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Tomáš Kadavý

Studium rezonancí v anomálním sektoru kvantové chromodynamiky

Ústav částicové a jaderné fyziky

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Tomáš Kadavý

On resonances in the anomalous sector of quantum chromodynamics

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Dedicated to my parents.

I declare that I carried out this bachelor thesis independently, and only with the cited sources, literature and other professional sources.

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Název práce: Studium rezonancí v anomálním sektoru kvantové chromodynamiky

Autor: Tomáš Kadavý

Katedra: Ústav částicové a jaderné fyziky

Vedoucí bakalářské práce: RNDr. Karol Kampf, Ph.D.

Abstrakt: V nedávné době byla ve článku [1] formulována báze operátorů pro nejdůležitější resonance v tzv. sektoru liché parity. Pomocí těchto operátorů můžeme zkonstruovat Lagrangiány odpovídajících Feynmanových diagramů, které nám dále umožňují popsat a pochopit různé interakční procesy. V tomto článku byly detailně studovány Greenovy funkce pro korelátory $\langle VVP \rangle$ a $\langle VAS \rangle$.
Tato práce by tak měla navázat na zmíněný článek a formulovat Feynmanova pravidla pro Greenovy funkce odpovídající dosud ne zcela známým korelátorům $\langle AAP \rangle$, $\langle VVA \rangle$ a $\langle AAA \rangle$. Jelikož těmto korelátorům nebylo dosud věnováno příliš pozornosti, a to jak teoretické, tak i experimentální, nebude možné blíže provést fenomenologickou analýzu, oproti článku, na který tato práce navazuje. Takové problematice pak může být věnována příslušná pozornost v některé z navazujících pracích během magisterského studia.

Klíčová slova: Kvantová chromodynamika, chirální poruchová teorie, resonanční chirální teorie.

Title: On resonances in the anomalous sector of quantum chromodynamics

Author: Tomáš Kadavý

Department: Institue of particle and nuclear physics

Supervisor: RNDr. Karol Kampf, Ph.D.

Abstract: The independent operator basis of the most important resonances in the so-called odd-intrinsic parity sector was formulated in [1]. Using these operators we can construct Lagrangians of corresponding Feynman diagrams, that allow us to describe and understand several interaction processes. In the mentioned article were studied Green functions for $\langle VVP \rangle$ and $\langle VAS \rangle$ correlators including their phenomenological aspects.
This thesis should establish the Feynman rules for not so much known Green functions corresponding to $\langle AAP \rangle$, $\langle VVA \rangle$ and $\langle AAA \rangle$ correlators. Because not enough theoretical or experimental attention has been paid to these correlators, we will not be able to perform phenomenological study. But we can return to this point in the future during master degree studies.

Keywords: Quantum chromodynamics, chiral perturbation theory, resonance chiral theory.

Contents

Introduction

Processes followed by rules of quantum chromodynamics can give us answers to fundamental questions of particle physics. However, understanding to these processes is very difficult because of the quark-gluon field properties, such as asymptotic freedom. This phenomena tells us that quarks are free on short distances but for increasing distances the mutual strength grows.

Because of that we need to use appropriate approaches based on the energy of studied processes. The methods we pay an attention to in this thesis are chiral perturbation theory and resonance chiral theory. Properties of both methods will be discussed later in details. For now let us just settle for the fact that using special Green functions in anomalous sector of QCD we can find a way to describe behaviour of some processes including, for example, rare decays of mesons which can be useful in determination of validity of some theories.

Chiral perturbation theory and resonance chiral theory offer several formalisms how to describe and calculate Green functions [2]. In this thesis we restrict ourselves to vector field formalism and antisymmetric tensor formalism.

The solution of any calculation should be independent on used formalism. However, because we work with perturbation theories, we calculate only up to some order and this makes different formalisms non-equivalent. Our task is then not only to find a solution but also to determine what formalism is more convenient in particular case.

We will start our calculations of $\langle AAP\rangle$ with vector formalism which is basically the most natural of all formalisms.

Motivation

The general motivation of this thesis is to finish systematic study of contributions of vector, axial-vector and pseudoscalar resonances into three-point Green functions. This thesis is supposed to be an extension of [1] for new correlators $\langle AAP\rangle$, $\langle VVA\rangle$ and $\langle AAA\rangle$ which have not been studied before, except for especially [3] in the case of $\langle AAP\rangle$ correlator.

Our intention is to calculate specific correlator in two different formalisms - antisymmetric tensor and vector field. Both formalisms start at different order which is important for satisfying OPE coupling constraints, as we will show later. The formalism, which satisfies these conditions better, gives us simplification of the particular calculated result.

Outline

This thesis has completely six chapters, including Conclusion and Appendix. In the first chapter we describe quantum chromodynamics (QCD), its basic properties and mathematical structures. We also pay an attention to different approaches to QCD, such as chiral perturbation theory and resonance chiral theory. Second chapter is dedicated to the description of resonances.

In the third chapter we study $\langle AAP\rangle$ correlator using two methods - vector field formalism, also known as the Proca field formalism, and antisymmetric tensor formalism. We also discuss behaviour of this correlator using OPE technique.

In the fourth chapter we show possible Lagrangians and Feynman diagrams contributing to $\langle VVA \rangle$ and $\langle AAA \rangle$ correlators. At the end of the thesis we sum up all obtained results for these correlators including final discussion.

The last chapter contains useful mathematical structures we have used in the thesis, including an example calculation of Feynman rules.

1. Quantum chromodynamics and chiral symmetry

Quantum chromodynamics (also known as QCD) is a quantum field theory of the strong interaction, a fundamental force describing the interactions between quarks and gluons. This theory is a special kind of a non-abelian gauge theory, consisting of a 'color field' mediated by a set of exchange particles - the gluons, and is based on $\mathrm{SU}(3)_C$ Yang–Mills theory of colour-charged fermions - the quarks.

QCD is an important part of the Standard model of particle physics and a lot of experimental evidences exist for its benefit.

Let us start this paper with basic introduction to mathematical structures of quantum chromodynamics, so we could build our calculations on this later.

It is well-known, that number of colours in the Standard model including QCD is $N_C = 3$. These colours are called red (r), green (g) and blue (b). We present the quark colour triplet as the basic building block

$$q_f = \begin{pmatrix} q_f^r \\ q_f^g \\ q_f^b \end{pmatrix}, \tag{1.1}$$

where f stands for flavour of the quark. In Standard model we have six types of flavour, according to the three two-quark generations, such as u, d, s, c, b, t.

The quark triplet transforms as

$$q_f \to U(x) q_f \tag{1.2}$$

where $U(x)$ is an element of $\mathrm{SU}(3)_C$ group.

We can write $\mathrm{SU}(3)_C$ invariant quark Lagrangian in the form [4]

$$\mathcal{L}_q = \sum_f \bar{q}_f (i\gamma^\mu \nabla_\mu - m_f) q_f, \tag{1.3}$$

where ∇_μ is the covariant derivative and

$$\nabla_\mu g_f = \partial_\mu q_f - iq\mathcal{A}_\mu(x) q_f, \tag{1.4}$$

where

$$\mathcal{A}_\mu(x) = \sum_{a=1}^{8} \frac{\lambda^a}{2} \mathcal{A}_\mu^a(x) \tag{1.5}$$

is the octet of $\mathrm{SU}(3)_C$ gauge fields. The octet transforms as

$$\mathcal{A}_\mu(x) \to U(x)\mathcal{A}_\mu(x)U^\dagger(x) - \frac{i}{g}\partial_\mu U(x) U^\dagger(x). \tag{1.6}$$

As an invariant object made of gluon fields we present nonabelian stress tensor [4]

$$\mathcal{G}_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c \tag{1.7}$$

with its transformation

$$\mathcal{G}_{\mu\nu} \to U(x)\mathcal{G}_{\mu\nu}U^\dagger(x). \tag{1.8}$$

The only nontrivial scalar, considering dimension ≤ 4, made of (1.3) and (1.7) is the contraction of two stress tensors. Thus we have the complete QCD Lagrangian [4]

$$\mathcal{L}_{QCD} = \sum_f \overline{q}_f (i\gamma^\mu \nabla_\mu - m_f) q_f - \frac{1}{4}\sum_{a=1}^{8} \mathcal{G}^a_{\mu\nu}\mathcal{G}^{a,\mu\nu}. \tag{1.9}$$

As we mentioned before, there are six quarks in the flavour sector. In the respect to their masses we can divide them into two parts consisting of quarks with masses less or greater than 1 GeV, which is so called hadron scale Λ_H. Schematically, we have

$$m_u, m_d, m_s \ll 1\,\mathrm{GeV} < m_c, m_b, m_t. \tag{1.10}$$

In the low-energy region only first three quarks are necessary to be taken into account. The approximation, based on the massless quarks, is called chiral limit. In this case (1.9) has the form [4]

$$\mathcal{L}^0_{QCD} = \sum_{f=u,d,s} \overline{q}_f i\gamma^\mu \nabla_\mu g_f - \frac{1}{4}\sum_{a=1}^{8} \mathcal{G}^a_{\mu\nu}\mathcal{G}^{a,\mu\nu}. \tag{1.11}$$

Massless QCD Lagrangian (1.11) is invariant under $SU(3)_C$ and even possesses $U(3)$ symmetry.

In order to exhibit the global symmetries of (1.11), we consider the chirality matrix γ_5, also known as the fifth Dirac matrix, and projection operators

$$P_L = \frac{1}{2}(1+\gamma_5), \qquad P_R = \frac{1}{2}(1-\gamma_5), \tag{1.12}$$

These operators satisfy their expected properties, such as idempotent

$$P_L^2 = P_L, \qquad P_R^2 = P_R, \tag{1.13}$$

orthogonality

$$P_L P_R = 0, \qquad P_R P_L = 0 \tag{1.14}$$

and completeness

$$P_L + P_R = 1. \tag{1.15}$$

The properties (1.13) - (1.15) guarantee that P_L and P_R project from the quark field q to its chiral components q_L and q_R,

$$q_L = P_L q, \qquad q_R = P_R q, \tag{1.16}$$

where

$$q = q_L + q_R. \tag{1.17}$$

In the respect to the properties of Dirac matrices (4.32), we can rewrite the massless QCD Lagrangian (1.11)

$$\mathcal{L}^0_{QCD} = \sum_{f=u,d,s} (\overline{q}_{R,f} i\gamma^\mu \nabla_\mu q_{R,f} + \overline{q}_{L,f} i\gamma^\mu \nabla_\mu q_{L,f}) - \frac{1}{4}\sum_{a=1}^{8} \mathcal{G}^a_{\mu\nu}\mathcal{G}^{a,\mu\nu}. \tag{1.18}$$

Now we can see that (1.11) is also invariant under the independent transformations of chiral components q_L and q_R,

$$q_L \to U_L q_L, \qquad q_R \to U_R q_R, \tag{1.19}$$

where U_L and U_R are unitary 3×3 matrices. We say that (1.18) has a classical global $U(3)_L \times U(3)_R$ symmetry.

The result of Noether's theorem is that there are 18 conserved currents associated with the mentioned transformations of left-handed and right-handed quarks

$$L^{a,\mu} = \overline{q}_L \gamma^\mu \frac{\lambda^a}{2} q_L, \qquad R^{a,\mu} = \overline{q}_R \gamma^\mu \frac{\lambda^a}{2} q_R \tag{1.20}$$

with

$$\partial_\mu L^{a,\mu} = 0, \qquad \partial_\mu R^{a,\mu} = 0. \tag{1.21}$$

Instead of (1.20) it is more useful to take into account its linear combinations

$$V^{a,\mu} = R^{a,\mu} + L^{a,\mu} = \overline{q} \gamma^\mu \frac{\lambda^a}{2} q \tag{1.22}$$

and

$$A^{a,\mu} = R^{a,\mu} - L^{a,\mu} = \overline{q} \gamma^\mu \gamma_5 \frac{\lambda^a}{2} q. \tag{1.23}$$

In addition to the vector (1.22) and axial-vector (1.23) currents we also define scalar density S^a

$$S^a = \overline{q} \frac{\lambda^a}{2} q \tag{1.24}$$

and pseudoscalar P^a density

$$P^a = i \overline{q} \gamma_5 \frac{\lambda^a}{2} q. \tag{1.25}$$

All mentioned currents and densities transform under parity transformations [4]

$$\begin{aligned} V^{a,\mu}(\mathbf{x}, t) \to V^a_\mu(-\mathbf{x}, t), \qquad & A^{a,\mu}(\mathbf{x}, t) \to -A^a_\mu(-\mathbf{x}, t) \\ S^a(\mathbf{x}, t) \to S^a(-\mathbf{x}, t), \qquad & P^a(\mathbf{x}, t) \to -P^a(-\mathbf{x}, t). \end{aligned} \tag{1.26}$$

Very useful is also to define external fields

$$\upsilon^\mu = \sum_{a=1}^{8} \frac{\lambda_a}{2} \upsilon^\mu_a, \qquad a^\mu = \sum_{a=1}^{8} \frac{\lambda_a}{2} a^\mu_a, \qquad s = \sum_{a=1}^{8} \frac{\lambda_a}{2} s_a, \qquad p = \sum_{a=1}^{8} \frac{\lambda_a}{2} p_a \tag{1.27}$$

that in the flavour sector are represented by Hermitian 3×3 matrices. Coupling vector and axial-vector currents to external fields we can get

$$\begin{aligned} \mathcal{L} &= \mathcal{L}^0_{QCD} + \mathcal{L}_{ext} = \\ &= \mathcal{L}^0_{QCD} + \overline{q} \gamma_\mu \left(\upsilon^\mu + \frac{1}{3} \upsilon^\mu_{(s)} + \gamma_5 a^\mu \right) q - \overline{q}(s - i\gamma_5 p) q. \end{aligned} \tag{1.28}$$

If we define vector and axial-vector currents in the form

$$\upsilon^\mu = \frac{1}{2}(r^\mu + l^\mu), \qquad a^\mu = \frac{1}{2}(r^\mu - l^\mu), \tag{1.29}$$

Lagrangian (1.28) has now the form

$$\mathcal{L} = \mathcal{L}^0_{QCD} + \overline{q}_L\gamma^\mu\left(l_\mu + \frac{1}{3}\upsilon^s_\mu\right)q_L + \overline{q}_R\gamma^\mu\left(r_\mu + \frac{1}{3}\upsilon^s_\mu\right)q_R - \\ - \overline{q}_R(s+ip)q_L - \overline{q}_L(s-ip)q_R. \tag{1.30}$$

This Lagrangian is invariant under the local group $SU(3)_L \times SU(3)_R \times U(1)_V$.

Depending on energy region, we have several approaches to QCD. First of them is Chiral perturbation theory.

1.1 Chiral perturbation theory

Spontaneous symmetry breaking in QCD leads to the presence of Goldstone bosons [4]. Identifying them with the octet of pseudoscalar mesons, as the lightest hadrons, we can construct so called Chiral perturbation theory (χPT), which is an effective theory for QCD in low-energy region, especially for energies less than 1 GeV.

According to (1.30) and its invariance, the eight pseudoscalar mesons transform as an octet under the subgroup $SU(3)_V$. The basic building block of χPT is [1]

$$u(\phi) = \exp\left(\frac{i}{\sqrt{2}F}\phi\right), \tag{1.31}$$

where

$$\phi = \frac{1}{\sqrt{2}}\phi^a\lambda^a \tag{1.32}$$

and

$$\phi(x) = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \overline{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 \end{pmatrix} \tag{1.33}$$

is the matrix of pseudoscalar meson fields.

In Chiral perturbation theory we assume that external momenta p is much smaller than an energy scale, which is typically $\Lambda \approx 1\,\text{GeV}$. After that, we can write an expansion of the Lagrangian in terms of p in the form

$$\mathcal{L} = \mathcal{L}^{(2)}_\chi + \mathcal{L}^{(4)}_\chi + \mathcal{L}^{(6)}_\chi + \ldots, \tag{1.34}$$

where we already considered symmetry conditions and some of the contributing terms. In (1.34) $\mathcal{L}^{(n)}$ stands for a part of the Lagrangian of the n-th order, i.e. $\mathcal{L}^{(n)} = \mathcal{O}^{(n)}$.

The Lagrangian of the lowest term is [1]

$$\mathcal{L}^{(2)}_\chi = \frac{F^2}{4}\langle u_\mu u^\mu + \chi_+\rangle, \tag{1.35}$$

where we have defined following covariant tensors

$$u_\mu = u^\dagger_\mu = i\left[u^\dagger(\partial_\mu - ir_\mu)u - u(\partial_\mu - i\ell_\mu)u^\dagger\right] \tag{1.36}$$

and

$$\chi_\pm = u^\dagger\chi u^\dagger \pm u\chi^\dagger u, \tag{1.37}$$

where

$$\chi = 2B_0(s + ip). \tag{1.38}$$

We have also defined

$$f_{\pm}^{\mu\nu} = uF_L^{\mu\nu}u^\dagger \pm u^\dagger F_R^{\mu\nu} u \tag{1.39}$$

with

$$F_R^{\mu\nu} = \partial^\mu r^\nu - \partial^\nu r^\mu - i\,[r^\mu, r^\nu]\,, \qquad F_L^{\mu\nu} = \partial^\mu l^\nu - \partial^\nu l^\mu - i\,[l^\mu, l^\nu] \tag{1.40}$$

and

$$h_{\mu\nu} = \nabla_\mu u_\nu + \nabla_\nu u_\mu. \tag{1.41}$$

The covariant derivative is defined by

$$\nabla_\mu X = \partial_\mu X + [\Gamma_\mu, X]\,, \tag{1.42}$$

where the chiral connection is

$$\Gamma_\mu = \frac{1}{2}\left[u^\dagger(\partial_\mu - ir_\mu)u + u(\partial_\mu - i\ell_\mu)u^\dagger\right]. \tag{1.43}$$

Because in the fourth chapter we will be interested only in the first terms of previous chiral building blocks and covariant tensors, we can simplify (1.31) and (1.36) - (1.43) as follows

$$u(\phi) \sim \mathbf{1} + \frac{i}{\sqrt{2}F}\phi, \qquad u^\dagger(\phi) \sim \mathbf{1} - \frac{i}{\sqrt{2}F}\phi, \tag{1.44}$$

$$\partial_\mu u(\phi) \sim \frac{i}{\sqrt{2}F}\partial_\mu\phi, \qquad \partial_\mu u(\phi) \sim -\frac{i}{\sqrt{2}F}\partial_\mu\phi, \tag{1.45}$$

$$u_\mu = u_\mu^\dagger \sim -\frac{\sqrt{2}}{F}\partial_\mu\phi, \tag{1.46}$$

$$u_\mu = u_\mu^\dagger \sim \begin{cases} -\dfrac{\sqrt{2}}{F}\partial_\mu\phi & \text{if } r_\mu = l_\mu = 0 \\ 2a_\mu & \text{if } u = u^\dagger = \mathbf{1}. \end{cases} \tag{1.47}$$

The difference in $u = u^\dagger$ in comparison to (1.31) depends on specific situation and we will discuss it later. Next we have

$$\chi_+ \sim \frac{2\sqrt{2}B_0}{F}\{\phi, p\}, \qquad \chi_- \sim 4B_0 ip, \tag{1.48}$$

$$f_+^{\mu\nu} \sim \frac{2\sqrt{2}i}{F}[\partial^\mu a^\nu - \partial^\nu a^\mu, \phi], \qquad f_-^{\mu\nu} \sim -2\big(\partial^\mu a^\nu - \partial^\nu a^\mu\big), \tag{1.49}$$

and

$$F_R^{\mu\nu} \sim \partial^\mu a^\nu - \partial^\nu a^\mu, \qquad F_L^{\mu\nu} \sim -\partial^\mu a^\nu + \partial^\nu a^\mu. \tag{1.50}$$

Now we also have $\nabla_\mu X = \partial_\mu X$, so

$$h_{\mu\nu} \sim \begin{cases} -\dfrac{\sqrt{2}}{F}\big(\partial_\mu\partial_\nu\phi + \partial_\nu\partial_\mu\phi\big) & \text{if } r_\mu = l_\mu = 0 \\ 2(\partial_\mu a_\nu + \partial_\nu a_\mu) & \text{if } u = u^\dagger = \mathbf{1}. \end{cases} \tag{1.51}$$

1.2 Resonance chiral theory

Taking number of colours in the limit $N_C \to \infty$ we can construct the effective theory of QCD for intermediate energy region that also satifies all symetries in the underlying theory [5]. This effective theory is called Resonance chiral theory (RχT) and is relevant for energies $1\,\mathrm{GeV} \leq E \leq 2\,\mathrm{GeV}$. For bigger energies R$\chi$T looses its convergence and can not be properly used because of the higher masses that become significant in hadron dynamics.

Up to the order $\mathcal{O}(p^6)$ we can write Lagrangian in the form

$$\begin{aligned}\mathcal{L}_{R\chi T} &= \mathcal{L}_{GB} + \mathcal{L}_{res} = \\ &= \mathcal{L}_{GB}^{(2)} + \mathcal{L}_{GB}^{(4)} + \mathcal{L}_{GB}^{(6)} + \mathcal{L}_{res}^{(4)} + \mathcal{L}_{res}^{(6)},\end{aligned} \tag{1.52}$$

where $\mathcal{L}_{GB}$ contains only Goldstone bosons and $\mathcal{L}_{res}$ is resonance Lagrangian.

On the other hand, after integrating out resonance fields from (1.52) and expanding to the given chiral order, we can get an effective χPT Lagrangian (1.34). Up to the order $\mathcal{O}(p^6)$ we can write

$$\begin{aligned}\mathcal{L}_{\chi PT} &= \mathcal{L}_{GB} + \mathcal{L}_{\chi,eff} = \\ &= \mathcal{L}_{GB}^{(2)} + \mathcal{L}_{GB}^{(4)} + \mathcal{L}_{GB}^{(6)} + \mathcal{L}_{\chi,eff}^{(4)} + \mathcal{L}_{\chi,eff}^{(6)}.\end{aligned} \tag{1.53}$$

That means

$$\mathcal{L}_{\chi PT}^{(2)} = \mathcal{L}_{GB}^{(2)}, \tag{1.54}$$

$$\mathcal{L}_{\chi PT}^{(4)} = \mathcal{L}_{GB}^{(4)} + \mathcal{L}_{\chi,R}^{(4)}, \tag{1.55}$$

$$\mathcal{L}_{\chi PT}^{(6)} = \mathcal{L}_{GB}^{(6)} + \mathcal{L}_{\chi,R}^{(6)}. \tag{1.56}$$

Resonance chiral theory increases the number of degrees of freedom of Chiral perturbation theory by including massive multiplets of vector $V(1^{--})$, axial-vector $A(1^{++})$, scalar $S(0^{++})$ and pseudoscalar $P(0^{-+})$ resonances, denoted generically as a nonet field R. We assume that this field can be decomposed into singlet R_0 and octet R_8 such as

$$R = \frac{1}{\sqrt{3}} R_0 + \sum_i \frac{\lambda_i}{\sqrt{2}} R_i, \tag{1.57}$$

where $R = V, A, S, P$.

In this thesis we are interested only in cases of $R = V, A, P$ so we can write specifically

$$V_{\mu\nu} = \begin{pmatrix} \frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{6}}\omega_8 + \frac{1}{\sqrt{3}}\omega_1 & \rho^+ & K^{*+} \\ \rho^- & -\frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{6}}\omega_8 + \frac{1}{\sqrt{3}}\omega_1 & K^{*0} \\ K^{*-} & \overline{K}^{*0} & -\frac{1}{\sqrt{6}}\omega_8 + \frac{1}{\sqrt{3}}\omega_1 \end{pmatrix}_{\mu\nu} \tag{1.58}$$

and

$$A_{\mu\nu} = \begin{pmatrix} \frac{1}{\sqrt{2}}a_1^0 + \frac{1}{\sqrt{6}}f_1^8 + \frac{1}{\sqrt{3}}f_1^1 & a_1^+ & K_{1A}^+ \\ a_1^- & -\frac{1}{\sqrt{2}}a_1^0 + \frac{1}{\sqrt{6}}f_1^8 + \frac{1}{\sqrt{3}}f_1^1 & K_{1A}^0 \\ K_{1A}^- & \overline{K}_{1A}^0 & -\frac{2}{\sqrt{6}}f_1^8 + \frac{1}{\sqrt{3}}f_1^1 \end{pmatrix}_{\mu\nu}. \tag{1.59}$$

Both octets have the same form in the antisymmetric tensor and vector formalism.

Resonance fields $R_{\mu\nu}$ transform under the following symmetries [4], [5], [7]

$$\begin{aligned} \mathrm{P} &: R^{\mu\nu} \longmapsto R^{\mu\nu}, \\ \mathrm{C} &: R^{\mu\nu} \longmapsto -R^{\mathrm{T}}_{\mu\nu}, \\ \mathrm{h.c.} &: R^{\mu\nu} \longmapsto R_{\mu\nu}. \end{aligned} \tag{1.60}$$

In the leading order in $1/N_C$ the interaction resonance Lagrangian has the form [1]

$$\begin{aligned} \mathcal{L}_R^{(4)} &= c_d\langle S u^\mu u_\mu\rangle + c_m\langle S\chi_+\rangle + id_m\langle P\chi_-\rangle + \frac{id_{m0}}{N_F}\langle P\rangle\langle\chi_-\rangle + \\ &+ \frac{F_V}{2\sqrt{2}}\langle V_{\mu\nu} f_+^{\mu\nu}\rangle + \frac{iG_V}{2\sqrt{2}}\langle V_{\mu\nu}[u^\mu, u^\nu]\rangle + \frac{F_A}{2\sqrt{2}}\langle A_{\mu\nu} f_-^{\mu\nu}\rangle \end{aligned} \tag{1.61}$$

and is invariant under symmetries (1.60). Of course, for our purposes we do not consider terms with scalar field S in (1.61).

2. Description of resonances

2.1 Wess-Zumino-Witten Lagrangian

The leading order of odd-intrinsic parity sector of resonance chiral theory starts at $\mathcal{O}(p^4)$ and the parameters are set only by the chiral anomaly. The Lagrangian is defined as [8]

$$\mathcal{L}_{WZW} = \frac{N_C}{48\pi^2}\left[\int_0^1 \mathrm{d}\xi \langle \sigma_\mu^\xi \sigma_\nu^\xi \sigma_\alpha^\xi \sigma_\beta^\xi \frac{\phi}{F}\rangle - i\langle W_{\mu\nu\alpha\beta}(U,l,r) - W_{\mu\nu\alpha\beta}(1,l,r)\rangle\right]\varepsilon^{\mu\nu\alpha\beta} \tag{2.1}$$

$$\simeq -\frac{iN_C}{48\pi^2}\langle W_{\mu\nu\alpha\beta}(U,l,r) - W_{\mu\nu\alpha\beta}(1,l,r)\rangle\varepsilon^{\mu\nu\alpha\beta}$$

with [1],[9]

$$\begin{aligned} W_{\mu\nu\alpha\beta} = {} & L_\mu L_\nu L_\alpha R_\beta + \frac{1}{4}L_\mu R_\nu L_\alpha R_\beta + iL_{\mu\nu}L_\alpha R_\beta + iR_{\mu\nu}L_\alpha R_\beta - \\ & - i\sigma_\mu L_\nu R_\alpha L_\beta + \sigma_\mu R_{\nu\alpha}L_\beta - \sigma_\mu\sigma_\nu R_\alpha L_\beta + \sigma_\mu L_\nu L_{\alpha\beta} + \\ & + \sigma_\mu L_{\nu\alpha}L_\beta - i\sigma_\mu L_\nu L_\alpha L_\beta + \frac{1}{2}\sigma_\mu L_\nu \sigma_\alpha L_\beta - i\sigma_\mu\sigma_\nu\sigma_\alpha L_\beta - \\ & - (L \leftrightarrow R)\,, \end{aligned} \tag{2.2}$$

where we have defined

$$\begin{aligned} L_\mu &= u\, l_\mu u^\dagger\,, \qquad & L_{\mu\nu} &= u\,\partial_\mu l_\nu u^\dagger, \\ R_\mu &= u^\dagger r_\mu u\,, \qquad & R_{\mu\nu} &= u\,\partial_\mu r_\nu u^\dagger \end{aligned} \tag{2.3}$$

and

$$\sigma_\mu = \{u^\dagger,\, \partial_\mu u\} \tag{2.4}$$

In (2.1) the power ξ stands for a change of u to $u^\xi = \exp(i\xi\phi/(F\sqrt{2}))$ and in (2.2) $(L \leftrightarrow R)$ stands also for $\sigma \leftrightarrow \sigma^\dagger$ interchange.

2.2 Vector formalism

First method we will use to describe resonances is vector formalism.

Vector fields describing the vector and axial vector resonances are labeled as $\widehat{V}_\mu$ and $\widehat{A}_\mu$. Building blocks are partially the same as in previous chapter, but we now have different notation, such as [3]

$$\widehat{R}_\mu = \sum_{a=1}^{8} \frac{\lambda^a}{\sqrt{2}}\widehat{R}^a_\mu, \tag{2.5}$$

and

$$\widehat{R}_{\mu\nu} = \nabla_\mu \widehat{R}_\nu - \nabla_\nu \widehat{R}_\mu. \tag{2.6}$$

The resonance Lagrangian in the vector formalism is given by [3]

$$\mathcal{L}_{res} = \mathcal{L}_V + \mathcal{L}_A + \mathcal{L}^{(2)}_{VV} + \mathcal{L}^{(2)}_{AA} + \mathcal{L}^{(2)}_{VA} \tag{2.7}$$

where individual terms are as following

$$\begin{aligned}\mathcal{L}_V =& -\frac{1}{4}\langle \widehat{V}_{\mu\nu}\widehat{V}^{\mu\nu} - 2M_V^2\widehat{V}_\mu\widehat{V}^\mu\rangle - \frac{1}{2\sqrt{2}}\big(f_V\langle \widehat{V}_{\mu\nu}f_+^{\mu\nu}\rangle + ig_V\langle \widehat{V}_{\mu\nu}[u^\mu, u^\nu]\rangle\big)+\\ &+ i\alpha_V\langle \widehat{V}_\mu[u_\nu, f_-^{\mu\nu}]\rangle + \beta_V\langle \widehat{V}_\mu[u^\mu, \chi_-]\rangle + i\theta_V\langle \widehat{V}^\mu u^\nu u^\alpha u^\beta\rangle\varepsilon_{\mu\nu\alpha\beta}+\\ &+ h_V\langle \widehat{V}^\mu\{u^\nu, f_+^{\alpha\beta}\}\rangle\varepsilon_{\mu\nu\alpha\beta},\end{aligned} \tag{2.8}$$

$$\begin{aligned}\mathcal{L}_A =& -\frac{1}{4}\langle \widehat{A}_{\mu\nu}\widehat{A}^{\mu\nu} - 2M_A^2\widehat{A}_\mu\widehat{A}^\mu\rangle - \frac{1}{2\sqrt{2}}f_A\langle \widehat{A}_{\mu\nu}f_-^{\mu\nu}\rangle + i\alpha_A\langle \widehat{A}_\mu[u_\nu, f_+^{\mu\nu}]\rangle+\\ &+ \gamma_A^{(1)}\langle \widehat{A}_\mu u_\nu u^\mu u^\nu\rangle + \gamma_A^{(2)}\langle \widehat{A}_\mu\{u^\mu, u^\nu u_\nu\}\rangle + \gamma_A^{(3)}\langle \widehat{A}_\mu u_\nu\rangle\langle u^\mu u^\nu\rangle+\\ &+ \gamma_A^{(4)}\langle \widehat{A}_\mu u^\mu\rangle\langle u^\nu u_\nu\rangle + h_A\langle \widehat{A}^\mu\{u^\nu, f_-^{\alpha\beta}\}\rangle\varepsilon_{\mu\nu\alpha\beta},\end{aligned} \tag{2.9}$$

$$\begin{aligned}\mathcal{L}^{(2)}_{VV} =& \frac{1}{2}\delta_V^{(1)}\langle \widehat{V}_\mu\widehat{V}^\mu u_\nu u^\nu\rangle + \frac{1}{2}\delta_V^{(2)}\langle \widehat{V}_\mu u_\nu\widehat{V}^\mu u^\nu\rangle + \frac{1}{2}\delta_V^{(3)}\langle \widehat{V}_\mu\widehat{V}_\nu u^\mu u^\nu\rangle+\\ &+ \frac{1}{2}\delta_V^{(4)}\langle \widehat{V}_\mu\widehat{V}_\nu u^\mu u^\nu\rangle + \frac{1}{2}\delta_V^{(5)}\langle \widehat{V}_\mu u^\mu\widehat{V}_\nu u^\nu + \widehat{V}_\mu u_\nu\widehat{V}^\nu u^\mu\rangle+\\ &+ \frac{1}{2}\kappa_V\langle \widehat{V}_\mu\widehat{V}^\mu\chi_+\rangle + \frac{1}{2}i\phi_V\langle \widehat{V}_\mu[\widehat{V}_\nu, f_+^{\mu\nu}]\rangle + \frac{1}{2}\sigma_V\langle \widehat{V}^\mu\{u^\nu, \widehat{V}^{\alpha\beta}\}\rangle\varepsilon_{\mu\nu\alpha\beta},\end{aligned} \tag{2.10}$$

$$\begin{aligned}\mathcal{L}^{(2)}_{AA} =& \frac{1}{2}\delta_A^{(1)}\langle \widehat{A}_\mu\widehat{A}^\mu u_\nu u^\nu\rangle + \frac{1}{2}\delta_A^{(2)}\langle \widehat{A}_\mu u_\nu\widehat{A}^\mu u^\nu\rangle + \frac{1}{2}\delta_A^{(3)}\langle \widehat{A}_\mu\widehat{A}_\nu u^\mu u^\nu\rangle+\\ &+ \frac{1}{2}\delta_A^{(4)}\langle \widehat{A}_\mu\widehat{A}_\nu u^\mu u^\nu\rangle + \frac{1}{2}\delta_A^{(5)}\langle \widehat{A}_\mu u^\mu\widehat{A}_\nu u^\nu + \widehat{A}_\mu u_\nu\widehat{A}^\nu u^\mu\rangle+\\ &+ \frac{1}{2}\kappa_A\langle \widehat{A}_\mu\widehat{A}^\mu\chi_+\rangle + \frac{1}{2}i\phi_A\langle \widehat{A}_\mu[\widehat{A}_\nu, f_+^{\mu\nu}]\rangle + \frac{1}{2}\sigma_A\langle \widehat{A}^\mu\{u^\nu, \widehat{A}^{\alpha\beta}\}\rangle\varepsilon_{\mu\nu\alpha\beta}\end{aligned} \tag{2.11}$$

and finally

$$\begin{aligned}\mathcal{L}^{(2)}_{VA} =& iA^{(1)}\langle \widehat{V}_\mu[\widehat{A}_\nu, f_-^{\mu\nu}]\rangle + iA^{(2)}\langle \widehat{V}_\mu[u_\nu, \widehat{A}^{\mu\nu}]\rangle + iA^{(3)}\langle \widehat{A}_\mu[u_\nu, \widehat{V}^{\mu\nu}]\rangle+\\ &+ B\langle \widehat{V}_\mu[\widehat{A}^\mu, \chi_-]\rangle + H\langle \widehat{V}^\mu\{\widehat{A}^\nu, f_+^{\alpha\beta}\}\rangle\varepsilon_{\mu\nu\alpha\beta} + iZ^{(1)}\langle u^\mu u^\nu\{\widehat{A}^\alpha, \widehat{V}^\beta\}\rangle\varepsilon_{\mu\nu\alpha\beta}+\\ &+ iZ^{(2)}\langle u^\mu\widehat{A}^\nu u^\alpha\widehat{V}^\beta\rangle\varepsilon_{\mu\nu\alpha\beta}.\end{aligned} \tag{2.12}$$

2.3 Antisymmetric tensor formalism

In the antisymmetric tensor formalism, we will use the independent operator basis, relevant in the odd-intrinsic parity sector, which was formulated in [1]. Every operator has the form

$$\mathcal{O}_i^X = \varepsilon^{\mu\nu\alpha\beta}\widehat{\mathcal{O}}^X_{i\,\mu\nu\alpha\beta}, \tag{2.13}$$

thus the Lagrangian is

$$\mathcal{L}^{(6,\mathrm{odd})}_{R\chi T} = \sum_X\sum_i \kappa_i^X\mathcal{O}_i^X, \tag{2.14}$$

where X stands for resonance fields contributing to the Lagrangian. For our purposes we only show operators contributing to $\langle AAP\rangle$, $\langle VVA\rangle$ and $\langle AAA\rangle$ correlators without further discussion. We will also use strictly the same notation as in [1] and consider this as a referent basis, especially with indices i standing for a serial number of the Lagrangian. For more details see [1].

3. $\langle AAP \rangle$ correlator

The standard definition of this correlator is

$$\Pi^{abc}_{\mu\nu}(p,q) = \int \mathrm{d}^4x\, \mathrm{d}^4y\, e^{i(px+qy)}\, \langle 0|T[A^a_\mu(x)A^b_\nu(y)P^c(0)]|0\rangle. \tag{3.1}$$

3.1 $\langle AAP \rangle$ correlator in vector formalism

3.1.1 Lagrangian basis

For our cases of three-point correlators we will need modificated Lagrangians (2.8) - (2.12), such as:

One axial-vector resonance field (A)

$$\mathcal{L}^I_A = -\frac{1}{2\sqrt{2}} f_A \langle \widehat{A}_{\mu\nu} f^{\mu\nu}_- \rangle \sim \frac{1}{2}(\partial_\mu \widehat{A}^a_\nu - \partial_\nu \widehat{A}^a_\mu)(\partial^\mu a^\nu_a - \partial^\nu a^\mu_a), \tag{3.2}$$

$$\begin{aligned}\mathcal{L}^{II}_A &= h_A \langle \widehat{A}^\mu \{u^\nu, f^{\alpha\beta}_-\}\rangle \varepsilon_{\mu\nu\alpha\beta} \sim \\ &\sim \frac{2\sqrt{2}h_A}{F}\left[\widehat{A}^\mu_a(\partial^\alpha a^\beta_b - \partial^\beta a^\alpha_b) + \widehat{A}^\mu_b(\partial^\alpha a^\beta_a - \partial^\beta a^\alpha_a)\right]\partial^\nu \phi_c \varepsilon_{\mu\nu\alpha\beta} d^{abc}.\end{aligned} \tag{3.3}$$

Two axial-vector resonance fields (AA)

$$\begin{aligned}\mathcal{L}^{(2)}_{AA} &= \frac{1}{2}\sigma_A \langle \widehat{A}^\mu \{u^\nu, \widehat{A}^{\alpha\beta}\}\rangle \varepsilon_{\mu\nu\alpha\beta} \sim \\ &\sim -\frac{\sigma_A}{F}\left[\widehat{A}^\mu_a(\partial^\alpha \widehat{A}^\beta_b - \partial^\beta \widehat{A}^\alpha_b) + \widehat{A}^\mu_b(\partial^\alpha \widehat{A}^\beta_a - \partial^\beta \widehat{A}^\alpha_a)\right]\partial^\nu \phi_c \varepsilon_{\mu\nu\alpha\beta} d^{abc}.\end{aligned} \tag{3.4}$$

3.1.2 Feynman rules

Vector propagator

Vector propagator is formed by kinetic and mass terms [6].

$$i\Delta_R(p)^{ab}_{\mu\nu} = -\frac{i}{p^2 - m^2}\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{m^2}\right)\delta^{ab} \tag{3.5}$$

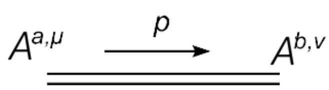

Figure 3.1: Feynman diagram of vector propagator

Pseudoscalar propagator

The kinetic term comes from $\mathcal{L}^{(2)}_\chi$. Pseudoscalar mesons are massless in chiral limit [6].

$$i\Delta_P(r) = \frac{i}{r^2}\delta^{ab}. \tag{3.6}$$

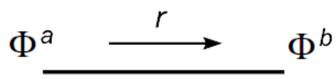

Figure 3.2: Feynman diagram of pseudoscalar propagator

Vertex χ (pseudoscalar source - pseudoscalar)

To this vertex contributes $\mathcal{L}_\chi^{(2)}$. The Feynman rule is

$$(V_\chi)^{cd} = iFB_0\delta^{cd}. \tag{3.7}$$

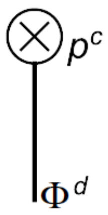

Figure 3.3: Feynman diagram of vertex χ

Vertex WZ (axial source - axial source - pseudoscalar)

To this vertex contributes the anomaly Wess-Zumino-Witten Lagrangian. The Feynman rule is

$$(V'_{WZ})^{abe}_{\mu\nu} = -i\frac{N_C}{48\pi^2 F}\varepsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta d^{abe}. \tag{3.8}$$

Due to Bose statistic we get

$$(V_{WZ})^{abe}_{\mu\nu} = -i\frac{N_C}{24\pi^2 F}\varepsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta d^{abe}. \tag{3.9}$$

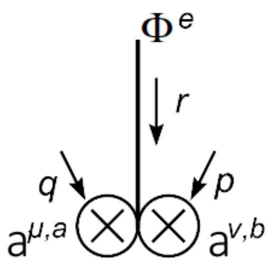

Figure 3.4: Feynman diagram of vertex WZ

Important note is that vertices χ and WZ have the same form even in the antisymmetric tensor formalism.

Vertex 1 (axial source - resonance)

To this vertex contributes Lagrangian $\mathcal{L}_A^I$. The Feynman rule is

$$(V_1)^{ad}_{\mu\alpha} = if_A(p^2 g_{\mu\alpha} - p_\mu p_\alpha)\delta^{ad}. \tag{3.10}$$

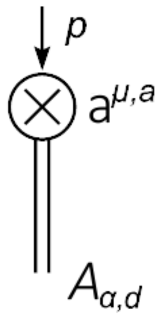

Figure 3.5: Feynman diagram of vertex 1

Vertex 2 (resonance - resonance - pseudoscalar)

To this vertex contributes Lagrangian $\mathcal{L}^{(2)}_{AA}$. The Feynman rule is

$$(V_2)^{def}_{\rho\sigma} = i\frac{\sigma_A}{F} r^\kappa (p^\alpha \varepsilon_{\sigma\kappa\alpha\rho} - p^\beta \varepsilon_{\sigma\kappa\rho\beta} + q^\alpha \varepsilon_{\rho\kappa\alpha\sigma} - q^\beta \varepsilon_{\rho\kappa\sigma\beta}) d^{def}. \quad (3.11)$$

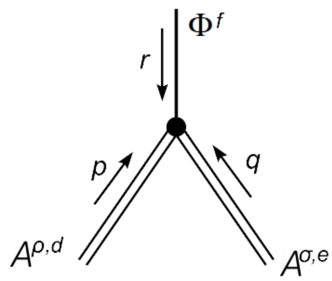

Figure 3.6: Feynman diagram of vertex 2

Vertex 3 (axial source - resonance - pseudoscalar)

To this vertex contributes Lagrangian $\mathcal{L}^{II}_{A}$. The Feynman rule is

$$(V_3^1)^{aed}_{\mu\alpha} = -\frac{2i\sqrt{2}h_A}{F} r^\nu (p^\kappa \varepsilon_{\alpha\nu\kappa\mu} - p^\beta \varepsilon_{\alpha\nu\mu\beta}) d^{aed}, \quad (3.12)$$

$$(V_3^2)^{fbd}_{\nu\beta} = -\frac{2i\sqrt{2}h_A}{F} r^\kappa (q^\alpha \varepsilon_{\beta\kappa\alpha\nu} - q^\lambda \varepsilon_{\beta\kappa\nu\lambda}) d^{fbd}. \quad (3.13)$$

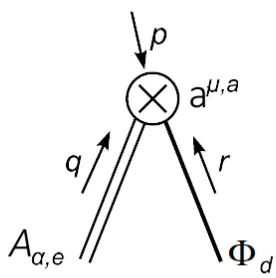

Figure 3.7: Feynman diagram of vertex 3 - first variant

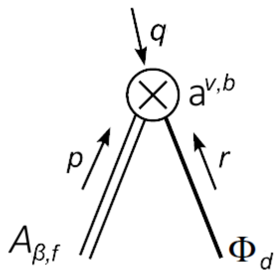

Figure 3.8: Feynman diagram of vertex 3 - second variant

Subdiagram 1 (vertex 1 - vector propagator)

This subdiagram consists of vertex 1 and vector propagator. The Feynman rule is

$$(S_1(p))^{af}_{\mu\beta} = (V_1)^{ad}_{\mu\alpha} i(\Delta_V(p))^{\alpha\beta,df} = \frac{f_A}{p^2 - m}(p^2 g_{\mu\beta} - p_\mu p_\beta)\delta^{af}. \qquad (3.14)$$

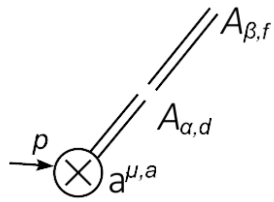

Figure 3.9: Feynman subdiagram 1

Subdiagram χ (vertex χ - pseudoscalar)

This subdiagram consists of vertex χ and pseudoscalar propagator. The Feynman rule is

$$(S_\chi)^{ce} = (V_\chi)^{cd} i\big(\Delta_P(r)\big)^{de} = -\frac{FB_0}{r^2}\delta^{ce}. \qquad (3.15)$$

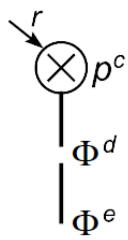

Figure 3.10: Feynman subdiagram χ

3.1.3 Feynman diagrams

Diagram χ

This diagram consists of one subdiagram and one vertex. The Feynman rule is

$$(\Pi_\chi)^{abc}_{\mu\nu} = (S_\chi)^{cd}(V_{WZ})^{abd}_{\mu\nu} = i\frac{B_0 N_C}{24\pi^2 r^2}\varepsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta d^{abc} \qquad (3.16)$$

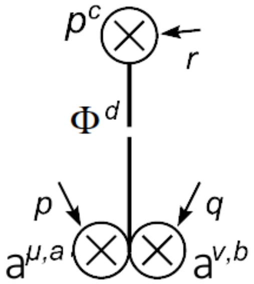

Figure 3.11: Feynman diagram χ

Diagram 1

This diagram consists of three subdiagrams and one vertex. The Feynman rule is

$$(\Pi_1)^{abc}_{\mu\nu} = (S_1(p))^{af}_{\mu\beta}(S_1(q))^{be}_{\nu\alpha}(S_\chi)^{cd}(V_2)^{\beta\alpha,def} = \\ = \frac{4iB_0 f_A^2 \sigma_A p^2 q^2}{(p^2 - M_A^2)(q^2 - M_A^2)r^2}\varepsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta d^{abc} \tag{3.17}$$

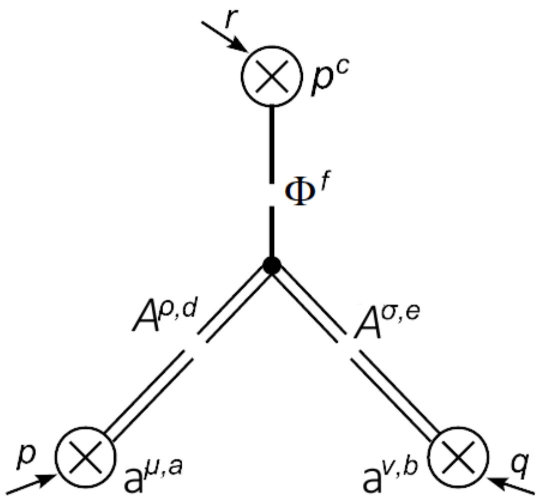

Figure 3.12: Feynman diagram 1

Diagram 2

This diagram consists of two subdiagrams and one vertex. The Feynman rule is

$$(\Pi_2^1)^{abc}_{\mu\nu} = (S_1(q))^{be}_{\nu\alpha}(S_\chi)^{cd}(V_3^1)^{aed}_{\mu\alpha} = \frac{-4i\sqrt{2}B_0 f_A h_A p^2}{(p^2 - M_A^2)r^2}\varepsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta d^{abc} \tag{3.18}$$

$$(\Pi_2^2)^{abc}_{\mu\nu} = (S_1(p))^{af}_{\mu\beta}(S_\chi)^{cd}(V_3^2)^{fbd}_{\nu\beta} = \frac{-4i\sqrt{2}B_0 f_A h_A q^2}{(q^2 - M_A^2)r^2}\varepsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta d^{abc} \tag{3.19}$$

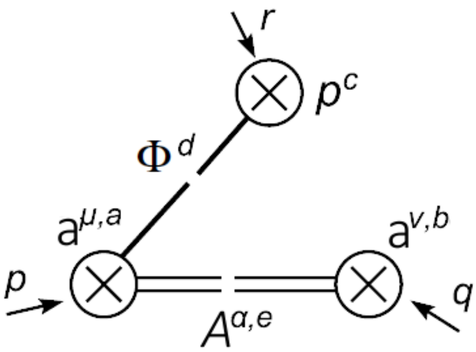

Figure 3.13: Feynman diagram 2 - first variant

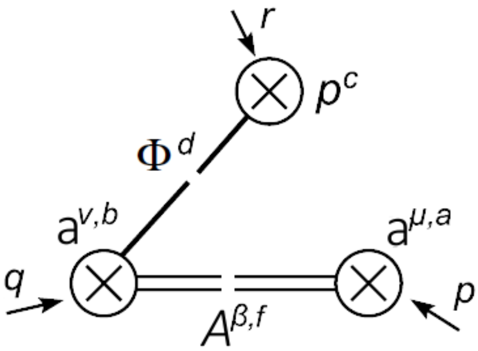

Figure 3.14: Feynman diagram 2 - second variant

Taking all contributions multiplied by i together, we get that in the vector formulation (3.1) has the form (3.60), where

$$\Pi_{AAP}(p^2, q^2, r^2) = -\frac{B_0 N_C}{24\pi^2 r^2} - \frac{4B_0 f_A^2 \sigma_A p^2 q^2}{(p^2 - M_A^2)(q^2 - M_A^2)r^2} + \\ + \frac{4\sqrt{2}B_0 f_A h_A p^2}{(p^2 - M_A^2)r^2} + \frac{4\sqrt{2}B_0 f_A h_A q^2}{(q^2 - M_A^2)r^2}. \tag{3.20}$$

3.2 $\langle AAP \rangle$ correlator in tensor formalism

For our purposes we will use only terms of (1.61) that include octets of pseudoscalar or axial-vector resonances. Hence the term proportional to $\langle P \rangle \langle \chi_- \rangle$ vanishes because of the null trace of χ_- in our case, we have

$$\mathcal{L}_R^{(4)} = i d_m \langle P \chi_- \rangle + \frac{F_A}{2\sqrt{2}} \langle A_{\mu\nu} f_-^{\mu\nu} \rangle. \tag{3.21}$$

3.2.1 Lagrangian basis

One axial-vector resonance field (A)

$$\mathcal{L}_3^A = \langle A^{\mu\nu}\{\nabla^\alpha h^{\beta\sigma}, u_\sigma\}\rangle \kappa_3^A \varepsilon_{\mu\nu\alpha\beta} \sim$$
$$\sim -\frac{2\sqrt{2}}{F}\Big\{ A_a^{\mu\nu}\Big[2a_b^\sigma\,\partial^\alpha\partial^\beta\partial^\sigma\phi_c + \big(\partial^\alpha\partial^\beta a_b^\sigma + \partial^\alpha\partial^\sigma a_b^\beta\big)\partial^\sigma\phi^c\Big] +$$
$$+ A_b^{\mu\nu}\Big[2a_a^\sigma\,\partial^\alpha\partial^\beta\partial^\sigma\phi_c + \big(\partial^\alpha\partial^\beta a_a^\sigma + \partial^\alpha\partial^\sigma a_a^\beta\big)\partial^\sigma\phi^c\Big]\Big\}\kappa_3^A\varepsilon_{\mu\nu\alpha\beta}, \quad (3.22)$$

$$\mathcal{L}_8^A = \langle A^{\mu\nu}\{f_-^{\alpha\sigma}, h^{\beta\sigma}\}\rangle \kappa_8^A \varepsilon_{\mu\nu\alpha\beta} \sim$$
$$\sim \frac{4\sqrt{2}}{F}\Big\{ A_a^{\mu\nu}\Big[\partial^\alpha a_b^\sigma\,\partial^\beta\partial^\sigma\phi_c - \partial^\sigma a_b^\alpha\partial^\beta\partial^\sigma\phi_c\Big] +$$
$$+ A_b^{\mu\nu}\Big[\partial^\alpha a_a^\sigma\,\partial^\beta\partial^\sigma\phi_c - \partial^\sigma a_a^\alpha\partial^\beta\partial^\sigma\phi_c\Big]\Big\}\kappa_8^A\varepsilon_{\mu\nu\alpha\beta}, \quad (3.23)$$

$$\mathcal{L}_{11}^A = i\langle A^{\mu\nu}\{f_-^{\alpha\beta}, \chi_-\}\rangle \kappa_{11}^A \varepsilon_{\mu\nu\alpha\beta} \sim$$
$$\sim 8\sqrt{2}B_0\big(\partial^\alpha a_a^\beta A_b^{\mu\nu} p_c + \partial^\alpha a_b^\beta A_a^{\mu\nu} p_c\big)\kappa_{11}^A\varepsilon_{\mu\nu\alpha\beta}d^{abc}, \quad (3.24)$$

$$\mathcal{L}_{12}^A = i\langle A^{\mu\nu}\{\nabla^\alpha\chi_-, u^\beta\}\rangle \kappa_{12}^A \varepsilon_{\mu\nu\alpha\beta} \sim$$
$$\sim -4\sqrt{2}B_0\big(A_a^{\mu\nu} a_b^\beta\partial^\alpha p_c + A_b^{\mu\nu} a_a^\beta\partial^\alpha p_c\big)\kappa_{12}^A\varepsilon_{\mu\nu\alpha\beta}d^{abc}, \quad (3.25)$$

$$\mathcal{L}_{15}^A = \langle A^{\mu\nu}\{\nabla^\alpha f_-^{\beta\sigma}, u_\sigma\}\rangle \kappa_{15}^A \varepsilon_{\mu\nu\alpha\beta} \sim$$
$$\sim \frac{2\sqrt{2}}{F}\Big[A_a^{\mu\nu}\big(\partial^\alpha\partial^\beta a_b^\sigma - \partial^\alpha\partial^\sigma a_b^\beta\big) + A_b^{\mu\nu}\big(\partial^\alpha\partial^\beta a_a^\sigma - \partial^\alpha\partial^\sigma a_a^\beta\big)\Big]\partial^\sigma\phi_c\kappa_{15}^A\varepsilon_{\mu\nu\alpha\beta}d^{abc}, \quad (3.26)$$

$$\mathcal{L}_{16}^A = \langle A^{\mu\nu}\{\nabla_\sigma f_-^{\alpha\sigma}, u^\beta\}\rangle \kappa_{16}^A \varepsilon_{\mu\nu\alpha\beta} \sim$$
$$\sim \frac{2\sqrt{2}}{F}\Big[A_a^{\mu\nu}\big(\partial^\sigma\partial^\alpha a_b^\sigma - \partial^\sigma\partial^\sigma a_b^\alpha\big) + A_b^{\mu\nu}\big(\partial^\sigma\partial^\alpha a_a^\sigma - \partial^\sigma\partial^\sigma a_a^\alpha\big)\Big]\partial^\beta\phi_c\kappa_{16}^A\varepsilon_{\mu\nu\alpha\beta}d^{abc}. \quad (3.27)$$

Pseudoscalar resonance field (P)

$$\mathcal{L}_1^P = \langle P\{f_-^{\mu\nu}, f_-^{\alpha\beta}\}\rangle \kappa_1^P \varepsilon_{\mu\nu\alpha\beta} \sim$$
$$\sim 8\sqrt{2}\big(\partial^\mu a_a^\nu\partial^\alpha a_b^\beta P_c + \partial^\mu a_b^\nu\partial^\alpha a_a^\beta P_c\big)\kappa_1^P\varepsilon_{\mu\nu\alpha\beta}d^{abc}. \quad (3.28)$$

Two resonance fields of the same kind (AA)

$$\mathcal{L}_2^{AA} = i\langle\{A^{\mu\nu}, A^{\alpha\beta}\}\chi_-\rangle \kappa_2^{AA}\varepsilon_{\mu\nu\alpha\beta} \sim -8B_0 A_a^{\mu\nu}A_b^{\alpha\beta}p_c\kappa_2^{AA}\varepsilon_{\mu\nu\alpha\beta}d^{abc}, \quad (3.29)$$

$$\mathcal{L}_3^{AA} = \langle\{\nabla_\sigma A^{\mu\nu}, A^{\alpha\sigma}\}u^\beta\rangle \kappa_3^{AA}\varepsilon_{\mu\nu\alpha\beta} \sim$$
$$\sim -\frac{2}{F}\big(A_a^{\alpha\sigma}\partial_\sigma A_b^{\mu\nu}\partial^\beta\phi_c + \partial_\sigma A_a^{\mu\nu}A_b^{\alpha\sigma}\partial^\beta\phi_c\big)\kappa_3^{AA}\varepsilon_{\mu\nu\alpha\beta}d^{abc}. \quad (3.30)$$

Two resonance fields of different kinds (AP)

$$\begin{aligned}\mathcal{L}_1^{AP} &= \langle\{A^{\mu\nu}, P\} f_-^{\alpha\beta}\rangle \kappa_1^{AP} \varepsilon_{\mu\nu\alpha\beta} \sim \\ &\sim -4(A_a^{\mu\nu}\partial^\alpha a_b^\beta P_c + A_b^{\mu\nu}\partial^\alpha a_a^\beta P_c)\kappa_1^{AP}\varepsilon_{\mu\nu\alpha\beta} d^{abc},\end{aligned} \tag{3.31}$$

$$\begin{aligned}\mathcal{L}_2^{AP} &= \langle\{A^{\mu\nu}, \nabla^\alpha P\} u^\beta\rangle \kappa_2^{AP} \varepsilon_{\mu\nu\alpha\beta} \sim \\ &\sim 2\big(A_a^{\mu\nu} a_b^\beta \partial^\alpha P_c + A_b^{\mu\nu} a_a^\beta \partial^\alpha P_c\big)\kappa_2^{AP}\varepsilon_{\mu\nu\alpha\beta} d^{abc}.\end{aligned} \tag{3.32}$$

Three resonance fields (AAP)

$$\mathcal{L}_3^{AAP} = \langle A^{\mu\nu} A^{\alpha\beta} P\rangle \kappa_3^{AAP} \varepsilon_{\mu\nu\alpha\beta} \sim \sqrt{2} A_a^{\mu\nu} A_b^{\alpha\beta} P_c \kappa_3^{AAP} \varepsilon_{\mu\nu\alpha\beta} d^{abc}. \tag{3.33}$$

3.2.2 Feynman rules

Here we calculate Feynman rules of all contributing Lagrangians.

Tensor propagator

The kinetic and mass terms form the tensor propagator [6].

$$i\big(\Delta_R(p)\big)^{ab}_{\alpha\beta,\rho\sigma} = -\frac{i\delta^{ab}}{M^2(p^2 - M^2)}\big[g_{\alpha\rho}g_{\beta\sigma}(M^2 - p^2) + g_{\alpha\rho}p_\beta p_\sigma - g_{\alpha\sigma}p_\beta p_\rho - (\alpha \leftrightarrow \beta)\big]. \tag{3.34}$$

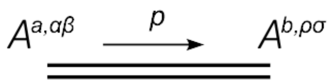

Figure 3.15: Feynman diagram of tensor propagator

Pseudoscalar mass propagator

The kinetic and mass terms form the mass pseudoscalar propagator.

$$i\Delta_P(r, M) = \frac{i}{r^2 - M^2}\delta^{ab}. \tag{3.35}$$

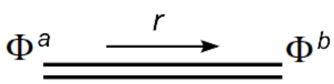

Figure 3.16: Feynman diagram of pseudoscalar mass propagator

Vertex 1 (pseudoscalar source - resonance)

To this vertex contributes resonance Lagrangian $\mathcal{L}_R^{(4)}$ (3.21) by its part proportional to d_m. Feynman rule is

$$(V_1)^{cd} = -2i\sqrt{2}B_0 d_m \delta^{cd}. \tag{3.36}$$

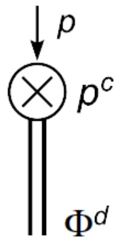

Figure 3.17: Feynman diagram of vertex 1

Vertex 2 (axial source - axial source - resonance)

To this vertex contributes Lagrangian $\mathcal{L}_1^P$. Due to Bose statistic the Feynman rule is

$$(V_2)^{abe}_{\mu\nu} = 16i\sqrt{2}\kappa_1^P p^\alpha q^\beta \varepsilon_{\mu\nu\alpha\beta} d^{abe} \tag{3.37}$$

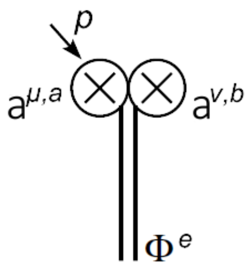

Figure 3.18: Feynman diagram of vertex 2

Vertex 3 (axial source - resonance)

To this vertex contributes resonance Lagrangian $\mathcal{L}_R^{(4)}$ (3.21) by its part proportional to F_A. Feynman rule is

$$(V_3)^{bd}_{\nu\alpha\beta} = \frac{F_A}{2}(g_{\nu\alpha}p_\beta - g_{\nu\beta}p_\alpha)\delta^{bd} \tag{3.38}$$

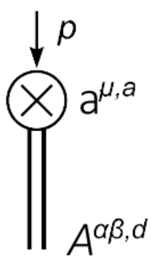

Figure 3.19: Feynman diagram of vertex 3

Vertex 4 (resonance - resonance - pseudoscalar)

To this vertex contributes Lagrangian $\mathcal{L}_3^{AA}$. Feynman rule is

$$(V_4^1)^{def}_{\rho\sigma\gamma\delta} = \frac{i}{F} r_\beta (q_\sigma \varepsilon_{\gamma\delta\rho\beta} - q_\rho \varepsilon_{\gamma\delta\sigma\beta} + p_\delta \varepsilon_{\rho\sigma\gamma\beta} - p_\gamma \varepsilon_{\rho\sigma\delta\beta}) \kappa_3^{AA} d^{def}. \tag{3.39}$$

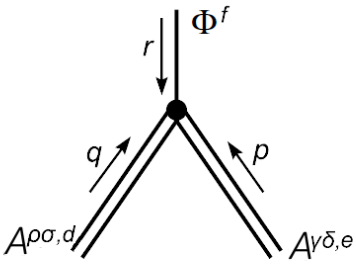

Figure 3.20: Feynman diagram of vertex 4

Vertex 5 (resonance - resonance - resonance)

To this vertex contributes Lagrangian $\mathcal{L}_3^{AAP}$. Feynman rule is

$$(V_5)^{def}_{\gamma\delta\rho\sigma} = i\sqrt{2}\kappa_3^{AAP} \varepsilon_{\gamma\delta\rho\sigma} d^{def}. \tag{3.40}$$

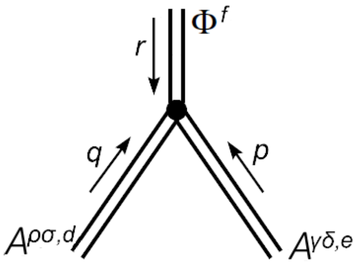

Figure 3.21: Feynman diagram of vertex 5

Vertex 6 (axial source - resonance - pseudoscalar)

To this vertex contribute Lagrangians $\mathcal{L}_3^A$, $\mathcal{L}_8^A$, $\mathcal{L}_{15}^A$ and $\mathcal{L}_{16}^A$. Feynman rule is

$$\begin{aligned}(V_6^1)^{ade}_{\mu\alpha\beta} = \frac{2\sqrt{2}}{F} \Big\{ & r^\kappa \varepsilon_{\alpha\beta\mu\kappa} \left[p^2 \kappa_{16} - (p^2 - q^2 + r^2)\kappa_8^A \right] - \\ & - \frac{1}{2} p^\lambda \varepsilon_{\alpha\beta\lambda\mu} (p^2 - q^2 + r^2)(\kappa_3^A + \kappa_{15}^A) \Big\} d^{ade}\end{aligned} \tag{3.41}$$

$$\begin{aligned}(V_6^2)^{bde}_{\nu\alpha\beta} = \frac{2\sqrt{2}}{F} \Big\{ & r^\kappa \varepsilon_{\alpha\beta\nu\kappa} \left[q^2 \kappa_{16} - (-p^2 + q^2 + r^2)\kappa_8^A \right] - \\ & - \frac{1}{2} q^\lambda \varepsilon_{\alpha\beta\lambda\nu} (-p^2 + q^2 + r^2)(\kappa_3^A + \kappa_{15}^A) \Big\} d^{bde}\end{aligned} \tag{3.42}$$

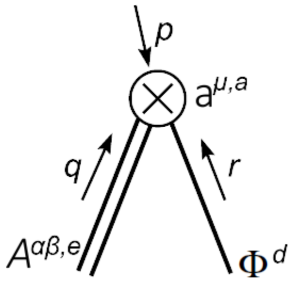

Figure 3.22: Feynman diagram of vertex 6 - first variant

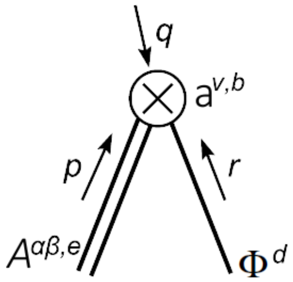

Figure 3.23: Feynman diagram of vertex 6 - second variant

Vertex 7 (axial source - resonance - resonance)

To this vertex contribute Lagrangians $\mathcal{L}_1^{AP}$ and $\mathcal{L}_2^{AP}$. Feynman rule is

$$(V_7^1)^{aed}_{\mu\alpha\beta} = 2(\kappa_2^{AP} r^\lambda - 2\kappa_1^{AP} p^\lambda) d^{aed} \varepsilon_{\alpha\beta\lambda\mu} \tag{3.43}$$

$$(V_7^2)^{bed}_{\nu\alpha\beta} = 2(\kappa_2^{AP} r^\lambda - 2\kappa_1^{AP} q^\lambda) d^{bed} \varepsilon_{\alpha\beta\lambda\nu} \tag{3.44}$$

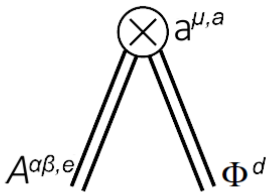

Figure 3.24: Feynman diagram of vertex 7 - first variant

Vertex 8 (pseudoscalar source - resonance - resonance)

To this vertex contributes Lagrangian $\mathcal{L}_2^{AA}$. Feynman rule is

$$(V_8)^{fce}_{\alpha\beta\rho\sigma} = -8iB_0\kappa_2^{AA}\varepsilon_{\alpha\beta\rho\sigma}d^{fce}. \tag{3.45}$$

Vertex 9 (axial source - pseudoscalar source - resonance)

To this vertex contribute Lagrangians $\mathcal{L}_{11}^A$ and $\mathcal{L}_{12}^A$. Feynman rule is

$$(V_9^1)^{acd}_{\rho\sigma\mu} = 4\sqrt{2}B_0(2\kappa_{11}^A p^\alpha - \kappa_{12}^A r^\alpha) d^{adc} \varepsilon_{\rho\sigma\alpha\mu}, \tag{3.46}$$

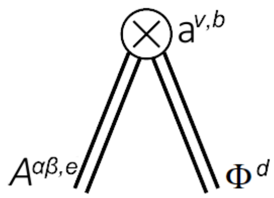

Figure 3.25: Feynman diagram of vertex 7 - second variant

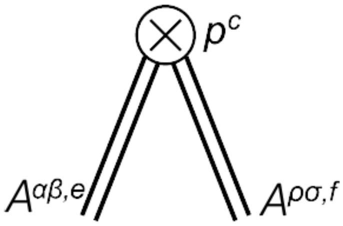

Figure 3.26: Feynman diagram of vertex 8

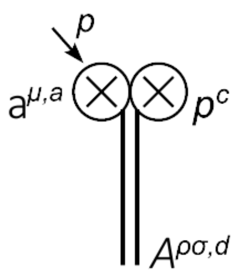

Figure 3.27: Feynman diagram of vertex 9 - first variant

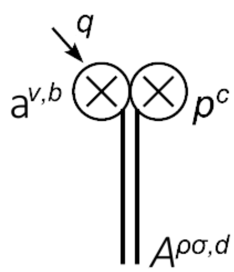

Figure 3.28: Feynman diagram of vertex 9 - second variant

$$(V_9^2)^{dbc}_{\rho\sigma\nu} = 4\sqrt{2}B_0(2\kappa^A_{11}q^\alpha - \kappa^A_{12}r^\alpha)d^{dbc}\varepsilon_{\rho\sigma\alpha\nu} \quad (3.47)$$

In order to simplify our calculations as much as possible, we now construct subdiagrams consisting of one vertex and one propagator. In the case of AAP correlator we have following subdiagrams.

Subdiagram 1 (vertex 1 - pseudoscalar mass propagator)

This subdiagram consists of one vertex and "pseudotensor" propagator. Feynman rule is

$$(S_1)^{ce} = (V_1)^{cd} i\big(\Delta_P(r,M)\big)^{de} = \frac{2\sqrt{2}B_0 d_m}{r^2 - M^2}\delta^{ce}. \quad (3.48)$$

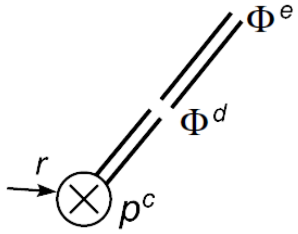

Figure 3.29: Feynman subdiagram 1

Subdiagram 2 (vertex 3 - tensor propagator)

This subdiagram consists of one vertex and tensor propagator. Feynman rule is

$$(S_2(p))^{ac}_{\mu\rho\sigma} = (V_3)^{ad}_{\mu\alpha\beta} i\big(\Delta_T(p)\big)^{\alpha\beta,\rho\sigma}_{dc} = -i\frac{F_A}{p^2 - M^2}(g_{\mu\rho}p_\sigma - g_{\mu\sigma}p_\rho)\delta^{ac}. \quad (3.49)$$

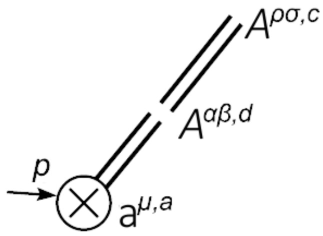

Figure 3.30: Feynman subdiagram 2

Now we have everything we need to construct complete Feynman diagrams and calculate their contributions. These diagrams consist of inner vertex which links to other subdiagrams.

3.2.3 Feynman diagrams

Diagram χ

First of all, in tensor formalism also contributes diagram χ and has the same form as (3.16).

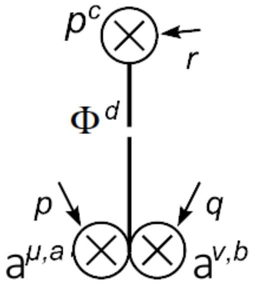

Figure 3.31: Feynman diagram χ

Diagram 1

This vertex consists of one subdiagram and one vertex. Feynman rules give us

$$(\Pi_1)^{abc}_{\mu\nu} = (S_1)^{ce}(V_2)^{abe}_{\mu\nu} = \frac{64iB_0 d_m}{r^2 - M_P^2}\kappa_1^P \varepsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta d^{abc} \tag{3.50}$$

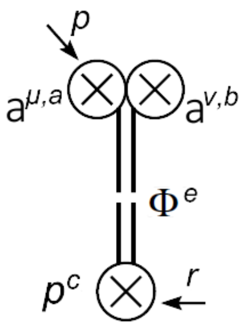

Figure 3.32: Feynman diagram 1

Diagram 2

This vertex consists of three subdiagrams ond one vertex. Feynman rules give us

$$\begin{aligned}(\Pi_2)^{abc}_{\mu\nu} &= (V_4)^{def}_{\rho\sigma\gamma\delta}(S_\chi)^{fc}(S_2(p))^{da}_{\mu\rho\sigma}(S_2(q))^{eb}_{\gamma\delta\nu} = \\ &= \frac{4iB_0F_A^2}{(p^2 - M_A^2)(q^2 - M_A^2)r^2}(-p^2 - q^2 + r^2)\kappa_3^{AA}\varepsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta d^{abc}\end{aligned} \tag{3.51}$$

Diagram 3

This vertex consists of three subdiagrams and one vertex. Feynman rules give us

$$\begin{aligned}(\Pi_3)^{abc}_{\mu\nu} &= (S_2(p))^{ad}_{\mu\rho\sigma}(S_2(q))^{be}_{\nu\gamma\delta}(S_1)^{fc}(V_5)^{def}_{\gamma\delta\rho\sigma} = \\ &= \frac{16iB_0F_A^2 d_m}{(p^2 - M_A^2)(q^2 - M_A^2)(r^2 - M_P^2)}\kappa_3^{AAP}\varepsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta d^{abc}\end{aligned} \tag{3.52}$$

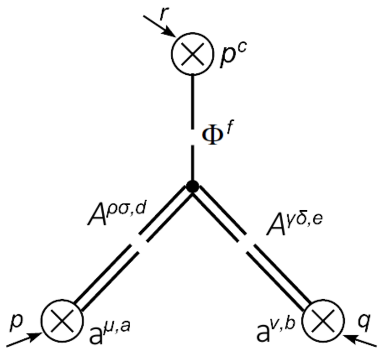

Figure 3.33: Feynman diagram 2

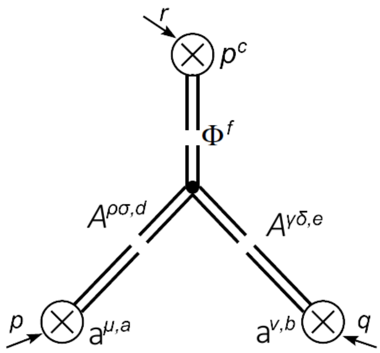

Figure 3.34: Feynman diagram 3

Diagram 4

This vertex consists of two subdiagrams and one vertex. Feynman rules give us

$$\begin{aligned}(\Pi_4^1)^{abc}_{\mu\nu} &= (S_2(q))^{be}_{\nu\alpha\beta}(S_\chi)^{dc}(V_6^1)^{ade}_{\mu\alpha\beta} = \\ &= \frac{4i\sqrt{2}B_0F_A}{r^2(q^2-M_A^2)}\left[\frac{1}{2}\big(-p^2+q^2-r^2\big)\big(\kappa_3^A+2\kappa_8^A+\kappa_{15}^A\big)+p^2\kappa_{16}^A\right]\varepsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta d^{abc}\end{aligned} \tag{3.53}$$

$$\begin{aligned}(\Pi_4^2)^{abc}_{\mu\nu} &= (S_2(p))^{ae}_{\mu\alpha\beta}(S_\chi)^{dc}(V_6^2)^{bde}_{\nu\alpha\beta} = \\ &= \frac{4i\sqrt{2}B_0F_A}{r^2(p^2-M_A^2)}\left[\frac{1}{2}\big(p^2-q^2-r^2\big)\big(\kappa_3^A+2\kappa_8^A+\kappa_{15}^A\big)+q^2\kappa_{16}^A\right]\varepsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta d^{abc}\end{aligned} \tag{3.54}$$

Diagram 5

This vertex consists of two subdiagrams and one vertex. Feynman rules give us

$$\begin{aligned}(\Pi_5^1)^{abc}_{\mu\nu} &= (S_1)^{cd}(S_2(q))^{be}_{\nu\alpha\beta}(V_7^1)^{aed}_{\mu\alpha\beta} = \\ &= \frac{8i\sqrt{2}B_0F_Ad_m}{(r^2-M_P^2)(q^2-M_A^2)}\big(2\kappa_1^{AP}+\kappa_2^{AP}\big)\varepsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta d^{abc}\end{aligned} \tag{3.55}$$

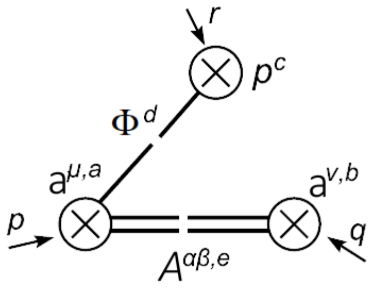

Figure 3.35: Feynman diagram 4 - first variant

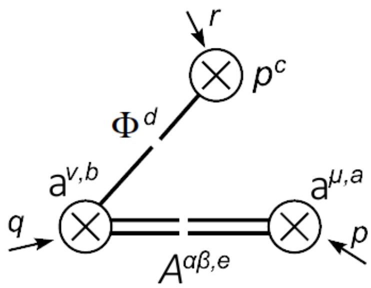

Figure 3.36: Feynman diagram 4 - second variant

$$
\begin{aligned}
(\Pi_5^2)^{abc}_{\mu\nu} &= (S_1)^{cd}(S_2(p))^{ae}_{\mu\alpha\beta}(V_7^2)^{bed}_{\nu\alpha\beta} = \\
&= \frac{8i\sqrt{2}B_0F_Ad_m}{(r^2-M_P^2)(p^2-M_A^2)}\big(2\kappa_1^{AP}+\kappa_2^{AP}\big)\varepsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta d^{abc}
\end{aligned} \tag{3.56}
$$

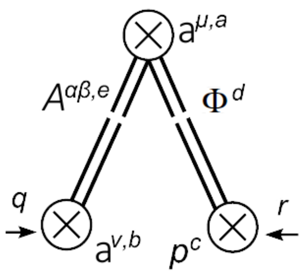

Figure 3.37: Feynman diagram 5 - first variant

Diagram 6

This vertex consists of two subdiagrams and one vertex. Feynman rules give us

$$
\begin{aligned}
(\Pi_6)^{abc}_{\mu\nu} &= (S_2(p))^{ae}_{\mu\alpha\beta}(S_2(q))^{bf}_{\nu\rho\sigma}(V_8)^{fce}_{\alpha\beta\rho\sigma} = \\
&= -\frac{32iB_0F_A^2}{(p^2-M_A^2)(q^2-M_A^2)}\kappa_2^{\mathrm{AA}}\varepsilon^{\mu\nu\alpha\beta}p^\alpha q^\beta d^{abc}
\end{aligned} \tag{3.57}
$$

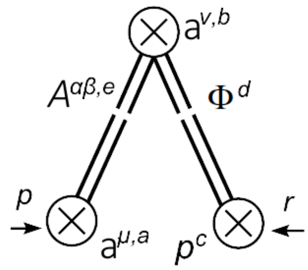

Figure 3.38: Feynman diagram 5 - second variant

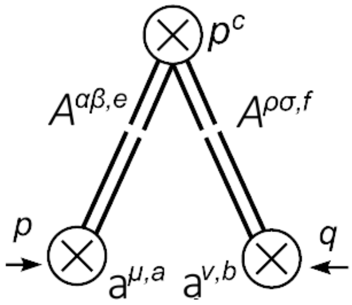

Figure 3.39: Feynman diagram 6

Diagram 7

This vertex consists of one subdiagram and one vertex. Feynman rules give us

$$(\Pi_7^1)^{abc}_{\mu\nu} = (S_2(q))^{bd}_{\nu\rho\sigma}(V_9^1)^{acd}_{\mu\rho\sigma} = \\ = -\frac{8i\sqrt{2}B_0F_A}{q^2-M_A^2}\left(2\kappa_{11}^A+\kappa_{12}^A\right)\varepsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta d^{abc} \tag{3.58}$$

$$(\Pi_7^2)^{abc}_{\mu\nu} = (S_2(p))^{ad}_{\mu\rho\sigma}(V_9^2)^{bcd}_{\nu\rho\sigma} = \\ = -\frac{8i\sqrt{2}B_0F_A}{p^2-M_A^2}\left(2\kappa_{11}^A+\kappa_{12}^A\right)\varepsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta d^{abc} \tag{3.59}$$

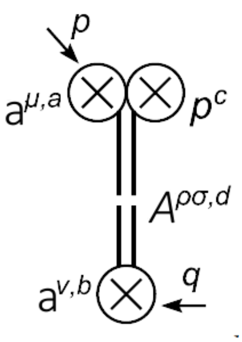

Figure 3.40: Feynman diagram 7 - first variant

Taking all calculated contributions multiplied by i together, we get that in the antisymmetric tensor formulation (3.1) has the form

$$\Pi_{AAP}(p,q)^{abc}_{\mu\nu} = \Pi_{AAP}(p^2,q^2,r^2)\varepsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta d^{abc}, \tag{3.60}$$

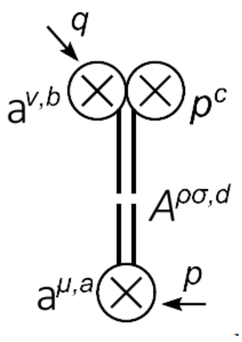

Figure 3.41: Feynman diagram 7 - second variant

where

$$
\begin{aligned}
\Pi_{AAP}(p^2,q^2,r^2) = &-\frac{B_0N_C}{24\pi^2r^2}-\frac{64B_0d_m}{r^2-M_P^2}\kappa_1^P-\\
&-\frac{4B_0F_A^2}{(p^2-M_A^2)(q^2-M_A^2)r^2}(-p^2-q^2+r^2)\kappa_3^{AA}-\\
&-\frac{16B_0F_A^2d_m}{(p^2-M_A^2)(q^2-M_A^2)(r^2-M_P^2)}\kappa_3^{AAP}-\\
&-\frac{4\sqrt{2}B_0F_A}{r^2(q^2-M_A^2)}\left[\frac{1}{2}\big(-p^2+q^2-r^2\big)\big(\kappa_3^A+2\kappa_8^A+\kappa_{15}^A\big)+p^2\kappa_{16}^A\right]-\\
&-\frac{4\sqrt{2}B_0F_A}{r^2(p^2-M_A^2)}\left[\frac{1}{2}\big(p^2-q^2-r^2\big)\big(\kappa_3^A+2\kappa_8^A+\kappa_{15}^A\big)+q^2\kappa_{16}^A\right]-\\
&-\frac{8\sqrt{2}B_0F_Ad_m}{(r^2-M_P^2)(p^2-M_A^2)}\big(2\kappa_1^{AP}+\kappa_2^{AP}\big)-\\
&-\frac{8\sqrt{2}B_0F_Ad_m}{(r^2-M_P^2)(q^2-M_A^2)}\big(2\kappa_1^{AP}+\kappa_2^{AP}\big)+\\
&+\frac{32B_0F_A^2}{(p^2-M_A^2)(q^2-M_A^2)}\kappa_2^{\mathrm{AA}}+\\
&+\frac{8\sqrt{2}B_0F_A}{p^2-M_A^2}\big(2\kappa_{11}^A+\kappa_{12}^A\big)+\frac{8\sqrt{2}B_0F_A}{q^2-M_A^2}\big(2\kappa_{11}^A+\kappa_{12}^A\big). \qquad (3.61)
\end{aligned}
$$

3.3 Operator product expansion

In this chapter we will pay our attention to the operator product expansion of $\langle AAP\rangle$ correlator. Operator product expansion (OPE) is an expression of products of composite operators (known also as local operators) at short distances. It means that in our case we will compute the term

$$\langle 0|T[A_\mu^a(x)A_\nu^b(y)P^c(0)]|0\rangle = \langle 0|T\Big[\Big(\overline{q}\gamma_\mu\gamma_5\frac{T^a}{\sqrt{2}}q\Big)\Big(\overline{q}\gamma_\mu\gamma_5\frac{T^b}{\sqrt{2}}q\Big)\Big(i\overline{q}\gamma_5\frac{T^c}{\sqrt{2}}q\Big)\Big]|0\rangle, \qquad (3.62)$$

where we have

$$A^{a\mu} = \frac{1}{\sqrt{2}}\overline{q}\gamma^\mu\gamma^5T^aq, \qquad P^a = \frac{1}{\sqrt{2}}\overline{q}i\gamma^5T^aq, \qquad (3.63)$$

and

$$T^a = \frac{1}{\sqrt{2}}\lambda^a. \tag{3.64}$$

Thus we have, only for non-numerical part of (3.62),

$$\begin{aligned}
&\langle 0|T\big(\bar{q}\gamma_\mu\gamma_5 T^a q\big)(x)\big(\bar{q}\gamma_\mu\gamma_5 T^b q\big)(y)\big(i\bar{q}\gamma_5 T^c q\big)(0)|0\rangle = \\
&= \langle 0| : \bar{q}(x)\gamma_\mu\gamma_5 T^a \underbrace{iS(x-y)}_{\frac{i}{-\not p}} \gamma_\nu\gamma_5 T^b \underbrace{iS(y)}_{\frac{i}{\not r}} i\gamma_5 T^c q(0) : |0\rangle + \\
&+\langle 0| : \bar{q}(y)\gamma_\nu\gamma_5 T^b \underbrace{iS(y)}_{\frac{i}{-\not q}} i\gamma_5 T^c \underbrace{iS(-x)}_{\frac{i}{\not p}} \gamma_\mu\gamma_5 T^a q(x) : |0\rangle + \\
&+\langle 0| : \bar{q}(0) i\gamma_5 T^c \underbrace{iS(-x)}_{\frac{i}{-\not r}} \gamma_\mu\gamma_5 T^a \underbrace{iS(x-y)}_{\frac{i}{\not q}} \gamma_\nu\gamma_5 T^b q(y) : |0\rangle + \\
&+\langle 0| : \bar{q}(x)\gamma_\mu\gamma_5 T^a \underbrace{iS(x)}_{\frac{i}{-\not p}} i\gamma_5 T^c \underbrace{iS(-y)}_{\frac{i}{\not q}} \gamma_\nu\gamma_5 T^b q(y) : |0\rangle + \\
&+\langle 0| : \bar{q}(0) i\gamma_5 T^c \underbrace{iS(-y)}_{\frac{i}{-\not r}} \gamma_\nu\gamma_5 T^b \underbrace{iS(y-x)}_{\frac{i}{\not p}} \gamma_\mu\gamma_5 T^a q(x) : |0\rangle + \\
&+\langle 0| : \bar{q}(y)\gamma_\nu\gamma_5 T^b \underbrace{iS(y-x)}_{\frac{i}{-\not q}} \gamma_\mu\gamma_5 T^a \underbrace{iS(x)}_{\frac{i}{\not r}} i\gamma_5 T^c q(0) : |0\rangle.
\end{aligned} \tag{3.65}$$

Using obvious identity $1/\not a = \not a/a^2$ and trace properties of Dirac matrices we can get that $\langle AAP\rangle$ correlator in OPE has the form

$$\Pi\big((\lambda p)^2, (\lambda q)^2, (\lambda r)^2\big)_{\mathrm{AAP}} = \frac{B_0 F^2}{2\lambda^4}\,\frac{p^2+q^2-r^2}{p^2q^2r^2} + \mathcal{O}\bigg(\frac{1}{\lambda^6}\bigg). \tag{3.66}$$

Our task is now to determine if tensor and vector formalism satisfy OPE result. Satisfying this behaviour in the case of antisymmetric tensor formalism gives us the following coupling constraints:

$$\kappa_3^{AAP} = 0, \qquad \frac{N_C}{24\pi^2} + 64 d_m \kappa_1^P + 8\sqrt{2}F_A\kappa_{16}^A = 0, \tag{3.67}$$

$$\frac{1}{2}(\kappa_3^A + 2\kappa_8^A + \kappa_{15}^A) - \kappa_{16}^A = 0, \qquad \kappa_3^{AA} - 8\kappa_2^{AA} = \frac{F^2}{8F_A^2}, \tag{3.68}$$

$$2(2\kappa_{11}^A + \kappa_{12}^A) + \kappa_{16}^A = 0, \qquad 2\kappa_1^{AP} + \kappa_2^{AP} = \frac{2\sqrt{2}F_A}{d_m}\kappa_2^{AA}. \tag{3.69}$$

For vector formalism we obtain the condition

$$-\frac{N_C}{24\pi^2} - 4f_A^2\sigma_A + 8\sqrt{2}f_A h_A = 0. \tag{3.70}$$

From complete calculation of (3.70) we know that vector formalism is not consistent with OPE at order $\sim 1/\lambda^4$ because of the missing terms $\sim 1/p^2q^2, 1/p^2r^2$ and $\sim 1/q^2r^2$ in (3.20). Now we will pay our attention back to antisymetric tensor formalism which OPE conditions satisfies.

By employing coupling constraints (3.67) - (3.69) into (3.61) we can get

$$\begin{aligned}\frac{1}{B_0}\Pi_{AAP}(p^2,q^2,r^2) = & \frac{8\sqrt{2}F_A\kappa_{16}^A}{r^2-M_P^2} + \frac{N_C}{24\pi^2}\frac{M_P^2}{r^2(r^2-M_P^2)} - \\ & - \frac{\left(\frac{F^2}{2}+32F_A^2\kappa_2^{AA}\right)(p^2+q^2-r^2)}{(p^2-M_A^2)(q^2-M_A^2)r^2} - \\ & - \frac{4\sqrt{2}F_A\kappa_{16}^A\left[2p^2q^2-M_A^2(p^2+q^2)\right]}{(p^2-M_A^2)(q^2-M_A^2)r^2} - \\ & - \frac{32F_A^2\kappa_2^A(p^2+q^2-r^2-2M_A^2+M_P^2)}{(p^2-M_A^2)(q^2-M_A^2)(r^2-M_P^2)}. \qquad (3.71)\end{aligned}$$

Considering only axial-vector interactions, which we can satisfy taking $M_P \to \infty$, we get

$$\begin{aligned}\frac{1}{B_0}\Pi_{AAP}(p^2,q^2,r^2) = & -\frac{\left(\frac{F^2}{2}+32F_A^2\kappa_2^{AA}\right)(p^2+q^2-r^2)}{(p^2-M_A^2)(q^2-M_A^2)r^2} - \\ & - \frac{4\sqrt{2}F_A\kappa_{16}^A\left[2p^2q^2-M_A^2(p^2+q^2)\right]}{(p^2-M_A^2)(q^2-M_A^2)r^2} - \\ & - \frac{32F_A^2\kappa_2^A(p^2+q^2-r^2-2M_A^2+M_P^2)}{(p^2-M_A^2)(q^2-M_A^2)(r^2-M_P^2)}. \qquad (3.72)\end{aligned}$$

The important note is that $\langle AAP \rangle$ correlator satisfies OPE only in antisymmetric tensor formalism. This formalism we will use in $\langle AAA \rangle$ and $\langle VVA \rangle$ correlators later.

4. $\langle AAA\rangle$ and $\langle VVA\rangle$ correlator

The standard definitions of these correlators are

$$\Pi^{abc}_{\mu\nu}(p,q) = \int \mathrm{d}^4x\,\mathrm{d}^4y\, e^{i(px+qy)}\,\langle 0|T[A^a_\mu(x)A^b_\nu(y)A^c_\eta(z)]|0\rangle, \tag{4.1}$$

$$\Pi^{abc}_{\mu\nu}(p,q) = \int \mathrm{d}^4x\,\mathrm{d}^4y\, e^{i(px+qy)}\,\langle 0|T[V^a_\mu(x)V^b_\nu(y)A^c_\eta(z)]|0\rangle. \tag{4.2}$$

In this chapter we will just show possible Lagrangians contributing to remaining correlators. We will not compute these Lagrangians now, because we will return to them in the future studies.

4.1 Lagrangian basis of $\langle AAA\rangle$

$$\mathcal{L}^A_3 = \langle A^{\mu\nu}\{\nabla^\alpha h^{\beta\sigma}, u_\sigma\}\rangle \kappa^A_3 \varepsilon_{\mu\nu\alpha\beta}, \tag{4.3}$$

$$\mathcal{L}^A_8 = \langle A^{\mu\nu}\{f^{\alpha\sigma}_-, h^{\beta\sigma}\}\rangle \kappa^A_8 \varepsilon_{\mu\nu\alpha\beta}, \tag{4.4}$$

$$\mathcal{L}^A_{15} = \langle A^{\mu\nu}\{\nabla^\alpha f^{\beta\sigma}_-, u_\sigma\}\rangle \kappa^A_{15} \varepsilon_{\mu\nu\alpha\beta}, \tag{4.5}$$

$$\mathcal{L}^A_{16} = \langle A^{\mu\nu}\{\nabla_\sigma f^{\alpha\sigma}_-, u^\beta\}\rangle \kappa^A_{16} \varepsilon_{\mu\nu\alpha\beta}, \tag{4.6}$$

$$\mathcal{L}^{AA}_3 = \langle \{\nabla_\sigma A^{\mu\nu}, A^{\alpha\sigma}\} u^\beta\rangle \kappa^{AA}_3 \varepsilon_{\mu\nu\alpha\beta}, \tag{4.7}$$

$$\mathcal{L}^{AA}_4 = \langle \{\nabla^\beta A^{\mu\nu}, A^{\alpha\sigma}\} u_\sigma\rangle \kappa^{AA}_4 \varepsilon_{\mu\nu\alpha\beta}. \tag{4.8}$$

4.2 Lagrangian basis of $\langle VVA\rangle$

$$\mathcal{L}^V_5 = i\langle V^{\mu\nu}[f^{\alpha\beta}_-, u_\sigma u^\sigma]\rangle \kappa^V_5 \varepsilon_{\mu\nu\alpha\beta}, \tag{4.9}$$

$$\mathcal{L}^A_3 = \langle A^{\mu\nu}\{\nabla^\alpha h^{\beta\sigma}, u_\sigma\}\rangle \kappa^A_3 \varepsilon_{\mu\nu\alpha\beta}, \tag{4.10}$$

$$\mathcal{L}^A_8 = \langle A^{\mu\nu}\{f^{\alpha\sigma}_-, h^{\beta\sigma}\}\rangle \kappa^A_8 \varepsilon_{\mu\nu\alpha\beta}, \tag{4.11}$$

$$\mathcal{L}^A_{15} = \langle A^{\mu\nu}\{\nabla^\alpha f^{\beta\sigma}_-, u_\sigma\}\rangle \kappa^A_{15} \varepsilon_{\mu\nu\alpha\beta}, \tag{4.12}$$

$$\mathcal{L}^A_{16} = \langle A^{\mu\nu}\{\nabla_\sigma f^{\alpha\sigma}_-, u^\beta\}\rangle \kappa^A_{16} \varepsilon_{\mu\nu\alpha\beta}, \tag{4.13}$$

$$\mathcal{L}^{AA}_3 = \langle \{\nabla_\sigma A^{\mu\nu}, A^{\alpha\sigma}\} u^\beta\rangle \kappa^{AA}_3 \varepsilon_{\mu\nu\alpha\beta}, \tag{4.14}$$

$$\mathcal{L}^{AA}_4 = \langle \{\nabla^\beta A^{\mu\nu}, A^{\alpha\sigma}\} u_\sigma\rangle \kappa^{AA}_4 \varepsilon_{\mu\nu\alpha\beta}, \tag{4.15}$$

$$\mathcal{L}^{VA}_1 = i\langle V^{\mu\nu}[A^{\alpha\beta}, u^\sigma u_\sigma]\rangle \kappa^{VA}_1 \varepsilon_{\mu\nu\alpha\beta} \tag{4.16}$$

We will have to answer the question if there are any contributions from vertices with pion fields coming out of axial sources, i.e. vertices proportional to the terms with $f^{\mu\nu}_+$.

Conclusion

Our task at the beginning of the thesis was to study new correlators $\langle AAP\rangle$, $\langle AAA\rangle$ and $\langle VVA\rangle$. On the case of $\langle AAP\rangle$ we have established that vector formalism does not satisfy OPE conditions. We also stated that antisymmetric tensor formalism these conditions satisfies and then we decided to use this formalism in the cases of $\langle AAA\rangle$ and $\langle VVA\rangle$.

We have also suggested the part of the independent operator basis for $\langle AAA\rangle$ and $\langle VVA\rangle$ which we will pay attention to. One of the related questions needed to be answered is if there are any contributions from terms with $f_+^{\mu\nu}$.

As we just mentioned, we plan to continue in this work during future studies. Hopefully, some experimental data should be available in a couple of years so we could study phenomenology of these correlators, just like in [1] in the case of $\langle VVP\rangle$ and $\langle VAS\rangle$.

The main results of this work were partially presented at MesonNet meeting in Prague [12].

Appendix

In this chapter we present very useful properties of basic mathematical structures used in this thesis. The idea of this part is just to sum up these properties, without any proof.

Example of using Feynman rules

Here we explicit calculation of Feynman rules from contributing Lagrangians on some examples. Let us start for example with Lagrangian contributing to vertex of diagram 2 in tensor formulation. The Lagrangian is

$$\begin{aligned}\mathcal{L}_3^{AA} &= \langle\{\nabla_\sigma A^{\mu\nu}, A^{\alpha\sigma}\}u^\beta\rangle\kappa_3^{AA}\varepsilon_{\mu\nu\alpha\beta} \sim \\ &\sim -\frac{2}{F}\left(A_a^{\alpha\sigma}\partial_\sigma A_b^{\mu\nu}\partial^\beta\phi_c + \partial_\sigma A_a^{\mu\nu}A_b^{\alpha\sigma}\partial^\beta\phi_c\right)\kappa_3^{AA}\varepsilon_{\mu\nu\alpha\beta}d^{abc} \qquad (4.17)\end{aligned}$$

and let us assume that we want to have the Feynamn diagram of this vertex as follows:

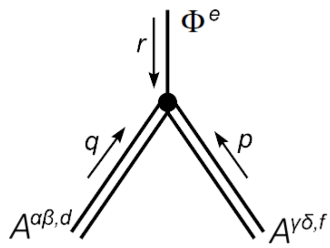

Figure 4.1: Feynman diagram

In a few following steps we will explain the procedure to calculate contribution of this vertex.

Step 1: changing indices

First of all we need to consider indices. On the figure we can see that we already used indices α and β which are same as in (4.17) - that means we can not use these indices in the Lagrangian. Let us use this substitution: $\alpha \to \kappa$ and $\beta \to \lambda$. (4.17) then becomes

$$\mathcal{L}_3^{AA} \sim -\frac{2}{F}\left(A_a^{\kappa\sigma}\partial_\sigma A_b^{\mu\nu}\partial^\lambda\phi_c + \partial_\sigma A_a^{\mu\nu}A_b^{\kappa\sigma}\partial^\lambda\phi_c\right)\kappa_3^{AA}\varepsilon_{\mu\nu\kappa\lambda}d^{abc}. \qquad (4.18)$$

Step 2: using basic Feynman rules

Next we can imply basic Feynman rules. By that we mean transformation that gives to all Lagrangian terms its mathematical representation. One of these rules is that whenever we have a derivation in Lagrangian, to this term belongs its impulse with index same as the index of the derivation. The sign of the impulse depends on the fact if this is incoming ($-$) or outgoing ($+$) impulse. Because of the different indices in the Lagrangian and in its Feynman diagram we have to multiply this impulse by metric tensor. Let us show this on particular term $A_a^{\kappa\sigma}$. Let us pressume that we want to identify term $A_a^{\kappa\sigma}$ from (4.18) with $A_a^{\alpha\beta}$, i.e. terms with Latin index a with terms with index d. Feynman rules in that case dictate that we have to write this mathematical representation of $A_a^{\kappa\sigma}$:

$$A_a^{\kappa\sigma} \xrightarrow{\text{Feynman rule}} \frac{1}{2}(g_{\kappa\alpha}g_{\sigma\beta} - g_{\sigma\alpha}g_{\kappa\beta})\delta^{ad} \tag{4.19}$$

and for the other term $A_a^{\mu\nu}$ with derivation,

$$\partial_\sigma A_a^{\mu\nu} \xrightarrow{\text{Feynman rule}} (-ip_\sigma)\frac{1}{2}(g_{\mu\alpha}g_{\nu\beta} - g_{\nu\alpha}g_{\mu\beta})\delta^{ab}. \tag{4.20}$$

Step 3: multiplication of particular Feynman rules

Using this example we can write Feynam rule for (4.18) after multiplication by i as

$$\begin{aligned} V^{ebf}_{\alpha\beta\gamma\delta} = -i\frac{2}{F}\Bigg[&\frac{1}{2}(g_{\kappa\alpha}g_{\sigma\beta} - g_{\sigma\alpha}g_{\kappa\beta})\delta^{ad}(-iq_\sigma)\frac{1}{2}(g_{\mu\gamma}g_{\nu\delta} - g_{\nu\gamma}g_{\mu\delta})\delta^{bf}(-ir_\lambda)\delta^{ce}+ \\ &+(-ip_\sigma)\frac{1}{2}(g_{\mu\alpha}g_{\nu\beta} - g_{\nu\alpha}g_{\mu\beta})\delta^{ad}\frac{1}{2}(g_{\kappa\gamma}g_{\sigma\delta} - g_{\sigma\gamma}g_{\kappa\delta})\delta^{bf}(-ir_\lambda)\delta^{ce}\Bigg]\times \\ &\times \kappa_3^{AA}\varepsilon_{\mu\nu\kappa\lambda}d^{abc}. \end{aligned} \tag{4.21}$$

Finally we get

$$V^{ebf}_{\alpha\beta\gamma\delta} = \frac{i}{F}r_\lambda(q_\beta\varepsilon_{\gamma\delta\alpha\lambda} - q_\alpha\varepsilon_{\gamma\delta\beta\lambda} + p_\delta\varepsilon_{\alpha\beta\gamma\lambda} - p_\gamma\varepsilon_{\alpha\beta\delta\lambda})\kappa_3^{AA}d^{def}. \tag{4.22}$$

Gell-Mann matrices

The Gell-Mann matrices are one of the possible representations of the infinitesimal generators of the special unitary group SU(3) with dimension eight. That means that it has a set with eight linearly independent generators g_i, wwhere $i1\ldots 8$. These generators satisfy conditions [11]

$$[g_i, g_j] = if^{ijk}g_k, \tag{4.23}$$

where f^{ijk} are completely antisymmetric structure constants. A particular choice of mentioned representation is $g_i = \lambda_i/2$, where [11]

$$\begin{gathered} \lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ \lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \\ \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \lambda_8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}. \end{gathered} \tag{4.24}$$

are known as the Gell-Mann matrices.

The most important properties are [11]:

$$\lambda_i^\dagger = \lambda_i, \qquad [\lambda^a, \lambda^b] = 2if^{abc}\lambda^c, \qquad \{\lambda^a, \lambda^b\} = \frac{4}{3}\delta^{ab} + 2d^{abc}\lambda^c \tag{4.25}$$

and

$$\langle\lambda^a\rangle = 0, \qquad \langle\lambda^a\lambda^b\rangle = 2\delta^{ab}, \qquad \langle\lambda^a\lambda^b\lambda^c\rangle = 2(d^{abc} + if^{abc}), \tag{4.26}$$

where d^{abc} is totally symmetric and f^{abc} is antisymmetric tensor. Non-zero elements of these tensors are shown in tables below.

abc	118	146	157	228	247	256	338	344
d_{abc}	$\frac{1}{\sqrt{3}}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{\sqrt{3}}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{\sqrt{3}}$	$\frac{1}{2}$
abc	355	366	377	448	558	668	778	888
d_{abc}	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$

Table 4.1: Totally symmetric non-vanishing d symbols of SU(3)

abc	123	147	156	246	257	345	367	458	678
f_{abc}	1	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{2}$

Table 4.2: Totally antisymmetric non-vanishing f symbols of SU(3)

Dirac γ-matrices

Dirac gamma matrices, $\{\gamma^0, \gamma^1, \gamma^2, \gamma^3\}$, are a set of conventional matrices with specific anticommutation relations. These matrices ensure that they generate a matrix representation of the Clifford algebra $C\ell_{1,3}(\mathcal{R})$ and they are defined as [11]

$$\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \qquad \gamma^1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix},$$
$$\gamma^2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix}, \qquad \gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}. \tag{4.27}$$

It is useful to define the product of the four gamma matrices as follows:

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}. \tag{4.28}$$

We can define γ^5 also as

$$\gamma^5 = \frac{i}{4!}\varepsilon_{\mu\nu\alpha\beta}\gamma^\mu\gamma^\nu\gamma^\alpha\gamma^\beta. \tag{4.29}$$

The most important properties of Dirac matrices are [11]:

$$\gamma^\mu\gamma_\mu = 4I_4, \qquad \gamma^\mu\gamma^\nu\gamma_\mu = -2\gamma^\nu, \qquad \gamma^\mu\gamma^\nu\gamma^\rho\gamma_\mu = 4\eta^{\nu\rho}I_4, \tag{4.30}$$

$$\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_\mu = -2\gamma^\sigma\gamma^\rho\gamma^\nu, \qquad \gamma^\mu\gamma^\nu\gamma^\lambda = \eta^{\mu\nu}\gamma^\lambda + \eta^{\nu\lambda}\gamma^\mu - \eta^{\mu\lambda}\gamma^\nu - i\varepsilon^{\sigma\mu\nu\lambda}\gamma_\sigma\gamma^5, \tag{4.31}$$

$$\langle\gamma^\mu\rangle = 0, \qquad \langle\gamma^\mu\gamma^\nu\rangle = 4\eta^{\mu\nu}, \qquad \langle\gamma^5\rangle = \langle\gamma^\mu\gamma^\nu\gamma^5\rangle = 0, \tag{4.32}$$

and

$$\langle\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\rangle = 4(\eta^{\mu\nu}\eta^{\rho\sigma} - \eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho}), \qquad \langle\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma^5\rangle = -4i\varepsilon^{\mu\nu\rho\sigma}. \tag{4.33}$$

Feynman slash notation

Let us assume that A is an covariant vector. Then a product of Dirac matrix and this vector we can define as

$$\not{A} = \gamma^\mu A_\mu. \tag{4.34}$$

Levi-Civita tensor

The Levi-Civita symbol, also called the permutation symbol, antisymmetric symbol, or alternating symbol, is a mathematical symbol used in particular in tensor calculus. In our case we use Levi-Civita symbol in four dimensions defined as

$$\varepsilon_{ijkl} = \varepsilon^{ijkl} = \begin{cases} +1 & \text{if } (i,j,k,l) \text{ is an even permutation of } (0,1,2,3) \\ -1 & \text{if } (i,j,k,l) \text{ is an odd permutation of } (0,1,2,3) \\ 0 & \text{otherwise} \end{cases} \tag{4.35}$$

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