j\{p_{H,t}(j)\} + o\left(\left||a|\right|^3\right)$$

Proof: First, we need to define price dispersion. From the definition of domestic price index

$$P_{H,t} = \left(\int_0^1 P_{H,t}(j)^{1-\varepsilon} dj\right)^{\frac{1}{1-\varepsilon}} \rightarrow \ 1 = \left(\int_0^1 \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{1-\varepsilon} dj\right) \rightarrow 1$$
$$= e^{\left(\int_0^1 (1-\varepsilon)(p_{H,t}(j)-p_{H,t})dj\right)}$$

Now take second-order log linearize domestic price index around zero inflation level ($P_{H,t} = P_{H,t}(j) = P$):

$$1 = e^0 + e^0(1-\varepsilon)\int_0^1 \left(p_{H,t}(j) - p_{H,t}\right)dj$$
$$+ \frac{(1-\varepsilon)^2}{2}\int_0^1 \left(p_{H,t}(j) - p_{H,t}\right)^2 dj + o\left(\left||a|\right|^3\right) \rightarrow$$

$$p_{H,t} - \int_0^1 p_{H,t}(j)dj = \frac{(1-\varepsilon)}{2}\int_0^1 \left(p_{H,t}(j) - p_{H,t}\right)^2 dj + o\left(\left||a|\right|^3\right) \qquad (*)$$

Let take expectations from the expression (*), where E_j denotes the expectation operator with respect to good j, then I can get:

$$p_{H,t} = E_j\{p_{H,t}(j)\} + \frac{(1-\varepsilon)}{2}\int_0^1 E_j\{p_{H,t}(j) - p_{H,t}\}^2 dj + o\left(\left||a|\right|^3\right) \qquad (**)$$

Where $E_j\{p_{H,t}(j)\} \equiv \int_0^1 p_{H,t}(j)dj$ is the cross-sectional mean of logarithm term price. Lets consider following expression:

$$\int_0^1 \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} dj = \int_0^1 exp\{(-\varepsilon)\left(p_{H,t}(j) - p_{H,t}\right)\}dj$$
$$= 1 - \varepsilon\int_0^1 \left(p_{H,t}(j) - p_{H,t}\right)dj + \frac{\varepsilon^2}{2}\int_0^1 \left(p_{H,t}(j) - p_{H,t}\right)^2 dj$$
$$+ o\left(\left||a|\right|^3\right)$$

Inserting (*) into the above expression, we get:

$$\int_0^1 \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} dj$$
$$= 1 + \frac{\varepsilon(1-\varepsilon)}{2}\int_0^1 \left(p_{H,t}(j) - p_{H,t}\right)^2 dj + \frac{\varepsilon^2}{2}\int_0^1 \left(p_{H,t}(j) - p_{H,t}\right)^2 dj$$
$$+ o\left(\left||a|\right|^3\right) = 1 + \frac{\varepsilon}{2}\int_0^1 \left(p_{H,t}(j) - p_{H,t}\right)^2 dj + o\left(\left||a|\right|^3\right)$$

Let take expectations from the above expression,I can get:

$$E_j\left\{\int_0^1 \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} dj\right\} = 1 + \frac{\varepsilon}{2}\int_0^1 E_j\left\{p_{H,t}(j) - E_j\{p_{H,t}(j)\}\right\}^2 dj + o\left(\left||a|\right|^3\right)$$
$$= 1 + \frac{\varepsilon}{2}\cdot var_j\{p_{H,t}(j)\} + o\left(\left||a|\right|^3\right)$$

Finally using definition of d_t,

$$d_t \equiv log\left[\int_0^1 \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} dj\right] = \frac{\varepsilon}{2}\cdot var_j\{p_{H,t}(j)\} + o\left(\left||a|\right|^3\right) \qquad \text{(A.49)}$$

By combining (A.48) and (A.49), the employment gap from steady state can be expressed as:

$$\widehat{n_t} = n_t - n = (y_t - y) - (a_t - a) + (d_t - d) + t.i.p$$
$$= \widehat{y_t} - a_t + \frac{\varepsilon}{2}\cdot var_j\{p_{H,t}(j)\} + o\left(\left||a|\right|^3\right)$$

$$\widehat{n_t} = \widehat{y_t} - a_t + \frac{\varepsilon}{2}\cdot var_j\{p_{H,t}(j)\} + o\left(\left||a|\right|^3\right) + t.i.p \qquad \text{(A.50)}$$

By inserting (A.47) into (A.50), the expression utility can be represented by output gap and price variance:

$$\begin{cases} \dfrac{U_t - U}{U_C C} = (1-\alpha)\widehat{y_t} + \dfrac{U_N N}{U_C C}\left[\widehat{n_t} + \left(\dfrac{1+\varphi}{2}\right)\widehat{n_t}^2\right] + o\left(\left||a|\right|^3\right) + t.i.p \\ \widehat{n_t} = \widehat{y_t} - a_t + \dfrac{\varepsilon}{2}\cdot var_j\{p_{H,t}(j)\} + o\left(\left||a|\right|^3\right) + t.i.p \end{cases} \rightarrow$$

$$\frac{U_t - U}{U_C C} = (1-\alpha)\widehat{y_t}$$
$$+ \frac{U_N N}{U_C C}\left[\widehat{y_t} - a_t + \frac{\varepsilon}{2}\cdot var_j\{p_{H,t}(j)\}\right.$$
$$\left. + \left(\frac{1+\varphi}{2}\right)\left(\widehat{y_t} - a_t + \frac{\varepsilon}{2}\cdot var_j\{p_{H,t}(j)\}\right)^2\right] + o\left(\left||a|\right|^3\right) + t.i.p$$

$$\frac{U_t - U}{U_C C} = (1-\alpha)\widehat{y_t} + \frac{U_N N}{U_C C}\left[\widehat{y_t} + \frac{\varepsilon}{2} \cdot var_j\{p_{H,t}(j)\} + \left(\frac{1+\varphi}{2}\right)(\widehat{y_t} - a_t)^2\right]$$
$$+ o\left(||a||^3\right) + t.i.p$$

Under the optimal subsidy scheme assumed, the optimality condition hold:

$$\frac{U_N N}{U_c C} = (\alpha - 1)$$

Furthermore, by combining the optimality condition with utility deviations, we get:

$$\frac{U_t - U}{U_C C} = (1-\alpha)\widehat{y_t} - (1$$
$$- \alpha)\left[\widehat{y_t} + \frac{\varepsilon}{2} \cdot var_j\{p_{H,t}(j)\} + \left(\frac{1+\varphi}{2}\right)\left(\widehat{y_t}^2 - 2a_t\widehat{y_t} + a_t^2\right)\right]$$
$$+ o\left(||a||^3\right) + t.i.p$$

$$\frac{U_t - U}{U_C C} = -\frac{(1-\alpha)}{2}\left[\frac{\varepsilon}{2} \cdot var_j\{p_{H,t}(j)\} + (1+\varphi)\widehat{y_t}^2 - 2(1+\varphi)a_t\widehat{y_t}\right] + o\left(||a||^3\right) + t.i.p \qquad \text{(A.51)}$$

Natural level of output gap is defined as in our special case:

$$y_t^n = -\frac{\mu}{1+\varphi} + a_t \rightarrow \ \widehat{y_t^n} = y_t^n - y^n = \left(a_t - \frac{\mu}{1+\varphi}\right) - \left(0 - \frac{\mu}{1+\varphi}\right) = a_t$$

$$\widehat{y_t^n} = a_t \qquad \text{(A.52)}$$

Insert (A.52) into the (A.51):

$$\frac{U_t - U}{U_C C} = -\frac{(1-\alpha)}{2}\left[\varepsilon \cdot var_j\{p_{H,t}(j)\} + (1+\varphi)\widehat{y_t}^2 - 2(1+\varphi)\widehat{y_t^n}\widehat{y_t}\right] + o\left(||a||^3\right)$$
$$+ t.i.p \rightarrow$$

$$\frac{U_t - U}{U_C C} = -\frac{(1-\alpha)}{2}\left[\varepsilon \cdot var_j\{p_{H,t}(j)\} + (1+\varphi)\left(\widehat{y_t} - \widehat{y_t^n}\right)^2 - (1+\varphi)\widehat{y_t^n}^2\right]$$
$$+ o\left(||a||^3\right) + t.i.p \rightarrow$$

$$\frac{U_t - U}{U_C C} = -\frac{(1-\alpha)}{2}\left[\varepsilon \cdot var_j\{p_{H,t}(j)\} + (1+\varphi)\widetilde{y_t}^2\right] + o\left(||a||^3\right) + t.i.p \qquad \text{(A.53)}$$

We can rewrite (A.53) as discounted sum of overall period:

$$\sum_{t=0}^{\infty}\beta^t\left\{\frac{U_t-U}{U_C C}\right\}=-\frac{(1-\alpha)}{2}\sum_{t=0}^{\infty}\beta^t\left\{\frac{\varepsilon}{2}\cdot var_j\{p_{H,t}(j)\}+(1+\varphi)\tilde{y}_t^{\,2}\right\}+o\left(||a||^3\right)+t.i.p \qquad \text{(A.54)}$$

Lemma 2:

$\sum_{t=0}^{\infty}\beta^t\, var_j\{p_{H,t}(j)\}=\frac{1}{\lambda}\sum_{t=0}^{\infty}\beta^t\,\pi_{H,t}^2$, where $\lambda=\frac{(1-\theta)(1-\beta\theta)}{\theta}$

Proof:

Let denote $\overline{p_{H,t}}$ the price set by a domestic firm j adjusting its price with probability $(1-\theta)$ in period t. Thus, the expected value of the price level is

$$E_j\{p_{H,t}(j)\}=(1-\theta)\overline{p_{H,t}}+\theta E_j\{p_{H,t-1}(j)\}\rightarrow\overline{p_{H,t}}$$
$$=\frac{1}{(1-\theta)}E_j\{p_{H,t}(j)\}-\frac{\theta}{(1-\theta)}E_j\{p_{H,t-1}(j)\}\rightarrow$$

$$\overline{p_{H,t}}-E_j\{p_{H,t-1}(j)\}=\frac{1}{(1-\theta)}\left(E_j\{p_{H,t}(j)\}-E_j\{p_{H,t-1}(j)\}\right) \qquad (*)$$

Also, we can define price variance as:

$$var_j\{p_{H,t}(j)\}=varj\left\{p_{H,t}(j)-E_j\{p_{H,t-1}(j)\}\right\}$$
$$=E_j\left\{\left[p_{H,t}(j)-E_j\{p_{H,t-1}(j)\}\right]^2\right\}-\left[E_j\{p_{H,t}(j)\}-E_j\{p_{H,t-1}(j)\}\right]^2$$

$$var_j\{p_{H,t}(j)\}=E_j\left\{\left[p_{H,t}(j)-E_j\{p_{H,t-1}(j)\}\right]^2\right\}-\left[E_j\{p_{H,t}(j)\}-E_j\{p_{H,t-1}(j)\}\right]^2 \qquad (**)$$

Additionally, I can compute the following expectation by using Calvo type price model:

$$E_j\left\{\left[p_{H,t}(j)-E_j\{p_{H,t-1}(j)\}\right]^2\right\}=\left[\theta\cdot E_j\left\{\left(p_{H,t-1}(j)-E_j\{p_{H,t-1}(j)\}\right)^2\right\}+(1-\theta)\cdot\left(\overline{p_{H,t}}-E_j\{p_{H,t-1}(j)\}\right)^2\right] \qquad (***)$$

Insert (*) and (**) into the (**), I get:

$$var_j\{p_{H,t}(j)\}=\left[\theta\cdot E_j\left\{\left(p_{H,t-1}(j)-E_j\{p_{H,t-1}(j)\}\right)^2\right\}+(1-\theta)\left(\overline{p_{H,t}}-E_j\{p_{H,t-1}(j)\}\right)^2\right]-\left[E_j\{p_{H,t}(j)\}-E_j\{p_{H,t-1}(j)\}\right]^2\rightarrow$$

$var_j\{p_{H,t}(j)\} =$

$\theta \cdot E_j\left\{\left(p_{H,t-1}(j) - E_j\{p_{H,t-1}(j)\}\right)^2\right\} + (1-\theta)\left(\frac{(E_j\{p_{H,t}(j)\} - E_j\{p_{H,t-1}(j)\})}{1-\theta}\right)^2 -$

$\left[E_j\{p_{H,t}(j)\} - E_j\{p_{H,t-1}(j)\}\right]^2 \rightarrow$

$var_j\{p_{H,t}(j)\} = \theta \cdot E_j\left\{\left(p_{H,t-1}(j) - E_j\{p_{H,t-1}(j)\}\right)^2\right\} + \frac{1}{(1-\theta)}\left(\left(E_j\{p_{H,t}(j)\} -\right.\right.$

$\left.\left.E_j\{p_{H,t-1}(j)\}\right)\right)^2 - \left[E_j\{p_{H,t}(j)\} - E_j\{p_{H,t-1}(j)\}\right]^2 \rightarrow$

$$var_j\{p_{H,t}(j)\} = \theta E_j\left\{\left(p_{H,t-1}(j) - E_j\{p_{H,t-1}(j)\}\right)^2\right\} + \frac{\theta}{(1-\theta)}\left(\left(E_j\{p_{H,t}(j)\} - E_j\{p_{H,t-1}(j)\}\right)\right)^2 \rightarrow$$

$$var_j\{p_{H,t}(j)\} \approx \theta var_j\{p_{H,t-1}(j)\} + \frac{\theta}{(1-\theta)}\pi_{H,t}^2$$

If I write the last result in backward iteration, it yields:

$$var_j\{p_{H,t}(j)\} \approx \theta^2\left(\theta var_j\{p_{H,t-3}(j)\} + \frac{\theta}{(1-\theta)}\pi_{H,t-2}^2\right) + \frac{\theta^2}{(1-\theta)}\pi_{H,t-1}^2 + \frac{\theta}{(1-\theta)}\pi_{H,t}^2 \rightarrow$$

$$var_j\{p_{H,t}(j)\} = \theta^3 var_j\{p_{H,t-3}(j)\} + \frac{\theta^3}{(1-\theta)}\pi_{H,t-2}^2 + \frac{\theta^2}{(1-\theta)}\pi_{H,t-1}^2 + \frac{\theta}{(1-\theta)}\pi_{H,t}^2 \rightarrow$$

$$var_j\{p_{H,t}(j)\} = \theta^t var_j\{p_{H,0}(j)\} + \sum_{s=0}^{t-1}\theta^s\frac{\theta}{(1-\theta)}\pi_{H,t-s}^2$$

If $t \rightarrow \infty$ the parameter $\theta^t \rightarrow 0 \implies \theta^t var_j\{p_{H,0}(j)\} \approx 0$.

$$var_j\{p_{H,t}(j)\} \approx \sum_{s=0}^{t-1}\theta^s\frac{\theta}{(1-\theta)}\pi_{H,t-s}^2 \qquad \text{(A.55)}$$

So, if we take the discounted value of these terms over all periods:

$$\sum_{t=0}^{\infty} \beta^t \, var_j\{p_{H,t}(j)\} = \sum_{t=0}^{\infty} (\theta\beta)^t \frac{\theta}{(1-\theta)} \pi_{H,t}^2 = \frac{\theta}{(1-\theta)(1-\theta\beta)} \sum_{t=0}^{\infty} \beta^t \, \pi_{H,t}^2 \qquad \text{(A.56)}$$

Now insert (A.56) (result of lemma 2) into the (A.54):

$$\sum_{t=0}^{\infty} \beta^t \left\{\frac{U_t - U}{U_C C}\right\} = -\frac{(1-\alpha)}{2}\left[\frac{\varepsilon}{\lambda}\sum_{t=0}^{\infty} \beta^t \, \pi_{H,t}^2 + (1+\varphi)\sum_{t=0}^{\infty} \beta^t \, \tilde{y}_t^{\,2}\right] + o\left(||a||^3\right) + t.i.p$$

Now we can write the second order approximation to the utility losses of the domestic representative consumer resulting from deviations in optimal allocation (policy) level which is expressed as a fraction of steady state consumption

$$\boldsymbol{W} = -\frac{(1-\alpha)}{2}\left[\frac{\varepsilon}{\lambda}\sum_{t=0}^{\infty} \beta^t \, \pi_{H,t}^2 + (1+\varphi)\sum_{t=0}^{\infty} \beta^t \, \tilde{y}_t^{\,2}\right] \qquad \text{(A.57)}$$

The expected period welfare losses of any policy that deviates from strict inflation targeting can be written as

$$\boldsymbol{L} = -\frac{(1-\alpha)}{2}\left[\frac{\varepsilon}{\lambda} var(\pi_{H,t}) + (1+\varphi) var(\tilde{y}_t)\right] \qquad \text{(A.58)}$$

Appendix C: Content of Enclosed DVD

There is a DVD enclosed to this thesis which contains empirical data and MatLab and Stata source codes.

- Folder 1: Source codes
- Folder 2: Empirical data of labor productivity