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Stressed Value-at-Risk: Assessing extended Basel II regulation

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Declaration of Authorship

The author hereby declares that he compiled this thesis independently, using only the listed resources and literature.

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Prague, May 17, 2013

Signature

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Abstract

This thesis investigates recently proposed enhancements to the Basel II market risk framework. The Basel Committee on Banking Supervision introduced a stressed Value-at-Risk, calculated from one year long period of financial stress, to be added to the current VaR as a reaction to the shortage in capital reserves of banks and thus their inability to cover extensive losses observed during the recent crisis. We present an empirical evidence that such an extension of the regulatory capital is not optimal. Firstly, supplementing an unconditional methods of VaR estimation, i.e. normal parametric VaR and historical simulation, by SVaR only lead to unnecessarily high capital requirements even in a low volatile periods whilst the same amount of capital during the crisis could be achieved using either the conditional GARCH VaR with student's-t innovations or the volatility weighted historical simulation. Moreover, we showed that all unconditional methods fail to capture volatility clusters such as the 2008 crisis.

JEL Classification	C51, C52, C53, C58, G01, G28, G32
Keywords	Value-at-Risk, Stressed Value-at-Risk, Basel II, Basel 2.5, GARCH

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Abstrakt

Tato práce se zabývá nedávným rozšířením rámce pro tržní riziko Baselu II v němž Basilejský výbor pro bankovní dohled představil takzvaný "stressed Value-at-Risk" model, jenž se odhaduje z jeden rok dlouhého období finančního stresu, který musí být přičten k současné denní hodnotě VaRu. Toto opatření je reakcí Basilejského výboru na nedostatečný kapitál bank a jejich neschopnost pokrýt ztráty způsobené nedávnou finanční krizí. Představujeme zde empirické důkazy, že navrhované rozšíření není optimální. Zaprvé, doplňování nepodmíněných metod odhadu VaRu, jako je například parametrická metoda předpokládající normální rozdělení denních ztrát nebo historická simulace, o hodnotu modelu SVaR má za následek zbytečně vysoké kapitálové požadavky a to i v dobách nízké volatility trhů. Zatímco stejné hodnoty kapitálových požadavků může být dosaženo pomocí podmíněných metod, jako například

GARCH model s použitím studentova rozdělení nebo volatilitou vážená historická simulace. Zadruhé, všechny nepodmíněné modely selhaly v zachycení klastrů během nedávné krize.

Klasifikace JEL C51, C52, C53, C58, G01, G28, G32
Klíčová slova Value-at-Risk, Stressed Value-at-Risk, Basel II, Basel 2.5, GARCH

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Acronyms

BIS	Bank for International Settlements
BCBS	Basel Committee on Banking Supervision
VaR	Value at Risk
SVaR	Stressed Value at Risk
HS	Historical Simulation
GARCH	Autoregressive Conditional Heteroskedasticity
EWMA	Exponentially Weighted Moving Average
iid	independent identically distributed

Chapter 1

Introduction

Value-at-Risk (VaR) has been established as a device for measuring market risk in the 1996 Amendment to the first Basel Accord. Banks who choose so called *internal model approach* to calculate the minimum capital requirements have to estimate VaR on a daily basis. Nevertheless, a particular method to obtain an accurate VaR forecast has not been prescribed yet, which led to wide adoption of models based on either normal distribution or historical simulation, the methods which have been criticized by many authors over the past decade. During the onset of the 2008 global financial crisis many of those banks did not manage to cover severe losses that they experienced. The capital which they have to set aside in case of adverse market movements was clearly insufficient and hence the Basel Committee on Banking Supervision (BCBS) in the latest enhancement to the capital adequacy framework proposed several changes. One of them is that a so called *stressed VaR* (SVaR) number have to be added to the ordinary VaR in order to ensure that this shortage in capital will not recur in the future. SVaR is intended to replicate VaR but it has to be calculated from one year long period of significant financial stress relevant to the institution's portfolio.

The aim of this thesis is twofold. At first we will review several current methods to estimate VaR and investigate their performance on daily returns of the S&P 500 index. Although similar researches were conducted in the past, most of them are outdated and with data from the recent crisis we will be able better assess each model's performance. Moreover, we included the conditional version of historical simulation of Hull & White (1998) which have not been tested on recent data yet (to our best knowledge).

The second target is to assess a contribution of a stressed VaR to the current market risk framework. To the author's best knowledge there is only one research concerning an application of proposed changes which were conducted by Rossignoloa *et al.* (2012). However, we propose a slightly different approach by assuming that these proposals have been applied in the beginning of 1990 allowing us to test SVaR performance not only during the 2008 crisis but also before and after this period when the markets were calm and thus required capital was lower.

Furthermore, we present a graphical analysis of each model performance during this crisis as well as comparisons of the new and the old minimum capital frameworks. The hypothesis we want to test is whether the additional measure really is essential for bank to meet market losses or if the desired level of capital could be achieved within old framework using conditional models and, most importantly, without overstating risk in non-stressed periods.

The Thesis is structured as follows. Chapter 2 at first briefly recaps development of the regulatory framework from the first Basel Accord to the latest proposal called Basel 2.5. Moreover, an academic response is discussed as well as a review of the literature concerning an application of VaR in practice. The last section is devoted to the stressed VaR literature. Chapter 3 introduces a value-at-risk concept and provides an overview of selected methods of its estimation. Also two formal tests for an evaluation of VaR performance are discussed. Chapter 4 conducts an empirical analysis which is divided into three parts. The first part investigates performance of selected VaR models, then the SVaR is estimated and finally all models are implemented into new and old regulatory framework. Finally, Chapter 5 concludes.

Chapter 2

Literature Review

This chapter is divided into three parts. Firstly, we review current regulatory framework focusing mainly on market risk, then we deal with a VaR application in practise and finally we review a stressed VaR related literature.

2.1 Regulatory framework

Before we begin discussing the latest Basel proposals, we should recap a development of international bank regulation in the past two decades. Since 1988, when the first Basel Accord has been introduced, there have been a structural change in the financial markets, especially a development of complex financial products such as off-balance sheet derivatives, swaps, mortgage securitization, etc. This process, commonly known as the financial engineering, is together with unsatisfactory regulation according to many people partly responsible for the recent financial crisis. Whilst this is a subject of discussion and definitely not the topic of this thesis, we will look how these changes affect the banking regulation.

We start with a short history lesson of the banking regulation since 1988 (major source for this part is Crouhy *et al.* (2006, chap. 3)), then we look at the academic response to the Basel II and finally we review the latest amendment called the Basel 2.5.

2.1.1 Basel I

The first Basel rules were published in 1988 after a discussion among bankers of the Federal Reserve Bank and the Bank of England. They started to concern persisting regulatory framework which they view as unsatisfactory. Due to the increasing investments of banks into off-balance-sheet products as well as loans to the third world countries, the uniform minimum capital requirement standard became insufficient.

The Basel I was implemented in 1992 and focused mainly on credit risk. The minimum regulatory capital was no longer uniform but depended on bank's credit exposures. Borrowers were divided into several groups based on their riskiness and each group had different capital requirements. But this division was in many cases illogical and did not express the true amount of risk of each group. Furthermore, the market risk was completely ignored which led to transforming credit risk into "capital cheaper" market risk instruments.

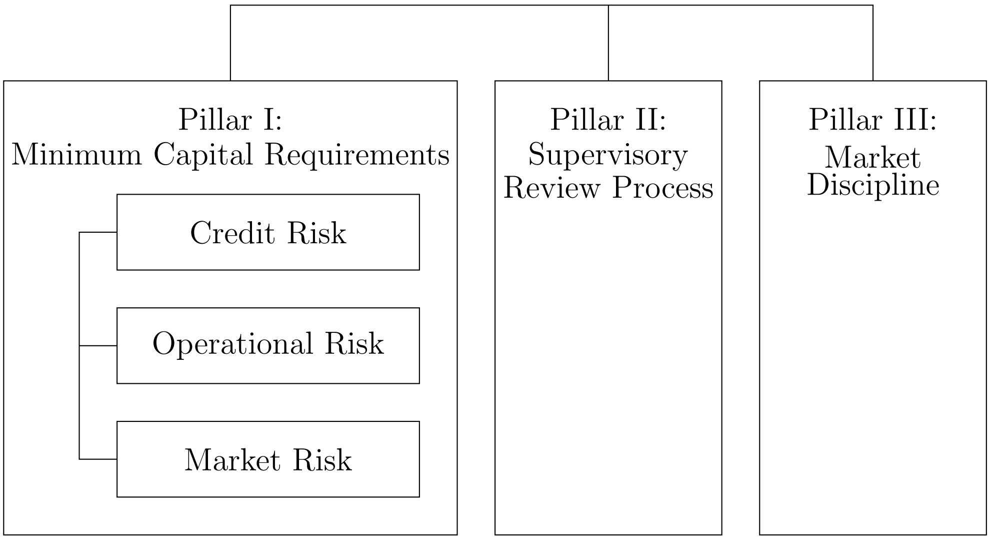
A step forward made the Basel Committee on Banking Supervision (1996) Basel I Amendment implemented in 1998. For the first time market risk was included in the regulatory capital computation. The institutions were allowed to choose between two methods to obtain the amount of capital to cover their exposures to market risk - the *standardized approach* and the *internal model approach*. Whilst the former method prescribes using the exact method as it is specified in the 1996 Amendment, the latter allows each bank to use its own value-at-risk model. Although the *internal model approach* does not specify the particular VaR model, it requires each bank to meet the qualitative standards such as the independent risk control unit responsible for the risk management system, a regular backtesting programme, a rigorous stress-testing programme, etc. It's also worth noting here that also the credit risk computation was revised in the 1996 Amendment but we will not go into details because it is not a topic of this thesis.

2.1.2 Basel II

The Basel I rules were flawed for several reasons connected mainly with credit risk (see Crouhy *et al.* 2006, pp. 68-70). These shortcomings resulted in a "regulatory arbitrage" which is a situation when bank "modify its behaviour so that it incurs lower capital charges while still incurring the same amount of actual risk" (Crouhy *et al.* 2006, p. 69) This is usually done by using securitization (for example mortgage backed securities such as collateralized debt obligations) and credit derivatives. In order to dispose of this problem regulators had to publish modified Basel rules which do not give banks incentives to bend the rules. In other words, the regulatory capital should better reflect the risk that a bank is taking.

The Basel II was released in 2004 with an intention to replace the 1988 Accord. New rules endeavoured to overcome above mentioned shortcomings through establishing so called "three pillars" capital regulation framework (see Figure 2.1).

Figure 2.1: Basel II three pillar framework



Source: Basel Committee on Banking Supervision (2006)

Pillar I is unambiguously the most important. Its objective is to revise the Basel I Accord of 1988, that is to introduce minimum capital requirements which better correspond to the bank's risk profile. The 1988 Accord accounted only for the credit risk and the market risk was further added in its 1996 Amendment, the Basel II goes step forward and introduces operational risk - losses caused by frauds by employees, computer failures, human errors, etc. The institutions are allowed to choose between three methods to obtain the credit risk capital: the standardized approach and two internal rating-based approaches (IRB). While the standardized approach relies on credit rating agencies, under the IRB approach banks could use their own assessment of credit risk, provided they meet qualitative criteria set by the Basel II Accord. Market risk is calculated using the same approach as it was defined in the 1996 Amendment.

Pillar II is designed to ensure that banks measure their risk correctly and its main objective is to dispose of the regulatory arbitrage. In other words, this pillar concerns mainly the supervisory approach to risk management.

Pillar III deals with banks disclosure of risk information to the investors.

2.1.3 Basel II Criticism

Despite the improvement of the regulatory framework in the Basel II proposal, many academics expressed their concerns about its ability to ensure stability of the global financial system. Danielsson *et al.* (2001) in their *Academic response to the Basel II* discuss several issues.

They view the risk as an endogenous variable whereas existing models (VaR) treat it as an exogenous process. They equate risk managers behaviour to the weather forecasting. The risk managers believe that risk forecasting will not affect the markets, similarly as the meteorologists does not affect the weather. But the markets (and so it is volatility) are partly determined by their behaviour. While this failure in assumption is relatively harmless in normal times when the participants behaviour is heterogeneous, during the crises when the behaviour of the market participants become more homogeneous this may cause extensive losses. Danielsson (2002) further discusses this issue and states that "a risk model breaks down when used for regulatory purposes." He illustrates this limitation of risk models on events during the 1998 Russia crisis. It was shortly after implementation of VaR models into the regulatory framework, therefore when volatility increased as the 1997 Asian crisis struck the broadly used VaR models led to application of similar trading strategies and worsened the crisis.

Another issue they discuss is the normality assumption which results in a misleading VaR estimates. It has been proven that daily returns (for example of equity indices) are not normally distributed and exhibit fat tails and clusters. This is further discussed in Section 3.3.

They also criticize the reliance on credit rating agencies for the standardised approach to credit risk measurement, controversial operational risk modelling and inherent procyclicality of financial regulations, but these topics are out of scope of this thesis.

2.1.4 Basel 2.5

As a reaction to the significant losses that occurred during the 2007-2008 crisis the Basel Committee on Banking Supervision (2009b) in the *Revisions to the Basel II market risk framework* suggest several improvements. They view the capital framework based on the 1996 Amendment as unsatisfactory and hence they introduce two main enhancements:

- An Incremental Risk Charge (IRC), which attempts to capture default as well as migration risk for unsecuritized credit products.
- A stressed Value-at-Risk (SVaR), which requires banks to calculate an additional VaR measure based on one-year data from a period of significant financial stress related to the banks portfolio.

The former is related to credit risk and we do not endeavour to assess its impact on banks in this thesis. The latter, on the other hand, is the major topic of our interest and it would be convenient to take up with this concept.

Under the 1996 Amendment banks that chose internal model approach to calculate market risk must have computed a capital requirements "as the higher of (i) its previous day's Value-at-Risk number (VaR_{t-1}) and (ii) an average of the daily value-at-risk measures on each of the preceding sixty business days (VaR_{avg}), multiplied by a multiplication factor (m_c)" (Basel Committee on Banking Supervision 2005, p. 41). Where the multiplication factor m_c has a value between 3 and 4 based on the model's *ex-post* performance in the past. Under the Basel 2.5 a capital requirement is expressed as the sum of the 1996 Amendment charge, which we just described, and "the higher of (i) latest available stressed Value-at-Risk number ($SVaR_{t-1}$) and (ii) an average of the stressed Value-at-Risk numbers over the preceding sixty business days ($SVaR_{avg}$), multiplied by a multiplication factor (m_s)" (Basel Committee on Banking Supervision 2009b, p. 15) We may summarize it using the following formula:

$$MCR = \max\{VaR_{t-1}, m_c \cdot VaR_{avg}\} + \max\{SVaR_{t-1}, m_s \cdot SVaR_{avg}\}, \quad (2.1)$$

The multiplication factor m_s is set by supervisory authorities but has minimal value of 3 plus an additional factor k which has value between 0 and 1 based on VaR (but not SVaR) performance in a backtest.

Stressed VaR measure is "intended to replicate a VaR calculation if the relevant market factors were experiencing a period of stress; and should be therefore based on the 10-day, 99th percentile, one-tailed confidence interval VaR measure of the current portfolio, with model inputs calibrated to historical data from continuous 12 month period of significant financial stress to the bank's portfolio" (Basel Committee on Banking Supervision 2009b, p. 14). The BCBS requires banks to calculate SVaR at least weakly and no particular model is prescribed. Furthermore, BCBS suggests the financial turmoil in

2007-2008 to be used as the stress period, although other periods must be also considered.

However, one issue discussed by Standard & Poor's Financial Services (2012) is that the banks have to use proxies for securities that did not exist in a used stress period (for example during the 2008 crisis) and this choice may result in different SVaR estimates. They also show on a survey of 11 large international banks that SVaR composes the largest part of the Basel 2.5 risk charge of 29%. On the contrary, they claim that the banks which use the standardised approach for the regulatory capital computation are less affected by the Basel 2.5.

Basel Committee on Banking Supervision (2009a) conducted the quantitative impact study of proposed Basel 2.5 revisions. Using data from 38 banks from 10 countries stressed VaR numbers were compared with non-stressed VaR measures. On average SVaR was 2.6 times higher than non-stressed VaR and its introduction on average resulted in 110% increase in the minimal capital requirements for market risk.

2.2 VaR in Practice

Although VaR has been pushed by regulators since 1996 Amendment, the first empirical research concerning the accuracy of VaR models used by the financial institutions was conducted by Berkowitz & O'Brien (2002). They showed on historical P&L and daily VaR estimates obtained from 6 large U.S. banks that these estimates are rather conservative and tend to overestimate risk. Only one bank reported 6 violations during 581 trading days, the others experienced only from 0 to 3 violations during the same period. Moreover, they applied the GARCH model to each bank's P&L and the results outperformed VaR forecasts obtained by banks. Similarly, Pérignon *et al.* (2008) examined VaR estimates of the six largest banks in Canada. The results shows popularity of a historical simulation and, most importantly, extremely low number of violations. There were only two violations out of the expected number of 74. This result even more support findings from the previous research. However, they claim that this overstatement of market risk is caused by bank's cautiousness rather than their inability to measure risk correctly. Pérignon & Smith (2010a) further investigated this overestimation of VaR and tested if it is caused by incomplete accounting for diversification effect among broad risk categories. Nevertheless, they did not find any signs of underestimation of this effect and hence they view VaR as a biased risk tool. It should be noted that above mentioned researches were conducted before the 2008 crisis and hence did not contain this most volatile period.

2.3 Stressed VaR literature

Probably the only research concerning the application of the new Basel proposals was conducted by Rossignoloa *et al.* (2012). They analysed the accuracy of several VaR models to compute minimum capital requirements within the old and the proposed framework on the data from Emerging and Frontier stock markets. The results suggest that using the Extreme value theory (EVT) may make SVaR redundant.

Different possibilities how to calculate a valid stressed VaR number are reviewed in Basel Committee on Banking Supervision (2011) working paper named *Messages from the academic literature on risk measurement for the trading book*. One method of conducting a stressed VaR estimate discussed here is stressing the correlation matrix used in all VaR methodologies. However, there is a problem with rising correlation among all risk-factors in the correlation matrix during crisis periods such as the 2008 financial turmoil.

Alexander & Ledermann (2012) introduce a random orthogonal matrix (ROM) simulation as a device for VaR and SVaR computation. They endeavour to overcome above mentioned issues through generating a large number of stress testing data. Their approach also accounts for higher moments such as skewness and kurtosis which are known to increase rapidly during crisis periods.

Chapter 3

Theoretical Framework

The aim of this chapter is to provide rigorous overview of a theoretical background required for the empirical part of this thesis. We start with a basic concepts, then we move to Value-at-Risk model - its definition and approaches to computation. The largest part of this chapter is devoted to describing each individual model that we are using in the empirical research. The last part explains formal statistical test to evaluate model's performance in a backtest.

3.1 Basic Concepts

3.1.1 Loss Distribution

Before we start describing individual models for financial risk, we must formally define basic concepts such as profit-and-loss (P&L) distribution, risk factors and mapping. We will adapt a theoretical framework from McNeil *et al.* (2005).

P&L distribution can be defined as a distribution of changes in a value of a portfolio, i.e. the changes in value $V_{t+1} - V_t$, where V_t and V_{t+1} are the portfolio values at time t and $t+1$, respectively. However, according to McNeil *et al.* (2005), in risk management we are concerned with the probability of large losses, therefore, we will use the loss distribution instead of P&L. Then

$$L_{t+1} := -(V_{t+1} - V_t) \tag{3.1}$$

is the loss of the portfolio from day t to day $t+1$.

McNeil *et al.* (2005) follows standard risk-management practice and models the value V_t as a function of time t and a d-dimensional vector of risk factors $\mathbf{Z}_t$ (logarithmic prices of financial assets, exchange rates, yields, etc.). Then a portfolio value is

$$V_t = f(t, \mathbf{Z}_t), \tag{3.2}$$

where f is a function $f : \mathbb{R}_+ \times \mathbb{R}^d \to \mathbb{R}$. This representation of the portfolio value is called a *mapping* of risks.

The series of risk factor changes $(\mathbf{X}_t)_{t\in\mathbb{N}}$ is defined by $\mathbf{X}_t := \mathbf{Z}_t - \mathbf{Z}_{t-1}$. Then we can write the portfolio loss as:

$$L_{t+1} = -(f(t+1, \mathbf{Z}_t + \mathbf{X}_{t+1}) - f(t, \mathbf{Z}_t)). \tag{3.3}$$

Because risk factors are observable at time t, so it is $\mathbf{Z}_t$. The series of risk-factor changes $\mathbf{X}_{t+1}$ therefore determine the loss distribution. Hence McNeil *et al.* (2005) introduces another piece of notation, the *loss operator* $l_{[t]} : \mathbb{R}^d \to \mathbb{R}$, which maps risk factor changes into losses.

$$l_{[t]}(\mathbf{x}) := -(f(t+1, \mathbf{Z}_t + \mathbf{x}) - f(t, \mathbf{Z}_t)), \qquad \mathbf{x} \in \mathbb{R}^d. \tag{3.4}$$

We can see that $L_{t+1} = l_{[t]}(\mathbf{X}_{t+1})$.

To represent loss as a linear function of the risk-factor changes, McNeil *et al.* (2005) uses a first-order approximation L^{Δ}_{t+1} of (3.3).Then

$$L^{\Delta}_{t+1} := -\left(f_t(t, \mathbf{Z}_t) + \sum_{i=1}^{d} f_{z_i}(t, \mathbf{Z}_t) X_{t+1,i} \right), \tag{3.5}$$

and similarly for the *loss operator*

$$l^{\Delta}_{[t]}(\mathbf{x}) := -\left(f_t(t, \mathbf{Z}_t) + \sum_{i=1}^{d} f_{z_i}(t, \mathbf{Z}_t) x_i \right), \tag{3.6}$$

where subscripts to f denote partial derivatives. This approximation is the best provided small risk-factor changes and portfolio value linear in the risk-factors.

In this thesis we consider portfolio of single stock (represented by stock index). Now we will derive a loss of this portfolio using an approach of McNeil *et al.* (2005, p. 29) but setting $d = 1$. Lets denote the price process of our stock by $(S_t)_{t\in\mathbb{N}}$. As a risk factor we use logarithmic prices, i.e. $Z_t = \ln(S_t)$, since this is a standard practise in risk management. Then we can write the risk factor changes as $X_{t+1} = \ln(S_{t+1}) - \ln(S_t)$ and the portfolio value as $V_t = \exp(Z_t)$. Then

$$L_{t+1} = -(V_{t+1} - V_t) = -S_t \cdot (\exp(X_{t+1}) - 1),$$

and the linearised loss L^{Δ}_{t+1} is given by

$$L^{\Delta}_{t+1} = -S_t \cdot X_{t+1}.$$

Since we have only one stock in our portfolio and most of daily returns are small, this approximation is quite well. Moreover, we set the total portfolio value $V_t = S_t = 1$. Then

$$L^{\Delta}_{t+1} = -X_{t+1}. \tag{3.7}$$

Therefore (L_{t+1}) is the process of negative log-returns.

This seems like a complicated approach to derive an obvious fact, but it is because we have a portfolio of single stock. A *mapping* of risk is very important when we have more then one risk factor. Consider an investment into USD/euro call option. Then the value of a position depends on the USD/euro exchange rate, the dollar and euro interest rates over the maturity of the option and on the USD/euro volatility (Crouhy *et al.* 2006, pp. 161-162). Although we are considering the portfolio represented by a single stock index (and hence we set $d = 1$ in (3.4), (3.5) and (3.6)), proper definitions are necessary to show what simplification are we using.

3.1.2 Conditional and Unconditional Loss Distribution

Both conditional and unconditional models are presented in this thesis, therefore it is convenient to properly define these concepts. Again, we will use theoretical scope presented by McNeil *et al.* (2005) to retain consistency.

Suppose that we have a stationary distribution $F_{\mathbf{X}}$ of the series of risk-factor changes $(\mathbf{X}_t)_{t\in\mathbb{N}}$ and $\mathcal{F}_t = \sigma(\{\mathbf{X}_s : s \leq t\})$ is the sigma algebra representing the publicly available information at time t. The distribution $F_{\mathbf{X}_{t+1}|\mathcal{F}}$ is the conditional distribution of $\mathbf{X}_{t+1}$ given current information $\mathcal{F}_t$. For example $\mathbf{X}_{t+1}$ can follow GARCH process (more in Section 3.3.3). On the other hand, $F_{\mathbf{X}}$ is not conditional on any past values of $\mathbf{X}_t$.

Now we can define the *conditional loss distribution* $F_{L_{t+1}|\mathcal{F}}$ as the distribution of the loss operator $l_{[t]}(\cdot)$ under $F_{\mathbf{X}_{t+1}|\mathcal{F}}$. Formally (McNeil *et al.* 2005, p. 28):

$$F_{L_{t+1}|\mathcal{F}}(l) = P(l_{[t]}(\mathbf{X}_{t+1}) \leq l \mid \mathcal{F}_t) = P(L_{t+1} \leq l \mid \mathcal{F}_t), \qquad l \in \mathbb{R}, \tag{3.8}$$

and the *unconditional loss distribution* $F_{L_{t+1}}$ as the distribution of the loss operator $l_{[t]}(\cdot)$ under stationary distribution $F_{\mathbf{X}}$ of risk-factor changes.

3.2 Value-at-Risk

Value at Risk (VaR) is an easy concept for measuring risk. Its definition is very straightforward and hence it is widely used among risk managers. In this section we will look closer at the VaR concept.

3.2.1 Value-at-Risk Definition

VaR is defined as a loss in a market value of a portfolio over the given time horizon t which will not be exceeded at a given probability p provided the portfolio is hold static over the time horizon. Mathematically VaR can be defined as (McNeil *et al.* 2005, p. 38):

Definition 3.1 (Value-at-Risk). Given some confidence level $\alpha \in (0,1)$. The VaR of our portfolio at the confidence level α is given by the smallest number l such that the probability that the loss L exceeds l is no larger than $(1-l)$. Formally:

$$VaR_\alpha = inf\{l \in \mathbb{R} : P(L > l) \leq 1 - \alpha\} = inf\{l \in \mathbb{R} : F_L(l) \geq \alpha\} \tag{3.9}$$

Where the $F_L(l) = P(L \leq l)$ is a distribution function of the underlying loss distribution. As McNeil *et al.* (2005) points out, VaR is simply a quantile of the corresponding loss distribution, thus can be computed easily. Values for the confidence level α are most often 0.95 and 0.99, and the time horizon t is typically 1 or 10 days.

One of the biggest drawbacks of VaR, that has been discussed by many academics in the past two decades, is that we do not have any information about losses which occur with a probability lower then $(1-\alpha)$. These could be far beyond the value of VaR_α. It is very important to understand that the VaR does not represent maximal loss over the time horizon. Instead it represents loss that will not be exceeded within the given probability. Therefore one can loose almost entire value of a portfolio although the VaR estimate was only few percent. This is given by the underlying distribution which is never known in practice.

3.2.2 Value-at-Risk Estimation Methods

Generally speaking, there are three steps in a VaR calculation. At first, the institution has to identify a restricted number of risk-factors which effects the institutions portfolio and derive the value of the portfolio in terms of risk-factor changes. This is called the *mapping of risk* and it is described in Section 3.1. Once risk-factors have been identified, the risk manager must derive the distribution of risk-factor changes - this is the most important step in VaR calculation and also the most problematic. The last step is straightforward - at a chosen significance level α the VaR number can be read off as a $1 - \alpha$ quantile of the loss distribution.

There are many different methods how the loss distribution can be derived, but the most of them stem from one of the three basic categories:

- Parametric models,
- Historical simulation,
- Monte Carlo simulation.

The first model makes an assumption of parametric distribution for the risk-factor changes, i.e. normal or student's-t distribution. Historical simulation uses large amount of historical data to estimate the empirical distribution but makes no parametric assumptions. The Monte Carlo model, in its basic form, makes similar assumptions to the normal linear VaR model.

3.3 Parametric VaR

This section focuses on parametric approaches to measure risk. These methods differ in the distribution that is fitted to data. Crouhy *et al.* (2006) explains that assumptions that we make about the distribution should be consistent with an empirical return process that is characterized by the *stylized facts* - a collection of empirical features of financial time series. We present here a version of McNeil *et al.* (2005, p. 117):

1. Return series are not iid although they show little serial correlation.
2. Series of absolute or squared returns show profound serial correlation.
3. Conditional expected returns are close to zero.

4. Volatility appears to vary over time.

5. Return series are leptokurtic or heavy-tailed.

6. Extreme returns appear in clusters.

The last two are also the most important because both are not satisfied by the unconditional normal distribution - the easiest way of VaR estimation. The first problem is that the Gaussian distribution ignores an extreme rare events (large profits and losses) and this is certainly not a property of a good risk measure. The solution for this could be to assume a fat-tailed student's-t distribution instead of the normal. But this will not solve the last issue - the *volatility clustering*. McNeil *et al.* (2005, p. 118) compared simulated normal and student's-t processes with DAX return data from 10 year period. It is shown that neither normal or student's-t model is able to simulate volatility clusters, i.e. a situation when an extreme return observed on one day is followed by another extreme return, which does not need to have the same sign. In other words, a large profit could be followed by large loss and vice versa. In order to take account of this phenomenon we should assume the distributions of returns which is conditional on a volatility process which is able to capture *volatility clustering*. For example the GARCH(1,1) process of Bollerslev (1987) seem to have this property.

In the rest of this section we further look at the normal and the student's-t model (both unconditional and conditional versions).

3.3.1 Unconditional Normal VaR

The easiest way to compute VaR is to assume that the loss distribution is normal with mean μ and variance σ^2. Fix $\alpha \in (0, 1)$, then (McNeil *et al.* 2005, p. 39)

$$VaR_\alpha = \mu + \sigma \Phi^{-1}(\alpha) \tag{3.10}$$

where $\Phi^{-1}(\alpha)$ denotes the α-quantile of the standard normal distribution function. As McNeil *et al.* (2005) remarks, if the time horizon is short and μ is close to zero, $VaR_\alpha^{mean} := VaR_\alpha - \mu$ is used for capital-adequacy purposes instead of ordinary VaR.

$$VaR_\alpha^{mean} = \sigma \Phi^{-1}(\alpha) \tag{3.11}$$

In this thesis we assume that mean is zero and therefore use formula 3.11.

The question is: why do we bother with explaining the unconditional normal VaR formula even though we criticized this model in the introduction to this section? Partly because we want to show these weaknesses in the empirical part of this thesis and compare this model with the other VaR models. But the main reason is to find out whether the model could be improved by adding some additional measure in form of a *stressed VaR* or if it only mitigates wrongly assumed normality.

3.3.2 Unconditional Student's-t VaR

One of the stylized facts of financial time series is that the distribution of daily returns (losses) is heavy tailed. Ignoring this feature and using normality assumption leads to an underestimation of VaR. One way to deal with heavy tails which are present in financial data is to use a student's-t distribution. Huisman *et al.* (1998) argues that t-distribution better fit to the empirical distribution than normal because of its ability to assign more probability to extreme events. Heaviness of tails is characterized by degrees of freedom ν of the student's-t distribution. As ν gets large the t-distribution converges to the normal.

To derive VaR formula based on student's t-distribution we apply the derivation of Dowd (2005, pp. 159-160). Since t-distribution has only one parameter - degrees of freedom ν, all moments could be written in terms of ν. Therefore the t-distribution with ν degrees of freedom has mean 0, provided $\nu > 1$; variance $\nu/(\nu-2)$, provided $\nu > 2$; zero skew, provided $\nu > 3$; and kurtosis of $3(\nu-2)/(\nu-4)$, provided $\nu > 4$. However, in risk measurement we prefer a *generalised t-distribution* defined as $t(a, b, \nu) = a + bt(\nu)$, where a and b are location parameters set by the user. This generalised t-distribution has mean a and variance $b^2\nu/(\nu-2)$, provided that they exist. Skew and kurtosis remains the same. Now we can assume that our losses are distributed as $L \sim t(\nu, \mu, \sigma^2)$, $E(L) = \mu$ and $var(L) = \sigma^2\nu/(\nu-2)$, provided that $\nu > 2$. Again we fix $\alpha \in (0, 1)$, then

$$VaR_\alpha = \mu + \sqrt{\frac{\nu-2}{\nu}}\,\sigma\, t_{\alpha,\nu}, \tag{3.12}$$

where μ, σ and ν are obtained from a sample of historical losses.

Huisman *et al.* (1998) compared performance of student's-t VaR with normal VaR using rolling window backtesting method over 5 year period of daily

S&P 500 returns. The results support the use of heavy tailed student's t-distribution. Discrepancies between two methods were more apparent at higher confidence levels which confirms that empirical returns have heavier tails than the normal distribution.

3.3.3 Conditional VaR Models

Up to this point we assumed that the loss distribution is unconditional and we calculated variance of this distribution using the historical moving average estimate. Dowd (2005, p. 128) mentions several issues of this method. One of them are equally weighted historical observations in this calculation - if we have a large data window (for example 1000 days) and an unusually high event occurs at time t, it will effect our volatility estimate for the next 1000 days, even though the high volatility period has passed a long time ago. One way to deal with this drawback is to use the exponentially weighted moving average (EWMA) model, which is based on estimating volatility from historical returns of the process but the weights decline exponentially as we move further to the past. Even though the EWMA model performs better than the equally weighted moving average volatility models, it has only one parameter and that is fixed over time. Therefore the model do not response to changes in market conditions satisfyingly. According to Dowd (2005, p. 131), the model will respond to the recent rises in volatility by predicting it to remain at current level, not to continue to rise, as would be more possible.

The best way to accommodate the last two stylised facts - heavy tails and volatility clustering - is to use the generalised autoregressive conditional heteroscedasticity (GARCH) model of Bollerslev (1987), which has been developed from the basic ARCH model suggested by Engle (1982). The introduction of GARCH model led to the birth of the whole family of GARCH-type models and even in its basic form with the Gaussian innovations is able to capture fatter than normal tails. The definition is (McNeil *et al.* 2005, p. 145):

Definition 3.2 (GARCH(p,q)). Let $(Z_t)_{t\in\mathbb{Z}}$ be SWN(0,1). The process $(X_t)_{t\in\mathbb{Z}}$ is a GARCH(p,q) process if it is strictly stationary and if it satisfies, for all $t \in \mathbb{Z}$ and some strictly positive-valued process $(\sigma_t)_{t\in\mathbb{Z}}$, the equations

$$X_t = \sigma_t \, Z_t, \qquad \sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i X_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2, \tag{3.13}$$

where $\omega > 0$, $\alpha_i \geq 0, i = 1, ..., p$, and $\beta_j \geq 0, j = 1, ..., q$.

The basic idea behind this model is allowing squared volatility σ_t^2 to depend on previous p squared values of the process X_{t-i}^2 (as the ARCH model) and also on previous q squared volatilities σ_{t-i}^2.

The most popular model is GARCH(1,1) with Gaussian innovations:

$$\sigma_t^2 = \omega + \alpha X_{t-1}^2 + \beta \sigma_{t-1}^2, \tag{3.14}$$

where $\omega \geq 0, \alpha, \beta \geq 0$ and $\alpha + \beta < 1$. The last condition is to ensure that this process is covariance-stationary. This model is popular because of its simplicity and a small number of parameters. Dowd (2005) explains the behaviour of the process through the parameters α and β - if the value of β is high, the volatility is persistent and takes long time to change, on the contrary, the high value of α implies a volatility which reacts very quickly to market changes.

In order to include the GARCH based volatility into our VaR model we must predict the volatility for the following day. Suppose that our data follow the GARCH(1,1) model. We therefore fit the model to our data and estimate model parameters ω, α and β using maximum likelihood estimation (MLE). A prediction of squared standard deviation is then given by:

$$\hat{\sigma}_{t+1}^2 = \omega + \alpha X_t^2 + \beta \hat{\sigma}_t^2, \tag{3.15}$$

where X_t is the last value of the process and $\hat{\sigma}_t$ is the last volatility estimate.

Now we derive conditional VaR using McNeil *et al.* (2005, p. 161). Suppose that we have the conditional loss distribution $F_{L_{t+1}|\mathcal{F}_t}$ (see Section 3.1.2) and the loss function $L_t = -X_t$, where $(X_t)_{t\in\mathbb{Z}}$ is a process of risk factor changes (i.e. log returns). Now we assume that the process of losses $(L_t)_{t\in\mathbb{Z}}$ follows a stationary model of the form $L_t = \mu_t + \sigma_t Z_t$, where Z_t is SWN(0,1) and μ_t and σ_t are based on $\mathcal{F}_{t-1}$ - publicly available information at time $t-1$. The conditional loss distribution is then (McNeil *et al.* 2005, p. 161):

$$F_{L_{t+1}|\mathcal{F}_t}(l) = P(\mu_{t+1} + \sigma_{t+1} Z_{t+1}) \leq l \mid \mathcal{F}) = G((l - \mu_{t+1})/\sigma_{t+1}) \tag{3.16}$$

Then the conditional VaR estimate is (McNeil *et al.* 2005, p. 161):

$$VaR_\alpha^t = \mu_{t+1} + \sigma_{t+1}\, q_\alpha(G) \tag{3.17}$$

where $q_\alpha(G)$ is an α-quantile of the function G and μ_{t+1} and σ_{t+1} are the estimates of mean and volatility. In the conditional normal VaR model $q_\alpha(G) = \Phi^{-1}(\alpha)$, in the conditional student's-t VaR $q_\alpha(G) = t_{\alpha,\nu}$ where ν are degrees of freedom estimated from the t-GARCH model. Moreover, we have to scale the quantile by the factor $\sqrt{(\nu-2)/\nu}$. Hence, we have:

$$VaR_\alpha^t = \mu_{t+1} + \sigma_{t+1}\,\Phi^{-1}(\alpha) \tag{3.18}$$

$$VaR_\alpha^t = \mu_{t+1} + \sigma_{t+1}\,\sqrt{(\nu-2)/\nu}\,t_{\alpha,\nu} \tag{3.19}$$

where (3.18) represents the normal GARCH VaR and (3.19) represents the t-GARCH VaR.

3.4 Historical VaR

Historical simulation is known to be one of the most popular among risk managers. Pérignon & Smith (2010b) in their survey of the level of VaR disclosure find out that 73% of the banks that disclose their methodology are using (unconditional) historical simulation. This popularity is according to Pérignon & Smith (2010b) caused by the size and complexity of the trading positions - banks have to deal with thousands of risk-factors and hence the parametric methods are difficult to use in practice. Another reason for the high popularity is the smoothness of risk forecasts. Because the unconditional historical simulation reacts slowly to changes in volatility, the VaR estimates does not change much from one day to the next.

As we mentioned earlier in the section (3.2.2) the historical simulation approach does not make any distribution assumptions. On the other hand, it requires significantly large amount of historical data to produce accurate results. Crouhy *et al.* (2006, pp. 165-166) describes the three steps involved in HS calculation:

1. Taking a sample of actual daily risk factor changes over a given period of time.

2. Applying those daily changes to the current value of the risk factors and revaluing the current portfolio as many times as the number of days in the historical sample.

3. Constructing the histogram of portfolio values and obtaining VaR as a $1-\alpha$ percentile.

To describe this method mathematically we again use McNeil *et al.* (2005, pp. 50-52) framework. The first step is getting a set of historically simulated losses by applying the loss operator to each of our historical observation:

$$\{\tilde{L}_s = l_{[t]}(\mathbf{X}_s) \ : \ s = t - n + 1, ..., t\}, \tag{3.20}$$

where the values $\tilde{L}_s$ show "what would happen to the current portfolio if the risk factor changes on day s were to recur" (McNeil *et al.* 2005, p. 50), and n is a number of historical observations in our sample. We can further distinguish between an unconditional and a conditional method. In the former we assume that the process of risk-factor changes $\mathbf{X}_s$ is stationary with distribution function $F_{\mathbf{X}}$. The latter could be done by several extended methods like the one described in Section 3.4.2.

3.4.1 Unconditional HS

Now assume that we have an unconditional series (3.20). Common method to estimate VaR in practice is called *empirical quantile estimation*, where we use a sample quantiles of our data to estimate VaR. Let

$$\tilde{L}_{n,n} \leq ... \leq \tilde{L}_{1,n} \tag{3.21}$$

be the ordered values of the data in (3.20). Then we can estimate $VaR_\alpha(L)$ as $\tilde{L}_{[n(1-\alpha)],n}$, where $[n(1-\alpha)]$ is the largest integer not exceeding $n(1-\alpha)$. In other words, we just order observations in our sample a take $n(1-\alpha)$-th observation as a VaR estimate.

This method is criticized by Pritsker (2006) because it assigns the same weight to each historical return which is the same as the i.i.d. assumption. In light of the stylised facts of financial time series presented in Section 3.3, especially the *volatility clustering*, is this unrealistic. Furthermore, Pritsker (2006) shows this disadvantage on a portfolio held during the 1987 equity crash. Although there were one significant fall in the portfolio and few high positive values, the HS approach still uses $n(1-\alpha)$-th lowest return, which is obviously not the lowest return observed on October 19, 1987 and does not take into account high positive returns at all.

3.4.2 Historical VaR with GARCH Volatility

The major drawback of the historical simulation model is its backward looking approach. We assume that the future changes in our portfolio will be drawn from a historical distribution of returns. On the other hand we do not have to make any further assumptions about distribution, which is the issue of parametric models. Another problem are equally weighted historical returns, which was mentioned above. If the volatility is high now but low in preceding months, current volatility estimate will be undervalued and vice versa (because in our historical distribution there is large number of low-volatility days compared to high-volatility days). The parametric models, on the other hand, allows for incorporating GARCH volatility which is an advantage over historical simulation.

Hull & White (1998) suggested to incorporate GARCH modelled volatility into the historical simulation model. Suppose that we have a historical distribution of certain market variable but each historical observation r_t is rescaled using GARCH volatility. If the σ_t^2 is the historical GARCH estimate of the daily variance made for day t at the end of day $t-1$, and the most recent GARCH estimate of the daily variance is σ_N^2. Hull & White (1998) assume that the probability distribution of r_t/σ_t is stationary and therefore replaces r_t by r_t' where:

$$r_t' = \sigma_N \cdot \frac{r_t}{\sigma_t}, \tag{3.22}$$

and daily variance is estimated via GARCH(1,1) model defined as 3.14.

This approach is sometimes referred as *Volatility-weighted Historical Simulation* (Dowd 2005, pp. 94 - 95) because of the adjustment of returns r_t. If the value of estimated volatility at time t is greater than current volatility estimate, $r_t' < r_t$ and vice versa. The VaR is then estimated in the usual way, i.e. obtaining a $(1-p)$ critical value of the probability distribution of r_t'.

3.5 Backtesting

Suppose that we are estimating VaR recurrently over a certain period of time. We should be able evaluate its performance somehow. This is done by *ex-post* comparison of the VaR estimates with actual losses observed on the same day - the process known as *backtesting*. VaR is defined as a $1-\alpha$ quantile of the underlying loss distribution thus we expect that there is a probability of α that this amount will not be exceeded. In other words, the probability that the VaR is *violated* is $1-\alpha$. Let

$$\hat{I}_{t+1} := I_{\{L_{t+1} > \widehat{VaR}_{\alpha}^{t}\}} \tag{3.23}$$

be an indicator notation for violation (McNeil *et al.* 2005, p. 55). Then this indicators should be Bernoulli distributed with success probability corresponding to $1-\alpha$, where α is the desired confidence level.

We would like to test if the empirical probability of violation correspond to the chosen confidence level α. In other words, whether, for example, 1% VaR really produce 1% of losses higher than the estimate. This could by done via *unconditional coverage* test.

Another issue we want to test is the volatility clustering. Even if the model passes the unconditional coverage test, it does not automatically mean that it is good. Danielsson (2011, p. 153) presents an example: what if all violations were squeezed into few weeks during some stress period? In such a situation the model will pass the unconditional coverage test (provided that only few violations occurred before the stress period) but the risk of bankruptcy is still very high. Within a good model the occurrences of violations should be spread out over a whole period of time. Therefore we would like to test independence of violations and this could be done via *independence test* of Christoffersen (1998).

3.5.1 Unconditional coverage test

We explained the basic idea behind this test, which was suggested by Kupiec (1995), earlier. Now we look at this test more closely using Christoffersen (1998) as the major source of information.

Suppose that we have a series $\{L_t\}_{t=1}^{T}$ of daily losses and a corresponding sequence of *out-of-sample* interval forecasts $\{VaR_{\alpha}^{t-1}\}_{t=1}^{T}$ where VaR_{α}^{t-1} is the VaR estimate for a day t made on the day $t-1$ at given significance level α. Then the indicator function I_t is defined as:

$$I_t = \begin{cases} 1 & \text{if} \quad L_t > VaR_{\alpha}^{t-1} \\ 0 & \text{if} \quad L_t \leq VaR_{\alpha}^{t-1} \end{cases} \tag{3.24}$$

Clearly, I_t indicates violations of VaR (L_t is equal to one if its value lies inside the interval and zero otherwise).

According to Christoffersen (1998), $\{VaR_{\alpha}^{t-1}\}_{t=1}^{T}$ has correct unconditional coverage if the sequence $\{I_t\}$ is i.i.d. Bernoulli distributed with parameter p.

In other words, if the number of VaR violations is the same as the expected probability of violation, i.e. $(1-\alpha)$ times a total number of observations. Hence, we can write the null hypothesis as:

$$H_0 : \{I_t\} \overset{iid}{\sim} B(p), \forall t, \tag{3.25}$$

where p is the probability of a violation.

We can test the null hypothesis through the use of a Likelihood Ratio (LR) test, provided that $I_t, I_{t-1}, \ldots$ are independent. The likelihood function is given by (Danielsson 2011, p. 154):

$$\mathcal{L}(p) = \prod_{1}^{T} (1-p)^{1-I_t} p^{I_t} = (1-p)^{v_0} p^{v_1}, \tag{3.26}$$

where $v_1 = \sum_1^T I_t$ and $v_0 = 1 - v_1$. Under the null hypothesis $p = q = 1 - \alpha$ and the likelihood functions is:

$$\mathcal{L}(q; I_1, \ldots, I_T) = (1-q)^{v_0} q^{v_1}, \tag{3.27}$$

and under the alternative $p = \pi = v_1/(v_0 + v_1)$ with likelihood function:

$$\mathcal{L}(\pi; I_1, \ldots, I_T) = (1-\pi)^{v_0} \pi^{v_1}. \tag{3.28}$$

The likelihood ratio test is then:

$$LR_{UC} = -2\log\left(\frac{\mathcal{L}(q; I_1, \ldots, I_T)}{\mathcal{L}(\pi; I_1, \ldots, I_T)}\right) = -2\log\left(\frac{(1-q)^{v_0} q^{v_1}}{(1-\pi)^{v_0} \pi^{v_1}}\right) \overset{asy}{\sim} \chi^2_1 \tag{3.29}$$

As Christoffersen (1998) points out, this test does not have any power to identify clusters. Only thing that matter is the total number of violations, i.e. number of ones in the indicator sequence. Their order does not matter.

3.5.2 Conditional coverage test

Christoffersen (1998) stresses that testing for the correct unconditional coverage is insufficient for the data that exhibit volatility clusters such as daily losses (returns). He suggest using two additional tests to overcome this deficiency - the *independence test* and the *jointly test for independence and conditional coverage* - giving a complete test of correct conditional coverage.

We begin with the *independence test.* Let

$$p_{ij} = P(I_t = j | I_{t-1} = i), \qquad i, j \in \{0, 1\} \tag{3.30}$$

be the probability of two consecutive events based on the values of i and j. If $i = j = 1$, p_{11} is the probability of consecutive violations. Similarly p_{01} is the probability of violation if the VaR on the previous day was not violated, and so on.

Christoffersen (1998) tests independence against specific first-order Markov alternative. Let I_t be a first-order Markov chain with transition probability matrix defined as:

$$\Pi_1 = \begin{bmatrix} 1 - p_{01} & p_{01} \\ 1 - p_{11} & p_{11} \end{bmatrix} \tag{3.31}$$

with the approximate likelihood function:

$$\mathcal{L}(\Pi_1; I_1, \ldots, I_T) = (1 - p_{01})^{n_{00}} p_{01}^{n_{01}} (1 - p_{11})^{n_{10}} p_{11}^{n_{11}} \tag{3.32}$$

where n_{ij} is the number of observations where i is followed by j. Then we maximize this function to obtain maximum likelihood estimate of Π_1:

$$\hat{\Pi}_1 = \begin{bmatrix} \frac{n_{00}}{n_{00} + n_{01}} & \frac{n_{01}}{n_{00} + n_{01}} \\ \frac{n_{10}}{n_{10} + n_{11}} & \frac{n_{11}}{n_{10} + n_{11}} \end{bmatrix} \tag{3.33}$$

Under the null hypothesis there is no volatility clustering, therefore the probability that VaR will be violated does not depend on whether or not there was a violation on the previous day ($p_{11} = p_{01} = p_2$). The transition matrix and likelihood function are then respectively:

$$\hat{\Pi}_2 = \begin{bmatrix} 1 - p_2 & p_2 \\ 1 - p_2 & p_2 \end{bmatrix} \tag{3.34}$$

$$\mathcal{L}(\Pi_2; I_1, \ldots, I_T) = (1 - p_2)^{n_{00} + n_{10}} p_2^{n_{01} + n_{11}} \tag{3.35}$$

and the ML estimate of Π_2 is

$$\hat{\Pi}_2 = \frac{n_{01} + n_{11}}{n_{00} + n_{10} + n_{01} + n_{11}}. \tag{3.36}$$

The likelihood ratio test is then calculated using likelihood functions (3.32) and

(3.35):

$$LR_{IND} = -2\log\left(\frac{\mathcal{L}(\Pi_1; I_1, \dots, I_T)}{\mathcal{L}(\Pi_2; I_1, \dots, I_T)}\right) \overset{asy}{\sim} \chi_1^2. \tag{3.37}$$

As Christoffersen (1998) remarks, this test does not depend on the probability of VaR violation p and test only the independence of violations.

However, there is one main problem with this test, which is mentioned by Danielsson (2011, p. 156), and that is the particularly specified independence. The departures from independence will not be detected if the probability depend on a violation two or more days ago, but not on the yesterday's violation.

The *jointly test for independence and conditional coverage* combines the *unconditional coverage test* and the *independence test* into a complete test of conditional coverage. The likelihood ratio test is:

$$LR_{CC} = -2\log\left(\frac{\mathcal{L}(p; I_1, \dots, I_T)}{\mathcal{L}(\Pi_2; I_1, \dots, I_T)}\right) \overset{asy}{\sim} \chi_2^2. \tag{3.38}$$

There is also following relationship between all three tests:

$$LR_{CC} = LR_{UN} + LR_{IND}. \tag{3.39}$$

However, this joint test is not preferred to previously derived tests as it may seem. The thing is, that, according to Danielsson (2011, p. 159), this test has less power to reject a VaR model satisfying only either unconditional coverage property or independence property. It could happen in such a case that this test will fail to reject the model which exhibits correct unconditional coverage but violates independence test. Therefore it is usually better to use individual tests.

Chapter 4

Empirical Research

4.1 Data

The performance of selected VaR models is tested on a portfolio represented by the S&P 500 equity index. Hence, the portfolio is influenced by the single risk factor which represents top 500 companies in leading industries of the United States and is known to capture 75% coverage of U.S. equities which makes it the best single gauge of the U.S. stock markets (Standard & Poor's Financial Services 2013).

Our dataset includes 5796 daily log returns from the beginning of 1990 to the end of 2012. This period selection was not random. During the past two decades there has been a rapid development in a financial engineering, i.e. an expansion of the off-balance-sheet products such as derivatives and swaps, as well as a development in the regulatory framework (the first Basel Accord was implemented in 1992). Moreover this data period covers three important crises: the 1997 Asian crisis; the 2000 dot-com bubble; and the 2007-2008 sub-prime mortgage crisis. This is important in a stressed VaR calculation because we need a one year long period of a significant financial stress.

In Table 4.1 there are summary statistics for the S&P 500 index. As we can see, the mean is statistically not different from zero hence we can assume that $\mu = 0$ in all our models. Negative skewness implies long left tail of a return series and kurtosis of 11.47 suggests that a series is heavy tailed. The biggest loss of 9.47% was observed on 15 October 2008, only 2 days after the biggest return of 10.96%. This is further evidence of volatility clustering.

Table 4.1: S&P500 daily log returns summary statistics

Observations	5796
Mean	0.00024
Std.	0.0117
Skewness	−0.2281
Kurtosis	11.47
Min	−0.0947
Max	0.1096

Source: author's computations.

4.2 Models

We calculate VaR using 6 different models, three of them are conditional and three are unconditional. Theoretical framework for each model is in Sections 3.3 and 3.4. Here we just briefly summarize each model's features:

Normal (N-UC). The standard unconditional parametric VaR model which assumes normally distributed risk-factor changes (i.e. daily log returns) as described in Section 3.3.1.

Student's-t (t-UC). The unconditional parametric VaR model which assumes student's-t distributed risk-factor changes as described in Section 3.3.2.

Historical Simulation (HS). Basic unconditional empirical method of VaR estimation as described in Section 3.4.1.

Normal-GARCH (N-G). A conditional version of the normal VaR method which assumes conditionally normally distributed risk factor changes (see Equation 3.18). To estimate the conditional variance of the risk-factor changes, the GARCH(1,1) model with normal innovation is used.

Student's-t-GARCH (t-G). The conditional student's-t VaR with variance estimated using the GARCH(1,1) model with student's-t innovations (see Equation 3.19).

Volatility weighted HS (WHS). The extended conditional version of historical simulation as described in Section 3.4.2. Again, the GARCH(1,1) model is fitted to historical losses to estimate the conditional variance of the risk-factor changes.

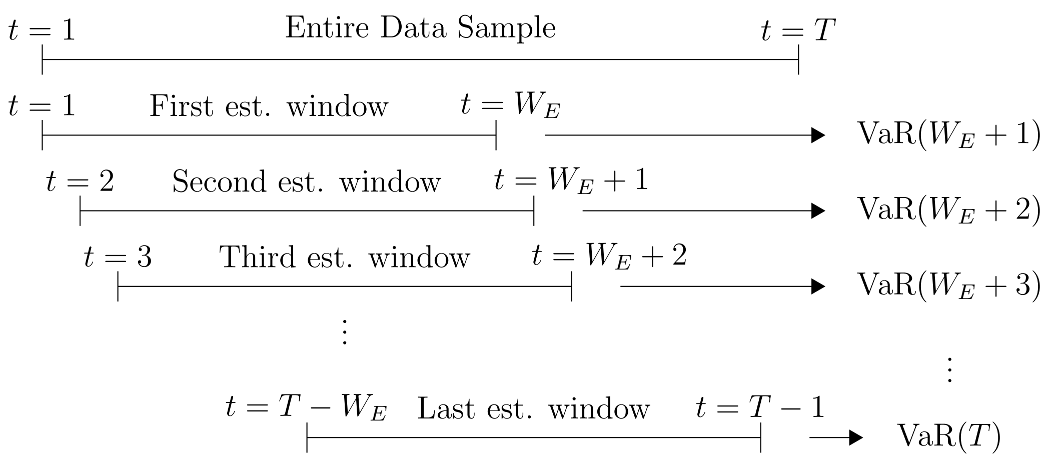
4.3 Methodology

The purpose of this thesis is to assess whether stressed VaR, as it is defined in the latest Basel amendment, is able to capture risk efficiently. In order to do it, we divide our research into three steps. In the first step we estimate an ordinary VaR models and comment its performance, then we add stressed VaR, and finally we try to implement these risk measures into the capital requirements computation.

4.3.1 VaR Estimation

In order to evaluate an ability of each VaR model to capture market risk we examine its performance over a period in the past. Hence, we apply backtesting method described in Danielsson (2011, pp. 143 - 147). This approach is visualised in Figure 4.1. Firstly, we take a first estimation window of length W_E and use the data in it to estimate VaR for day $t = W_E + 1$. Then we move our window by one day and repeat the estimation and so on, until the data are exhausted. We have decided to set $W_E = 500$ days because longer periods could cause problems for unconditional models (see Dowd 2005, p. 128). Altogether, we have $(T - W_E) = 5296$ VaR estimates. Note that the last VaR forecast is made on day $T - 1$ for the day T. We do not estimate VaR for the day $T + 1$ because we do not have an actual loss observed on this day in our dataset and hence we cannot compare it with VaR forecast.

Figure 4.1: Backtest Methodology



Source: (Danielsson 2011, p. 144)

Hereafter, we record the violations on each day (VaR is violated if actual loss on corresponding day is higher than estimated VaR) and obtain the *violation* *ratio*. The closer to one this ratio is the better, but without formal test we cannot ascertain whether values not equal to one are statistically significant. However, Danielsson (2011, p. 147) mentions a rule of thumb: if $VR \in \langle 0.8, 1.2 \rangle$ it is probably a good estimate and $VR < 0.5$ or $VR > 1.5$ suggest that the model is imprecise. This analysis help us to get basic information about each model before we apply the unconditional coverage test and the independence test.

The unconditional coverage test and the independence test are discussed in Section 3.5.1 and Section 3.5.2, respectively. Here we just summarize the null and the alternative hypotheses.

The unconditional coverage test. Under the null hypothesis the observed number of violations is the same as the expected number of violations which is equal to $(1 - \alpha)$ times the number of observations. Under the alternative this number is either lower or higher than it was expected.

The independence test. Under the null hypothesis the probability of VaR violation does not depend on whether or not there was a violation on the previous day. Under the alternative there is a dependence and therefore model probably does not account for the volatility clusters.

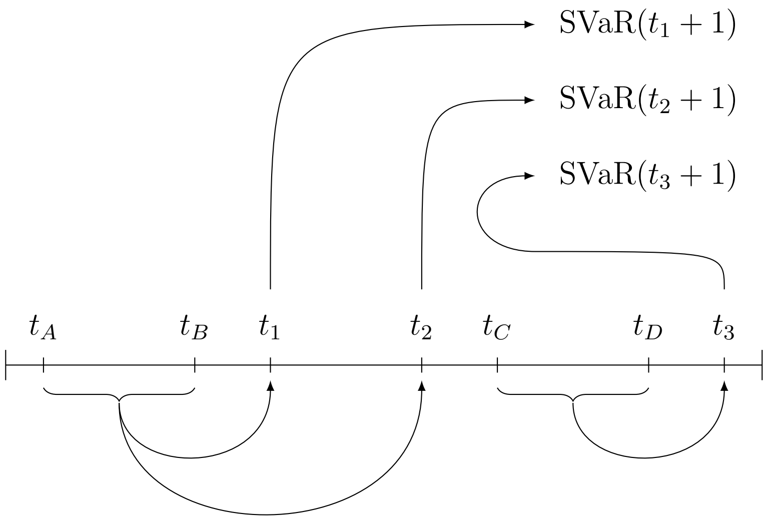
We afterwards graphically evaluate each model performance during the 2008 crisis because we believe that this period represents the worst segment of our dataset.

4.3.2 Stressed VaR Estimation

Given lack of academic literature concerning SVaR estimation, this part is undoubtedly the most challenging. Lets split the process into two steps. First step is a selection of a period that represents a significant stress for institutions portfolio. This period should be 250 days long. Then we use this period as an estimation window (similarly as in VaR estimation) for Stress VaR forecast.

Whole process is visualised in Figure 4.2. Suppose that the identified stress period for time $t \in \langle t_B, t_D)$ is the period $\langle t_A, t_B \rangle$; and for time $t \geq t_D$ it is $\langle t_C, t_D \rangle$. This means that, for example, on the days $t_1, t_2 \in \langle t_B, t_D)$ we calculate SVaR using the first stress period. And on the day $t_3 \geq t_D$ we use the second stress period as the estimation window. Notice that SVaR$(t_1 + 1)$ is the same as SVaR$(t_2 + 1)$ since the portfolio is constant over time and both estimates are

Figure 4.2: SVaR Methodology



Source: Author

calculated from the same stress period. This quite simplifies SVaR estimation - we only need to forecast SVaR once for each stress period.

Stress Period Identification

At first, we have to identify a stress period. According to the *Revisions to the Basel II market risk framework*, we need 250 days long (approximately 12 months) period of financial stress (more in Section 2.1.4). European Banking Authority (2012) suggests two approaches of the stress period identification: *a risk-factor based approach* and *a VaR based approach*.

The first one consists of a identifying the risk factors relevant to institution's portfolio and finding a period of highest volatility. Since we have only one risk factor in this thesis - the equity index - we just find the period when the standard deviation was the highest.

The VaR based approach identifies the stress period by running either full VaR model or an approximation over a historical period. The stress period is then a period of 250 days preceding the highest VaR estimate.

We chose the former approach for two reasons: (1) the stress period will be the same for all six models, and (2) the SVaR is calculated from 250 long estimation window but the VaR models assumes 500 days long estimation window therefore the highest VaR estimate would not imply the highest SVaR estimate. A solution for this could be running VaR model based on 250 days long period but it would be time demanding and the first option is sufficient for our research.

SVaR models

Once the stress period has been identified, a model for SVaR calculation must be chosen. The Basel Committee on Banking Supervision (2009b) does not prescribe any particular model therefore we can freely choose any method for VaR estimation from Section 4.2. To preserve simplicity we will retain consistency between VaR and SVaR estimates. In other words, we will not, for example, add normal SVaR to student's-t VaR and so on.

4.3.3 VaR and SVaR within the Basel framework

At first we calculate minimal capital requirements as defined in the Basel II, that is:

$$MCR_{OLD} = \max\left\{VaR_{\alpha}^{t-1}, m_c \cdot \sum_{i=1}^{60} VaR_{\alpha}^{t-i}\right\} \tag{4.1}$$

where $\alpha = 99\%$ and m_c is multiplication factor which has minimum value of 3 plus an additional factor k which rises according to model's performance in a backtest, i.e. the number of violations in preceding 250 days. Value of the factor k is obtained from Table 4.2. Consequently, we obtain minimal capital

Table 4.2: Basel II multiplication factor

Zone	Violations	Value of factor k	Multiplication factor m_c
Green Zone	0 - 4	0.00	3.00
Yellow Zone	5	0.40	3.40
	6	0.50	3.50
	7	0.65	3.65
	8	0.75	3.75
	9	0.85	3.85
Red Zone	≥ 10	1.00	4.00

Source: (Basel Committee on Banking Supervision 2006, p. 321)

requirements once again but within the Basel 2.5 framework, i.e. we account for stressed VaR:

$$MCR_{NEW} = \max\left\{VaR_{\alpha}^{t-1}, m_c \cdot \sum_{i=1}^{60} VaR_{\alpha}^{t-i}\right\} + \max\left\{SVaR_{\alpha}^{t-1}, m_s \cdot \sum_{i=1}^{60} SVaR_{\alpha}^{t-i}\right\} \quad (4.2)$$

where m_s has again minimum value of 3 plus a factor k obtained from Table 4.2. Note that backtesting results are based on VaR performance only and not SVaR.

We do not use 10-days long *holding period* as it is required by BCBS in the Basel II (and the Basel 2.5) but we use 1-day period instead. One reason is that the daily estimates are better for purposes of our thesis and, more importantly, a calculation of the 10-day VaR (and SVaR) is problematic. Danielsson (2002) criticize both possibilities of this computation; the first one is using non-overlapping 10 day returns to forecast VaR but this require 10 times larger dataset to produce statistically accurate results, the second option is applying the $square-root-of-time$ scaling rule which is recommended by the Basel Committee on Banking Supervision (2009b). However, Danielsson (2002) argues that this rule depends on strong distribution assumption such as normally distributed returns and time-independent volatility. This is obviously not true and using this rule for scaling conditional VaR models does not make any sense which pose a serious problems for risk managers who are using these models. Nevertheless, this problem is more complex and it is not a topic of this thesis.

We afterwards assess the model's performance using several criteria:

1. An amount of capital requirements.
2. Percentage of time when the model was in a green, yellow and red zone (see Table 4.2).
3. A range of capital requirements, i.e. minimum and maximum values.
4. An increase in requirements caused by applying new rules.

Obviously, the models with low capital requirements and with contemporaneous low number of violations (high number of days in a green zone) are preferred. A wide range suggest that the model better reacts to market movements, on the contrary a narrow range imply that model probably fails to capture these movements. The last criterion is rather informative.

4.4 VaR Estimation

All VaR models were estimated and numbers of violations for each year are collected in Table A.1. This section is divided into two parts, the first part is devoted to unconditional and the second to conditional models.

4.4.1 Unconditional models

The violation ratios and volatilities of all three unconditional modes are in Table 4.3. We also include test statistics and p-values of the unconditional coverage test and the independence test.

Table 4.3: Performance of the unconditional VaR models

			Unc. Coverage		Independence	
Model	VR	Volatility	TS	P-value	TS	P-Value
N-UC	2.0770	0.0106	47.3504	0.0000	63.0047	0.0000
t-UC	1.4917	0.0151	11.2355	0.0008	32.4913	0.0000
HS	1.4162	0.0149	8.2055	0.0042	25.3294	0.0000

Source: author's computations.

The normal model performed worst of all. The violation ratio is higher than 2 which is well over suggested value 1.2. On the other hand, the volatility of the normal VaR is the lowest of all unconditional models. In this light is not surprising that we can strongly reject the null hypothesis of the unconditional coverage test. Moreover, the null of the independence test is also rejected which indicate that this model fail to capture volatility clusters.

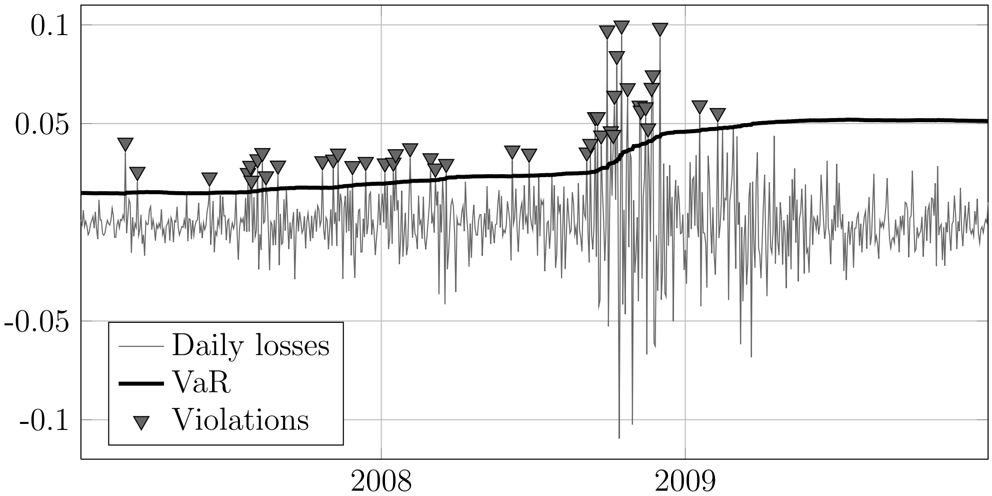
Although the student's-t VaR shows better results, the violation ratio is still almost 1.5, which suggest that the model is imprecise. And this is further confirmed by both the unconditional coverage and the independence tests, where the null hypotheses are rejected.

The Historical simulation performed the best of all unconditional models but its results are similar to student's-t model and far from being good. The model did not pass the unconditional coverage test nor the independence test.

Now lets focus on the 2007-2008 crisis to show the ability of the unconditional models to respond to abrupt volatility surges.

The normal VaR resulted in 15 violations in 2007 followed by 28 violations in 2008 (see Table A.1) - that is more than 10 times higher then it is expected.

Figure 4.3: Normal VaR estimate during the 2008 crisis. There were 15 violations in 2007, 28 in 2008, and 2 in 2009.



Source: author's computations.

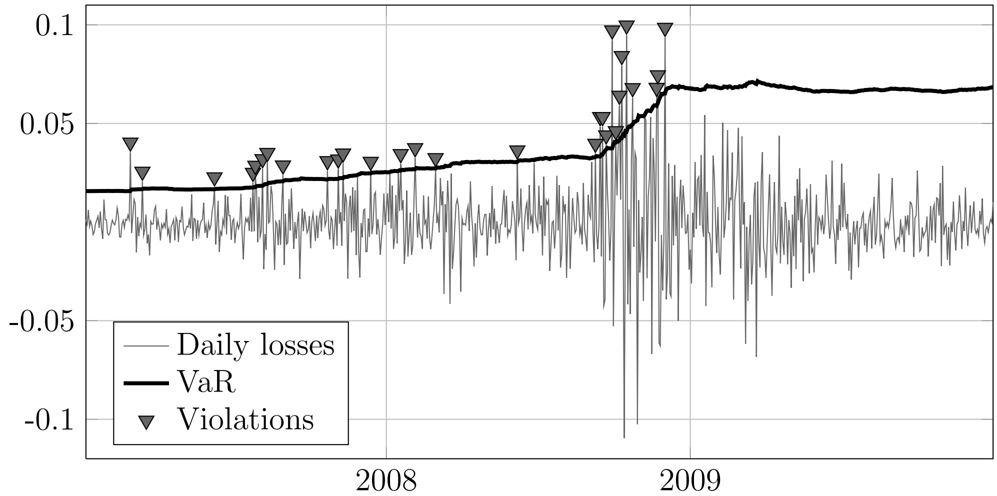
The situation is captured in Figure 4.3. It is apparent that the model does not react to sudden jumps in volatility during 2008 and during the first half of 2009. Moreover, when the volatility increases rapidly in the second half of 2008, VaR forecasts move upwards only very slowly and reach its maximal value not before the worst period is over.

Although the other two unconditional methods performed little better, during this high-volatile period both models suffer from the similar problems as the normal VaR - very slow reaction to sudden changes in volatility.

The Student's-t VaR performance is shown in Figure 4.4. The number of violations is slightly lower than in the previous case - 12 in 2007 and 17 in 2008 - but it is still very high. This results suggest that only accounting for heavy tails which are present in financial data is not enough if we base our model on the unconditional distribution.

The historical simulation method resulted in 11 violations in 2007 and 18 violations in 2008. The model reacts to the crisis similarly as the previous two as it is shown in Figure 4.5. We observe here an issue discussed by Pritsker (2006) - despite numerous volatility jumps in 2007 VaR estimate remains low because the 10th lowest return which correspond to 0.01 quantile is still higher than the returns that originated from these jumps. Furthermore, from the second quarter of 2009, when the markets resumed its normal conditions, VaR estimate remains high. Explanation for this is identical: the 10th largest value is now much higher than it was before the crisis due to the many volatile days

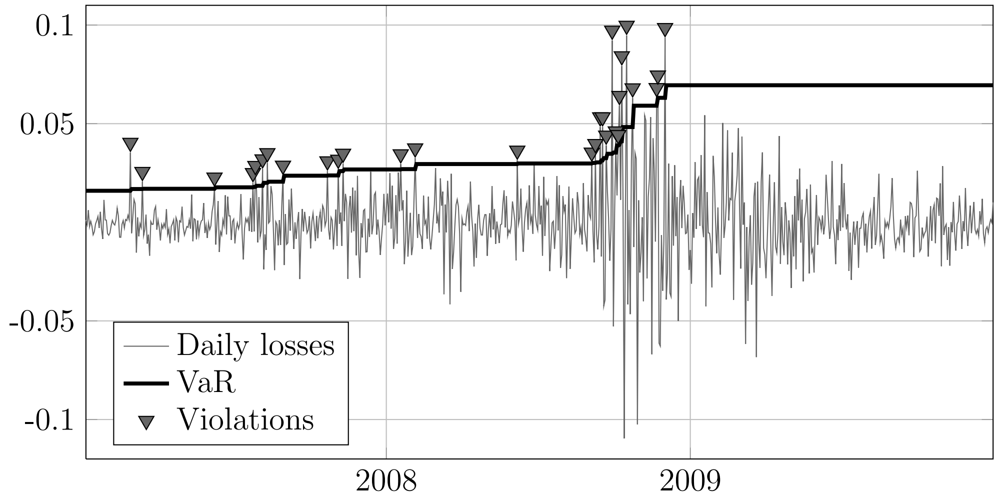
Figure 4.4: Student's-t VaR estimate during the 2008 crisis. There were 12 violations in 2007, 17 in 2008, and 0 in 2009.



Source: author's computations.

in the second half of 2008. And this will not change until this period is not present in the estimation window, i.e. 500 days later.

Figure 4.5: HS VaR estimate during the 2008 crisis There were 11 violations in 2007, 28 in 2008, and 0 in 2009.



Source: author's computations.

All the unconditional models suffer from similar issue - inability to sufficiently quickly react to changes in volatility. Not only causes problems sudden volatility increases, but also the returning to the normal market conditions is not followed by lowering the estimates which leads to overestimation of risk during low volatility periods. Evidence for this is clearly visible in Table A.1 - there is no single violation from 2004 till 2006 for all three unconditional models (preceded by one violation in 2003). This results correspond to empirical evidence of Berkowitz & O'Brien (2002) and Pérignon *et al.* (2008) who discovered that banks tend to overstate market risk. However, these researches are outdated because they do not account for the 2008 crisis where these models failed.

If the risk is underestimated during crises periods and overestimated when the markets are in normal conditions, the idea of supplementing this measure with some additional stressed VaR model does not seem very reasonable.

4.4.2 Conditional models

Now lets look at the conditional models. Again, we collected the violations rations, volatilities, test statistics and p-values of both the conditional coverage and independence tests in Table 4.4.

Table 4.4: Performance of the conditional VaR models

			Unc. Coverage		Independence	
Model	VR	Volatility	TS	P-value	TS	P-Value
N-G	1.9826	0.01347	40.167	0.0000	23.839	0.0000
t-G	1.2651	0.01509	3.469	0.0626	0.027	0.8706
WHS	1.0952	0.01843	0.470	0.4930	0.183	0.6692

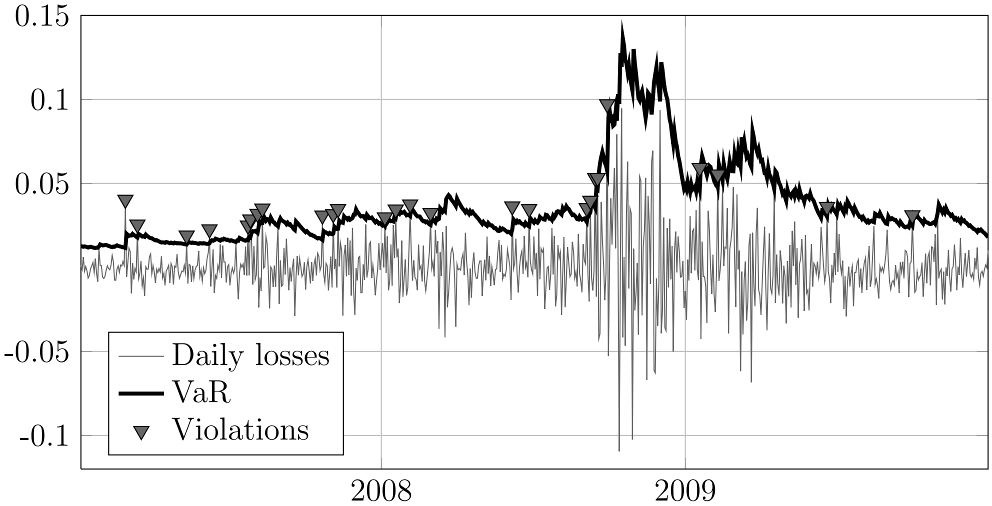
Source: author's computations.

Although the normal GARCH VaR performed better than the unconditional normal model, the volatility ratio is still almost two, which is worse result than the unconditional student's-t VaR and the historical simulation. Also we can reject the null hypothesis of the unconditional coverage test and of the independence test. The former is not surprising given the volatility ratio but he latter is rather unexpected since the GARCH model was used.

The GARCH model with student's-t innovations resulted in lower volatility ratio than the normal model; although it is still over 1.2, there are only 14 more violations within 18 years than it was expected, which is very good result. We cannot reject the null hypothesis of the unconditional coverage at 1% nor 5% significance levels but we manage to reject it at 10% level. Also we cannot reject the null hypothesis of the independence test which suggests that the model managed to capture volatility clusters.

The volatility weighted historical simulation model outperformed the other two conditional as well as all the unconditional models. The volatility ratio lies in the desired interval and we cannot reject the null hypothesis of the unconditional coverage test which means that this model almost succeeded in covering estimated number of violations. Moreover, we cannot reject the null hypothesis of the independence test.

Figure 4.6: Normal GARCH VaR estimate during the 2008 crisis There were 11 violations in 2007, 11 in 2008, and 4 in 2009.



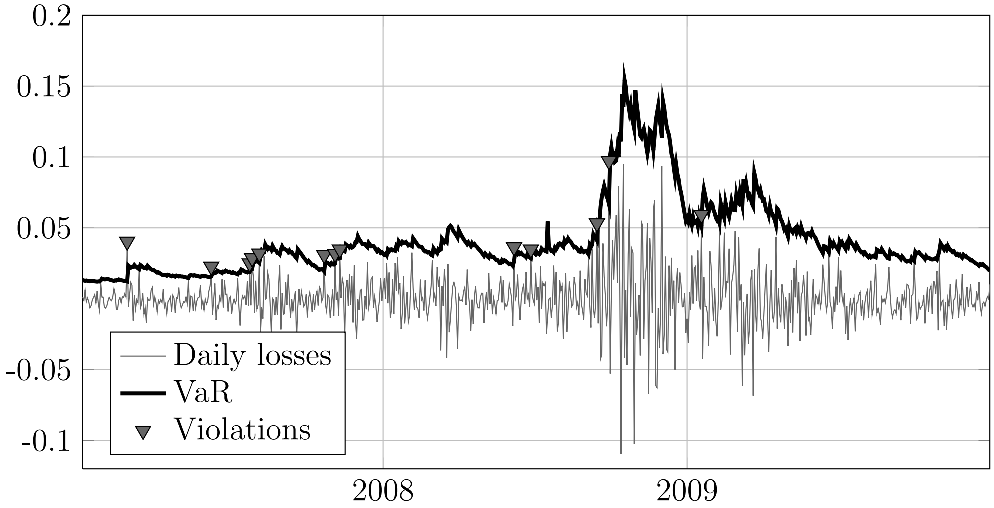
Source: author's computations.

During the 2007-2008 crisis the normal model does not performed very well concerning the number of violations - 11 in 2007 and 11 in 2008. However, from Figure 4.6 we can see that responds better to volatility cluster in the second half of 2008 than the unconditional models. The problem is that the normal distribution is thin-tailed and use of heavy-tailed distribution such as student's-t would be more appropriate.

The Student's-t and the volatility weighted HS models performed much better during this crisis period. Although both the models resulted in 8 violations in 2007, in 2008 there are only 4 and 3 violations for the student's-t and the volatility weighted HS models, respectively. As we can see in Figure 4.7 and in Figure 4.8 the models quickly reacts to volatility cluster in 2008.

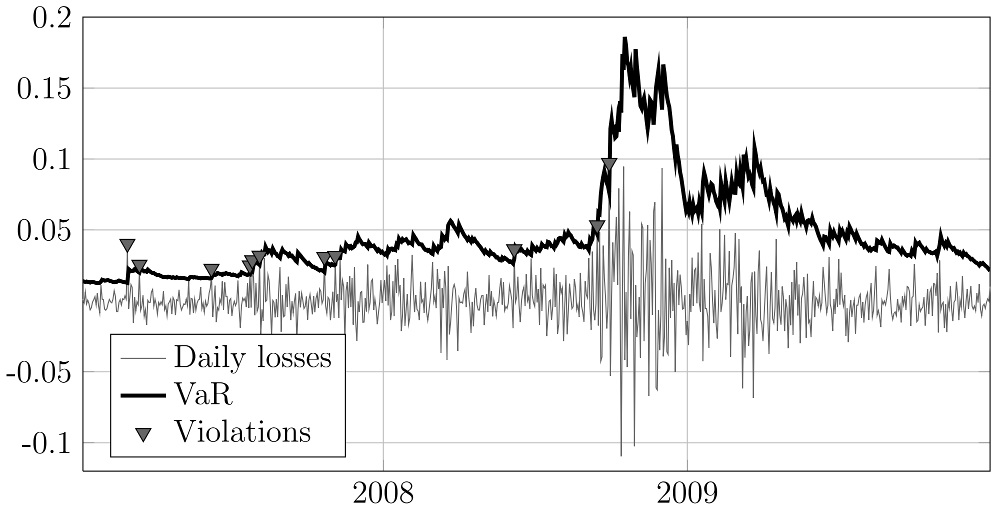
Generally, the conditional models outperformed the unconditional. This is especially visible during the crisis period when the unconditional models failed to sufficiently quickly respond to volatility surge. On the contrary, not only more quickly the conditional models responded, but also after the crisis the estimates did not remain to high.

Figure 4.7: Student's GARCH VaR estimate during the 2008 crisis
There were 8 violations in 2007, 4 in 2008, and 1 in 2009.



Source: author's computations.

Figure 4.8: Volatility weighted HS estimate during the 2008 crisis
There were 8 violations in 2007, 3 in 2008, and 0 in 2009.



Source: author's computations.

4.5 Stressed VaR Estimation

First issue in stressed VaR estimation is finding an appropriate stress period. From our dataset we identified five such periods using approach defined in Section 4.3.2, such that each new stress period has a higher standard deviation than the previous one. We afterwards calculated stressed VaR with the estimation window from this period for all six models (both unconditional and conditional). Results are collected in Table 4.5.

An issue occurred in SVaR estimation for fourth stress period. The historical simulation, the t-GARCH, and the volatility weighted HS resulted in lower SVaR number than for the previous period. An explanation for this is that higher standard deviation during this stress period does not imply higher estimate from the historical simulation. Similarly for the other two models. For further computations, we will use the higher values.

Table 4.5: Stressed VaR

Stress period			SVaR					
Start	End	Std. (%)	N-UC (%)	t-UC (%)	HS (%)	N-G (%)	t-G (%)	WHS (%)
10/07/1990	03/07/1991	1.08	2.50	2.60	3.04	2.39	2.41	3.22
23/07/1998	20/07/1999	1.42	3.30	3.61	3.91	2.80	2.98	3.96
10/10/2000	11/10/2001	1.46	3.40	3.69	4.41	3.45	3.67	4.28
01/05/2002	28/04/2003	1.74	4.04	4.12	3.91	3.52	3.54	3.52
22/07/2008	17/07/2009	2.88	6.70	7.99	9.35	4.02	4.30	6.27

Source: author's computations.

Notice that there is a big discrepancy between the last period, which correspond to the recent crisis, and those before. Explanation for this is simple: since 1990 there has not been such crisis as the one in 2008. Obviously, we could extend our dataset to the late 80' to include the 1987 equity crash or even more into the past. However, this would cause problems because the stress period has to be relevant to institution's portfolio and using such old data would require using proxies for position which not existed in those periods and this may result in very different SVaR estimates (see Standard & Poor's Financial Services 2012). We consequently add SVaR numbers to our VaR estimates from the Section 4.4 and collected the numbers of violations in the Table 4.6. The number of violations decreased rapidly for each model therefore it does not make sense to conduct any formal tests (and it is not even possible with such a

Table 4.6: VaR and SVaR comparison

Model	Violations		
	Expected	VaR	VaR + SVaR
N-UC	52.96	110	6
t-UC	52.96	79	4
HS	52.96	75	4
N-G	52.96	105	3
t-G	52.96	67	2
WHS	52.96	58	1

Source: author's computations.

small dataset). We present it here though, because it shows the enormous size of this buffer.

4.6 VaR and Basel Rules

We calculated minimal capital requirements using Formulas 4.1 and 4.2 and collected results in Tables 4.7 and 4.8, respectively. In these tables there are only average statistics for whole period; yearly results for each model are collected in Tables A.2, A.3, A.4, A.5, A.6, and A.7 in Appendix A.

Table 4.7: The Basel II minimal capital requirements

Model	Mean (%)	Min (%)	Max (%)	Green (%)	Yellow (%)	Red (%)
N-UC	8.51	3.82	20.71	54.4	33.8	11.8
t-UC	9.81	4.39	27.69	69.8	21.4	8.8
HS	9.66	4.26	27.79	73.4	17.9	8.7
N-G	8.27	3.44	40.58	51.4	42.0	6.6
t-G	8.39	3.69	34.84	76.9	23.1	0.0
WHS	8.99	3.49	41.53	82.1	17.9	0.0

Source: author's computations.

The unconditional normal model resulted in the lowest capital requirements among the unconditional models; however, the model was almost 12% of time in the red zone which is worst of all models. Unambiguously worst period is the 2008 crisis - the model was in the red zone for whole year 2008 followed by

202 days in 2009 (see Table A.2). This failure is not surprising given model's performance during this crisis discussed in Section 4.4.1.

Minimal capital requirements computed from the student's-t and the HS model are higher than those from the normal. This is given by using fat-tailed (t-UC) and empirical (HS) distributions. Nevertheless, both the models were in the red zone almost 9% of time and all these violations were in 2007 - 2009 period.

The conditional models achieved to capture the volatility cluster in 2008 much better. As we can see in Table 4.7, average capital requirements are generally lower whilst maximal observed values are far higher. By contrast, the highest value of capital requirements computed using unconditional models is 27.8 (HS) and 27.7 (t-UC) and this is even further raised by an additional factor k.

Despite the lowest capital requirements, the normal GARCH model fell into the red zone in 6.6% of time. Furthermore this model was in the green zone only 51% of time, which is the lowest of all models. On the contrary, the student's-t GARCH model as well as volatility weighted HS model managed to avoid the red zone entirely. The former also resulted in the second lowest capital requirements.

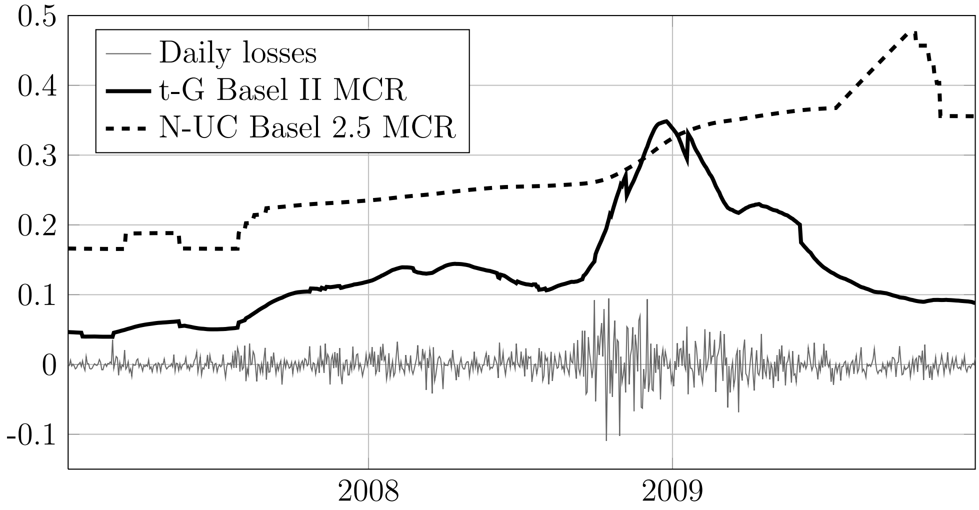
Table 4.8: The Basel 2.5 minimal capital requirements

Model	Mean (%)	Min (%)	Max (%)	Increase (%)
N-UC	21.23	11.33	47.48	149.47
t-UC	23.04	12.18	54.57	134.86
HS	24.55	13.39	61.59	154.14
N-G	18.68	10.63	54.67	125.88
t-G	18.59	10.94	45.84	121.57
WHS	22.01	13.16	54.37	122.58

Source: author's computations.

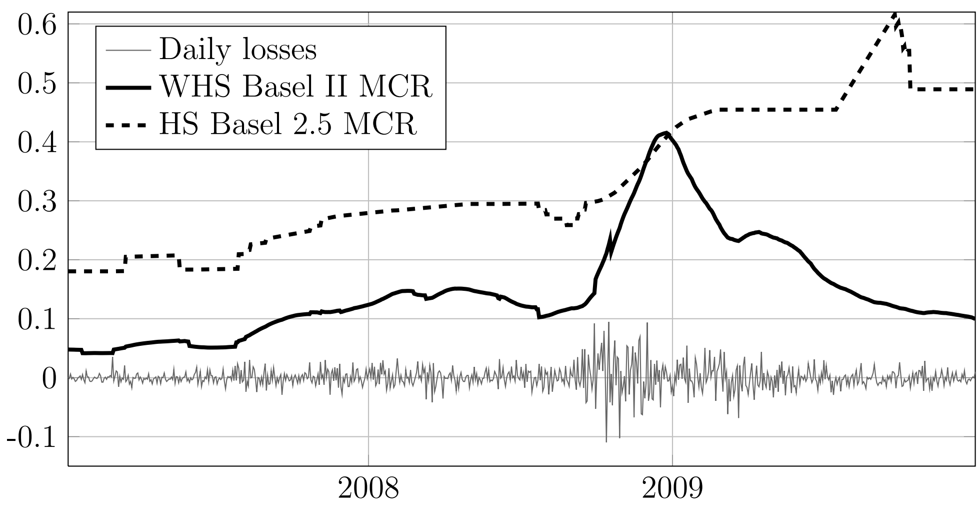
The implementation of the Basel 2.5 rules caused substantial increase in the minimal capital requirements. As can be seen in Table 4.8, this surge is at average highest for the unconditional models, especially the normal and the HS. The average capital requirements are now lowest for the conditional student's-t model, moreover, its maximal value of 45.8% is also the lowest thus this model is preferable to the volatility weighted HS which has higher average capital requirements and maximal value of 10% higher. However, this method performed the best in the initial VaR analysis (see Section 4.4) and this increase is probably caused by higher SVaR estimates (see Table 4.5).

Figure 4.9: Comparison of N-UC Basel 2.5 MCR with t-G Basel II MCR



Source: author's computations.

Figure 4.10: Comparison of HS Basel 2.5 MCR with WHS Basel II MCR



Source: author's computations.

The situation during the 2008 crisis is showed in Figures B.1, B.2, B.3, B.4, B.5, B.6 in Appendix B. In each figure there are both the Basel II and Basel 2.5 minimum capital requirements. When looking at unconditional models performance it is apparent that the last stressed period have impacted the models only when the worst period was over which was already too late. Consequently the minimum capital requirements remained unnecessarily high. Whereas the conditional t-GARCH VaR managed to reach the same level during the second half of 2008 but without any SVaR enhancements (see Figure 4.9). Similar situation is shown in Figure 4.10 where we compared widely used HS method under the new Basel 2.5 proposals with the volatility weighted HS within the old framework. This is clear evidence that the SVaR number added to the parametric model based on unconditional distributions only overestimates the risk during the normal market conditions whilst the same results during the crises periods could be achieved via the conditional t-GARCH or the volatility weighted HS model without this overestimation.

4.7 Summary

We presented here a clear evidence against usage of the unconditional models, especially under normality assumption. All these models failed in a conducted backtest and thus were penalised in the minimum capital requirements calculation. Furthermore, these models tend to overestimate risk in a common market conditions which lead to overstating of the regulatory capital. Conversely, during the crisis the models reacted only slowly and thus the minimum capital requirements remained at similar levels which resulted in large number of violations and possible inability of a bank to meet the losses.

On the other hand, the conditional models (with an exception of normal GARCH model) performed way better. Surprising results was achieved by the volatility weighted historical simulation of Hull & White (1998) although this model have not been used in any recent study (to the author's best knowledge). This model even managed to outperform the student's-t GARCH VaR regarding the number of violations and conditional coverage test. However, the latter model resulted in slightly lower minimum capital requirements which could be an advantage for many risk managers.

The addition of SVaR creates such a buffer that most of the days stay without violations. The danger is that this can bring false sense of security to bank before a huge loss hits them.

We do not see any improvements by an addition of the stressed VaR into the minimum capital requirements framework. When applied on the unconditional models, it only results in even higher overstating of risk during standard market conditions and during the crisis the same amount of capital could be estimated via both conditional models mentioned above.

Chapter 5

Conclusion

Recently proposed changes to the capital adequacy framework for market risk, as a reaction to the extensive losses which occurred in the course of the 2008 financial turmoil, have raised essential questions regarding its ability to forego such a high volatile periods. Whilst the Basel Committee on Banking Supervision believe that further enhancement of the minimum capital requirements through a newly required stressed Value-at-Risk number, calculated from a one year long period of financial stress, will help to eliminate losses caused by vigorous movements in financial markets, we presented here an empirical evidence that such an approach is not optimal. Furthermore, many issues discussed by academics in recent past decade remains.

We examined a performance of six models for Value-at-Risk estimation on approximately 20 years of daily financial returns of the S&P 500 index. Moreover, we graphically investigated their behaviour during the recent crisis. The normal as well as other two unconditional models proved to be insufficient for daily VaR estimation. Although fat tailed student's-t distribution and historical simulation showed little improvements in VaR performance, all unconditional models suffer from similar problems and that is slow reaction to sudden volatility surge on the onset of the crisis. Furthermore, the GARCH model with normal innovations did not produce desired improvement compared to unconditional models. This is further evidence against the normal distribution. On the other hand, the student's-t conditional model as well as volatility weighted historical simulation of Hull & White (1998) proved to be more effective. The latter almost managed to cover expected number of violations and both models succeeded in capturing volatility clusters.

The most important contribution of this thesis is a practical implementation, assessment and real-life performance evaluation of VaR models, discussed above, to the latest Basel capital adequacy framework. Although a few authors examined impact of these proposals, we are, to the author's best knowledge, the first who attempted to evaluate its performance in a backtest. Our research is carried out as if the these new proposals have existed since 1990 and this allowed us to assess whether these extensions of the minimum capital requirements fulfilled aims set by the BCBS.

We calculated stressed VaR via the same models as for daily VaR computation but using data from five stress periods. Each new stress period had higher standard deviation than the preceding one which made them a better representation of financial stress. Hereafter, we implemented each VaR and SVaR model into the old Basel II and the new Basel 2.5 minimum capital framework. Results confirmed unsuitability of the unconditional distributions to compute VaR. Although an average capital ratios were slightly higher to those obtained from conditional models, the large number of violations especially in the recent crisis resulted in entering the red zone and thus further increased the capital requirements through the penalty factor. Moreover, enhancing the minimum requirements by the SVaR led to even higher required capital. The average increase was between 135% to 155% which is higher than the 110% presented in the quantitative impact study of Basel Committee on Banking Supervision (2009a).

On the other hand, the conditional models (with an exception of normal GARCH) performed way better. The minimum capital requirements were, in average, lower and both models avoided entering the Red Zone (i.e. there were less than 10 violations in preceding 250 days) entirely and thus were not further penalised. Furthermore, those models managed to predict the same amount of required capital during the crisis as unconditional models but without any additional enhancements such as SVaR and, more importantly, without overestimation of risk during the low volatile periods.

Therefore, we do not see any improvements by adding the SVaR to the current daily VaR, because the same level of capital can be obtained via some conditional methods, such as stundent's-t GARCH or volatility weighted historical simulation within the old framework. We showed that this enhancement produces much higher capital requirements while the real problems such as wrongly assumed normal distribution of daily returns or using unconditional distribution instead of conditional still remains.

Although we managed to cover wide range of models we simplified our research by assuming univariate portfolio represented by single stock index. However, it is known that during the crisis the correlations among risk factors tend to rise. Therefore there is a possibility of conducting similar research but allowing portfolio to depend on several risk factors such as exchange rates or interest rates.

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Appendix A

Tables

Table A.1: Violations of the 99% VaR estimate for daily S&P 500 log returns

		Number of violations					
		VaR model					
Year	Expected	N-UC	t-UC	HS	N-G	t-G	WHS
1992	2.54	0	0	0	1	1	1
1993	2.53	2	2	2	4	2	3
1994	2.52	9	8	6	9	8	6
1995	2.52	3	1	0	3	2	2
1996	2.54	10	8	6	10	5	4
1997	2.53	8	7	7	6	5	2
1998	2.52	7	6	5	6	4	4
1999	2.52	0	0	0	2	0	0
2000	2.52	4	3	3	4	2	2
2001	2.48	3	3	3	3	2	2
2002	2.52	5	4	4	4	3	3
2003	2.52	1	0	1	1	0	0
2004	2.52	0	0	0	2	2	3
2005	2.52	0	0	1	2	2	3
2006	2.51	4	4	4	5	5	5
2007	2.51	15	12	11	11	8	8
2008	2.53	28	17	18	11	4	3
2009	2.52	2	0	0	4	1	0
2010	2.52	0	0	0	7	3	2
2011	2.52	9	4	4	6	6	5
2012	2.5	0	0	0	4	2	0

Source: author's computations.

Table A.2: Normal VaR Minimum capital requirements

Year	Basel II MCR Mean (%)	Min (%)	Max (%)	Zone Gr.	Yell.	Red	Basel 2.5 MCR Mean (%)	Min (%)	Max (%)	Inc. (%)
1993	4.82	4.3	5.5	253	0	0	12.33	11.8	13	155.89
1994	4.58	4	5.2	64	188	0	13.16	11.5	14.8	187.42
1995	4.28	3.8	5.2	126	126	0	12.49	11.3	14.8	192.15
1996	5.12	3.9	5.6	7	247	0	14.33	11.4	15.3	180
1997	6.57	5.4	8.3	0	200	53	16.17	14.6	18.3	146.23
1998	8.29	6.8	10.2	48	204	0	17.05	14.3	19.3	105.67
1999	9.64	8.6	10.3	89	163	0	18.77	17.4	19.7	94.66
2000	9	8.5	9.6	252	0	0	18.91	18.4	19.5	110.04
2001	9.24	8.7	9.6	248	0	0	19.18	18.7	19.8	107.52
2002	10.1	8.9	12	149	103	0	20.87	19.1	23.5	106.59
2003	11.12	9.8	12.2	116	136	0	23.27	21.9	25.7	109.27
2004	8.78	6.6	9.8	252	0	0	20.89	18.7	21.9	138
2005	5.35	4.7	6.6	252	0	0	17.47	16.9	18.7	226.25
2006	4.62	4.5	4.7	251	0	0	16.74	16.6	16.9	262.24
2007	5.49	4.4	7.4	97	68	86	19.54	16.5	23.5	255.74
2008	9.82	7.4	16.3	0	0	253	25.97	23.5	32.4	164.57
2009	19.12	15.5	20.7	30	20	202	37.97	32.5	47.5	98.58
2010	14.87	11.6	15.5	252	0	0	34.96	31.7	35.6	135.21
2011	9.28	7.3	11.5	159	93	0	31.09	27.4	37.3	234.89
2012	10.25	8.3	11.7	93	157	0	33.82	28.4	37.5	229.85

Source: author's computations.

Table A.3: Student's-t VaR Minimum capital requirements

	Basel II MCR			Zone			Basel 2.5 MCR			
Year	Mean (%)	Min (%)	Max (%)	Gr.	Yell.	Red	Mean (%)	Min (%)	Max (%)	Inc. (%)
1993	5.47	4.9	6.1	253	0	0	13.26	12.6	13.9	142.27
1994	4.99	4.5	5.8	121	131	0	13.51	12.3	15.5	170.67
1995	4.77	4.4	5.8	190	62	0	12.99	12.2	15.6	172.28
1996	5.45	4.5	6.3	86	168	0	14.34	12.3	16	163.26
1997	7.06	5.9	8.8	0	253	0	16.62	15	18.6	135.52
1998	8.76	7.4	11	100	152	0	17.46	15.2	20.1	99.22
1999	10.49	9.4	11.2	106	146	0	19.92	18.1	20.6	89.91
2000	9.84	9.2	10.5	252	0	0	20.68	20.1	21.4	110.07
2001	10.1	9.3	10.6	248	0	0	20.95	20.1	21.6	107.57
2002	10.49	9.6	11.9	244	8	0	21.61	20.7	24.4	105.89
2003	11.07	10.7	11.5	252	0	0	22.87	22.3	23.4	106.48
2004	9.81	7.1	10.7	252	0	0	22.16	19.5	23	125.84
2005	5.51	4.7	7.1	252	0	0	17.86	17.1	19.4	224.1
2006	4.7	4.6	4.8	251	0	0	17.05	16.9	17.2	262.84
2007	6.24	4.7	9.4	97	114	40	20.33	17.1	25.8	225.67
2008	13.04	9.4	23.5	0	27	226	29.42	25.8	40	125.64
2009	25.23	20	27.7	55	17	180	45.09	40.2	54.6	78.71
2010	20.73	15.6	21.9	252	0	0	44.71	39.6	45.9	115.65
2011	11.05	9	15.5	252	0	0	35.03	33	39.5	217.04
2012	11.46	10.7	12.2	250	0	0	35.44	34.7	36.2	209.27

Source: author's computations.

Table A.4: HS VaR Minimum capital requirements

Year	Basel II MCR Mean (%)	Basel II MCR Min (%)	Basel II MCR Max (%)	Zone Gr.	Zone Yell.	Zone Red	Basel 2.5 MCR Mean (%)	Basel 2.5 MCR Min (%)	Basel 2.5 MCR Max (%)	Basel 2.5 MCR Inc. (%)
1993	5.3	4.4	5.7	253	0	0	14.43	13.5	14.9	172.28
1994	4.77	4.3	5.7	181	71	0	14.27	13.4	16.3	199.32
1995	4.92	4.7	5.7	193	59	0	14.36	13.8	16.4	191.97
1996	5.49	4.7	6.3	130	124	0	15.36	13.8	17	179.66
1997	7.34	5.4	9.1	6	247	0	17.98	14.5	20.2	145.1
1998	8.87	7.4	10.5	100	152	0	18.98	16.5	21.2	114
1999	9.31	9.2	10.5	248	4	0	19.34	18.4	21	107.83
2000	9.64	8.4	11.1	252	0	0	21.37	20.1	22.8	121.8
2001	9.36	8.4	10.5	248	0	0	21.26	20.2	23.6	127.02
2002	10	9.5	11.5	244	8	0	23.3	22.7	26.5	132.99
2003	10.86	10.4	11.9	179	73	0	24.62	23.6	26.9	126.59
2004	9.52	7	10.4	252	0	0	22.76	20.2	23.6	139.07
2005	5.2	4.6	7	252	0	0	18.44	17.8	20.2	254.81
2006	4.73	4.6	4.9	251	0	0	17.97	17.8	18.1	279.95
2007	6.52	4.8	10.3	97	114	40	21.63	18	27.9	231.79
2008	12.86	10.3	24.3	0	39	214	30.3	25.9	41.9	135.64
2009	25.95	20.8	27.8	55	12	185	48.18	42.1	61.6	85.66
2010	20.14	14.7	20.8	252	0	0	48.2	42.8	48.9	139.35
2011	11.07	9.5	14.6	252	0	0	39.13	37.5	42.7	253.52
2012	11.56	11.2	11.9	250	0	0	39.62	39.3	40	242.72

Source: author's computations.

Table A.5: Normal GARCH VaR Minimum capital requirements

	Basel II MCR			Zone			Basel 2.5 MCR			
Year	Mean (%)	Min (%)	Max (%)	Gr.	Yell.	Red	Mean (%)	Min (%)	Max (%)	Inc. (%)
1993	4.37	3.6	4.9	253	0	0	11.55	10.8	12.1	164.49
1994	4.79	3.4	5.5	35	198	19	13.45	10.6	14.9	180.71
1995	4.16	3.5	5.5	126	126	0	12.02	10.7	14.7	188.94
1996	5.85	3.8	6.9	7	247	0	14.66	11	16.1	150.53
1997	8.32	5.5	11.6	0	251	2	17.18	14.5	20.6	106.42
1998	9.29	5.9	14.8	91	161	0	17.27	13.1	23.2	85.84
1999	9.27	7.8	10.4	89	163	0	17.58	15.6	18.8	89.63
2000	9.38	7.5	12.4	171	81	0	18.14	15.9	21.9	93.34
2001	9.57	8.1	10.7	248	0	0	18.17	16.5	19.7	89.82
2002	10.39	8	15.3	244	8	0	20.79	18.4	27	100.17
2003	8.52	5.9	11.4	252	0	0	19	16.5	21.7	122.97
2004	5.17	4.9	5.9	252	0	0	15.74	15.4	16.4	204.16
2005	4.73	4.5	5	252	0	0	15.29	15.1	15.6	223.44
2006	4.61	4	5.2	228	23	0	15.3	14.6	16.6	231.98
2007	6.54	3.8	10.2	29	173	49	19.2	14.4	24.3	193.71
2008	16.46	10.2	40.6	0	20	233	30.5	23.9	54.7	85.32
2009	17.55	8.2	39.1	75	148	29	30.69	20.2	53.2	74.84
2010	9.56	6.5	14.5	79	173	0	23.4	18.5	29.2	144.8
2011	9.8	5.8	17.2	36	216	0	23.78	17.9	31.3	142.64
2012	7.25	5.3	13.7	119	131	0	20.18	17.4	27.8	178.26

Source: author's computations.

Table A.6: Student's-t GARCH VaR Minimum capital requirements

	Basel II MCR			Zone			Basel 2.5 MCR			
Year	Mean (%)	Min (%)	Max (%)	Gr.	Yell.	Red	Mean (%)	Min (%)	Max (%)	Inc. (%)
1993	4.68	3.9	5.3	253	0	0	11.93	11.1	12.6	154.75
1994	4.85	3.7	6.2	121	131	0	12.78	10.9	15.2	163.43
1995	4.64	3.9	6.2	167	85	0	12.38	11.2	15.2	166.6
1996	5.74	4.3	7	122	132	0	13.58	11.5	15.4	136.81
1997	7.67	5	11.7	187	66	0	15.17	12.2	19.9	97.69
1998	9.45	6.5	14.3	147	105	0	17.14	13.7	22.5	81.45
1999	9.04	8.5	9.6	252	0	0	16.86	16	18.2	86.53
2000	9.59	7.9	11.8	252	0	0	18.53	16.9	20.7	93.26
2001	10.28	8.7	11.5	248	0	0	19.44	17.6	21.1	89.06
2002	10.91	8.5	15.1	252	0	0	21.91	19.5	26.1	100.76
2003	8.71	6	11.8	252	0	0	19.7	16.9	22.8	126.32
2004	5.2	4.9	5.9	252	0	0	16.19	15.9	16.9	211.66
2005	4.75	4.6	5	252	0	0	15.75	15.6	16	231.6
2006	4.64	4.2	5	228	23	0	15.77	15.2	17.3	239.83
2007	7.16	4	11.9	75	176	0	19.52	15	25.6	172.68
2008	16.51	10.6	34.8	39	214	0	29.41	23.1	45.8	78.15
2009	16.85	8.8	33.6	158	94	0	29.04	21.7	45.5	72.38
2010	8.94	6.9	12.7	252	0	0	21.83	19.7	25.6	144.15
2011	10.48	6	19.4	147	105	0	24.26	18.9	34.4	131.52
2012	7.83	5.9	15.7	217	33	0	20.98	18.8	30.7	167.89

Source: author's computations.

Table A.7: Volatility weighted HS VaR Minimum capital requirements

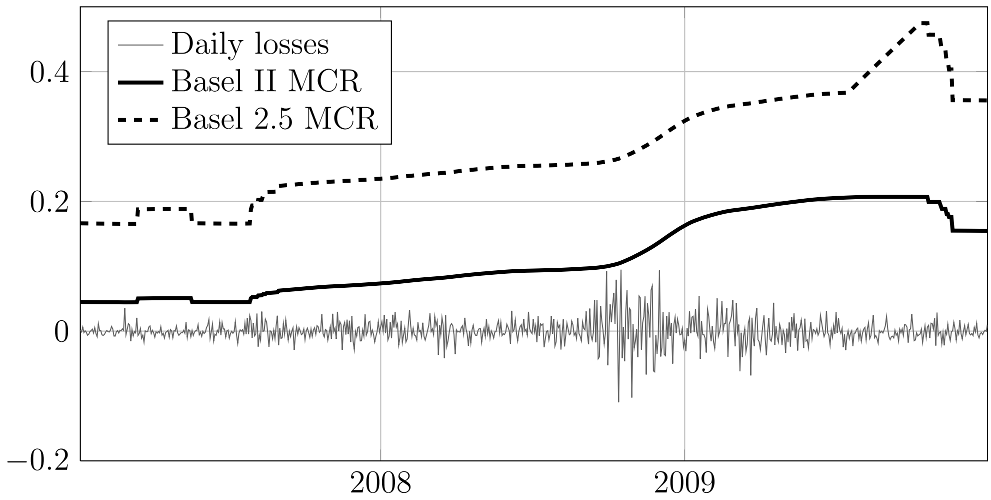
	Basel II MCR			Zone			Basel 2.5 MCR			
Year	Mean (%)	Min (%)	Max (%)	Gr.	Yell.	Red	Mean (%)	Min (%)	Max (%)	Inc. (%)
1993	4.31	3.6	5.1	253	0	0	13.98	13.2	14.7	224.39
1994	4.82	3.5	6.2	121	131	0	15.24	13.2	17.5	215.78
1995	4.79	4.2	6.2	193	59	0	14.79	13.9	17.5	208.73
1996	6.35	4.6	7.9	148	106	0	16.56	14.3	18.9	160.71
1997	8.66	5.6	12.2	253	0	0	18.33	15.2	21.9	111.73
1998	11.32	8	19	214	38	0	21.19	17.6	30	87.12
1999	11.13	9.1	12.1	252	0	0	21.55	20.4	23.2	93.56
2000	10.38	8.1	13.9	252	0	0	22.25	20	25.8	114.32
2001	10.14	8.5	11.3	248	0	0	22.11	20.4	23.3	118.05
2002	10.77	8.3	14.9	252	0	0	23.61	21.1	27.8	119.21
2003	8.78	6	12.1	252	0	0	21.62	18.8	25	146.29
2004	5.1	4.7	5.9	252	0	0	17.94	17.6	18.8	251.92
2005	4.64	4.4	5	252	0	0	17.48	17.2	17.8	277.05
2006	4.98	4.2	6.3	144	107	0	18.55	17.1	20.8	272.68
2007	7.37	4.1	12.4	65	186	0	22.11	17	28.4	200.23
2008	17.85	10.2	41.5	99	154	0	32.15	23.1	54.4	80.1
2009	19.03	10	40	252	0	0	33.92	28.5	52.8	78.24
2010	9.67	6.6	14.2	252	0	0	28.48	25.4	33	194.55
2011	11.14	5.8	21.8	149	103	0	30.98	24.6	43.1	177.96
2012	8.76	6.7	17.4	233	17	0	27.74	25.5	38.8	216.63

Source: author's computations.

Appendix B

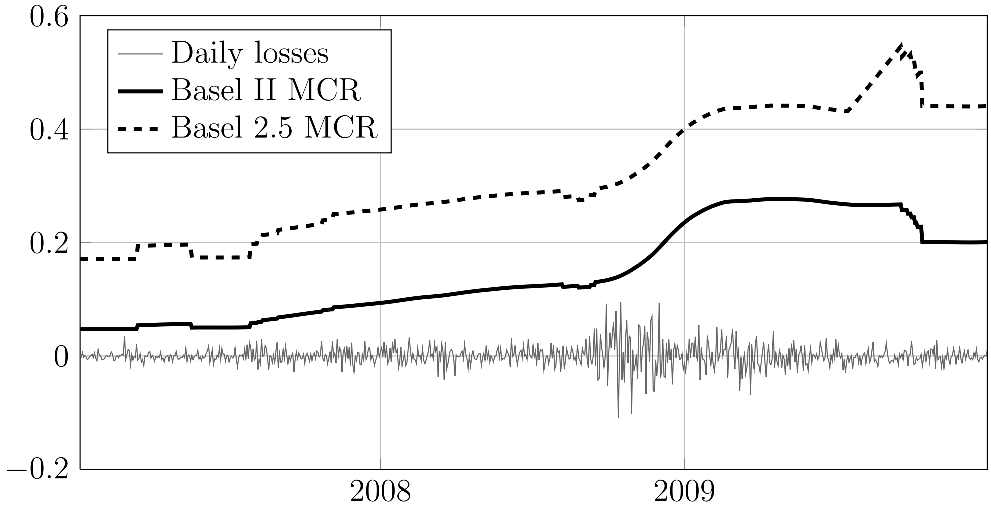
Figures

Figure B.1: Minimum capital requirements - Unconditional Normal VaR



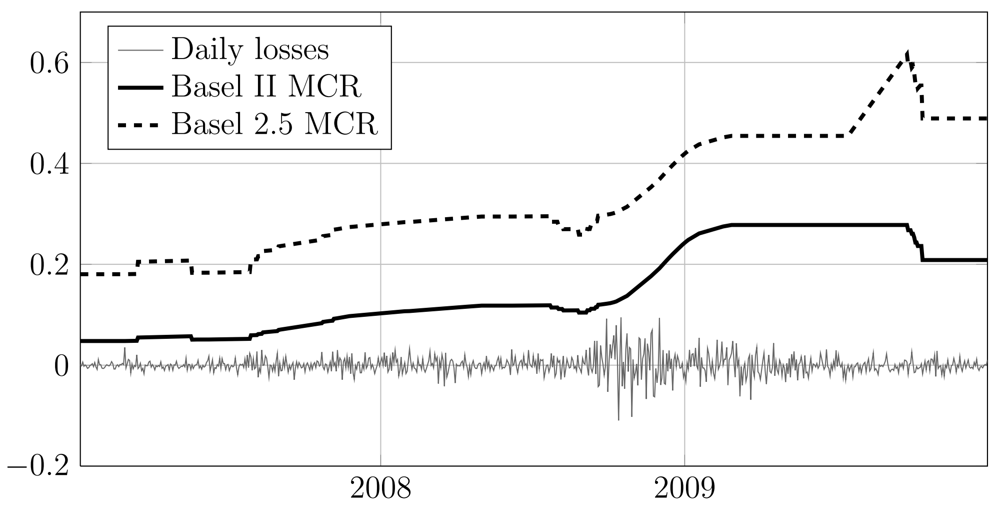
Source: author's computations.

Figure B.2: Minimum capital requirements - Unconditional Student's-t VaR



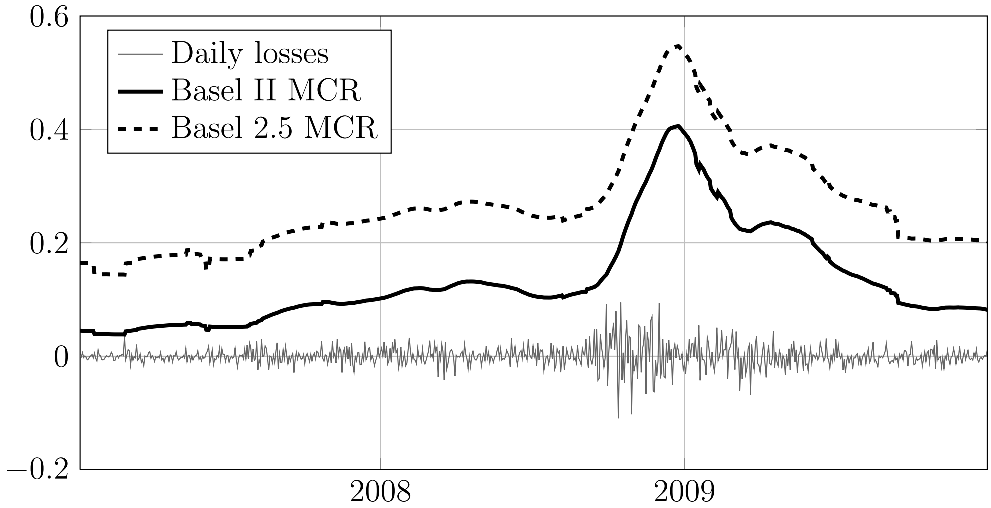
Source: author's computations.

Figure B.3: Minimum capital requirements - Unconditional Historical Simulation



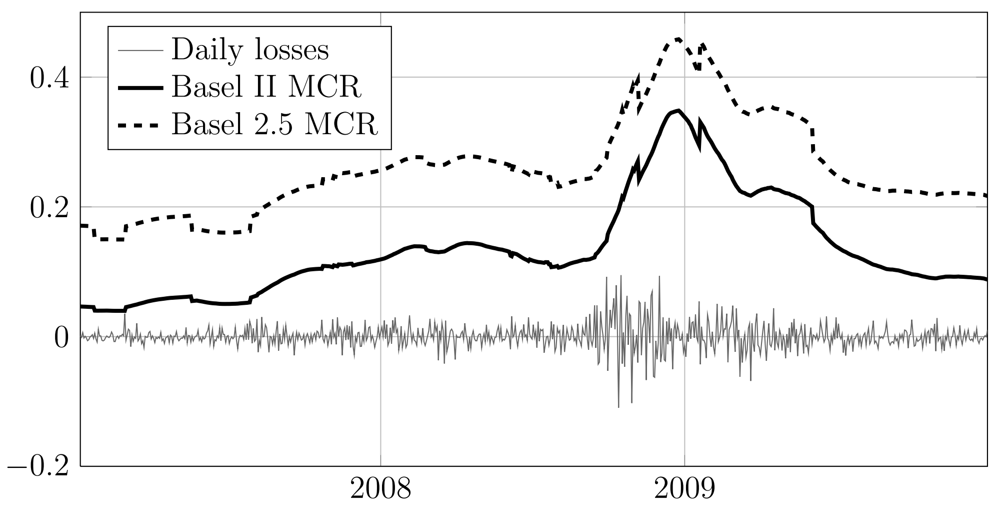
Source: author's computations.

Figure B.4: Minimum capital requirements - Normal GARCH VaR



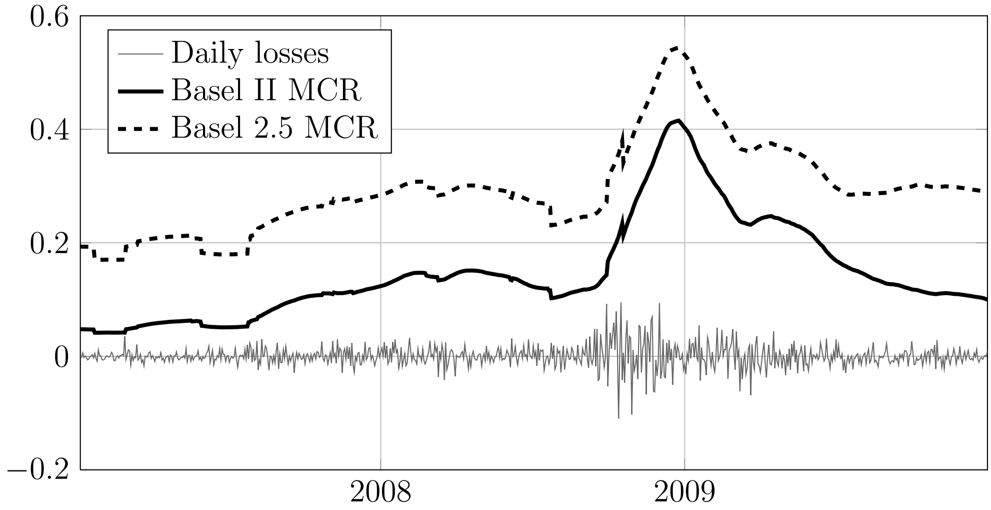
Source: author's computations.

Figure B.5: Minimum capital requirements - t-GARCH VaR



Source: author's computations.

Figure B.6: Minimum capital requirements - Volatility weighted HS



Source: author's computations.

Bachelor Thesis Proposal

Author	Vojtěch Pižl
Supervisor	PhDr. Boril Šopov, MSc., LL.M.
Proposed topic	Stressed Value-at-Risk: Assessing extended Basel II regulation

Topic characteristics Value at Risk (VaR) is a popular tool of risk management used to measure maximal losses of a specific portfolio at a given significance level. Because of its simplicity (outcome is one number) it has been set by regulators for IRB (Internal rating based) banks as a device to calculate their minimal capital requirements. But recent crisis revealed several problems. Although VaR measures maximal loss of a portfolio, it does not tell us anything about losses beyond chosen significance level. Another issue is a common assumption of normally distributed profits and losses which tend to underestimate VaR.

In reaction to significant bank losses which were higher than the minimum capital requirements the Basel Committee for Banking Supervision (BCBS) suggested adding the Stress Value-at-Risk (SVaR) to the current VaR. Stress VaR is a measure of portfolio risk which takes into account long time horizon (1 year) of financial stress. This additional requirement should according to BCBS reduce the pro-cyclicality of the minimum capital requirements.

First aim of this thesis will be review of the VaR model. We will analyze advantages of this model and its weaknesses as well. Then we will focus on the Stress VaR and attempt to find out whether it is a useful measure for an extreme risk and evaluate its performance in comparison with the VaR model. After that several models for VaR under fat tail distributions will be analyzed and compared with the Gaussian VaR.

Hypotheses

1. Normal Value-at-Risk is not sufficient tool for measuring risk, especially in the stress period (e.g. crisis).
2. Stress VaR (SVaR) is an efficient device to assess and report financial risk.
3. We endeavour to assess whether the higher capital requirements implied by Stress VaR only mitigate underlying wrongly assumed normality. .

Outline

1. Introduction
2. Literature review
3. Theoretical background
   (a) Value-at-Risk review
   (b) Stress VaR
4. Models application
5. Discussion on results
6. Conclusion

Core bibliography

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Supervisor