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Doctoral Dissertation

Developing the SIMPLE Approach for Teaching Mathematical Word Problems: A
Design-Based Study of Grade 8 Pupils in Kosovo

Metoda "SIMPLE Approach" pro výuku matematických slovních úloh:
Konstruktivní výzkum žák 8. ročníku v Kosovu

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I, the author, declare that the work “Developing the SIMPLE Approach for Teaching Mathematical Word Problems: A Study of Grade 8 Pupils in Kosovo” is my own and has not been submitted for a degree at any other institution. None of the work has previously been published in this form. I declare that I have not used AI for any part of research and writing up the thesis. To improve the English language used in the thesis, Writefull and Grammarly have been used.

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ABSTRACT

Word problems often cause difficulties for lower-secondary pupils and have thus been the subject of much research. This study explores the implementation of the SIMPLE Approach, a constructivist-based intervention for enhancing word problem-solving skills through bar models and metacognitive steps in lower-secondary mathematics classrooms. In Kosovo, teachers tend to neglect word problems despite their inclusion in the curriculum. Pupils have little to no experience with visual representations of word problems, and the development of metacognitive skills is usually not the focus of teaching mathematics. Based on the relevant literature, a specific teaching approach was designed to suit the educational context in Kosovo.

The study was conducted over three cycles of design-based research methodology, each refining and iterating the intervention based on feedback and observed outcomes. The intervention was tested in a classroom setting, with data collected through pre-tests, post-tests, observations, the teacher's diary, audio recordings, and interviews with pupils and teachers. The intervention involved three mathematics teachers and 62 pupils.

A precondition of this study was the teachers' and pupils' appreciation of the SIMPLE Approach and teachers' willingness to pause their regular curriculum and carry out the intervention. Despite this, some barriers were uncovered during the implementation of the intervention. One such was the teachers' usual teaching style. Different from the second and third teachers, the first teacher's teaching style was quite distinct from the required constructivist underpinning of the intervention, which made adapting the SIMPLE approach more difficult. Furthermore, the pupils' lack of familiarity with strategies in WP solutions posed a significant challenge. Introducing them directly to the block model method and the SIMPLE steps was demanding.

Despite these challenges, both teachers and pupils recognised the effectiveness and usefulness of the new approach. Even the pupils, after five or six lessons of practising utilising the SIMPLE approach, improved their performance on complex two-step word problems, as evidenced by increased post-test scores. While generalisability is limited, the findings suggest the SIMPLE Approach holds promise in improving pupils' understanding and ability to solve mathematical word problems but highlights the need for comprehensive teacher training and ongoing support, particularly when introducing novel methods to ensure effective implementation and high fidelity.

Keywords: Bar model, metacognition, pupils, SIMPLE approach, teachers, word problems.

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LIST OF ACRONYMS

BM – Bar Model

DBR – Design Based Research

MEST – Ministry of Education, Science and Technology (in Kosovo)

Px_1 – Pupil x (Cycle 1) / where x is 1,2 and 3

Px_2 – Pupil x (Cycle 2) / where x is 1,2 and 3

Px_3 – Pupil x (Cycle 3) / where x is 1,2 and 3

WP – Word Problem

WPs – Word Problems

WPx_E – Word Problem x of the pre-test / where x is 1, 2, 3, 4, 5 and 6

WPx_O – Word problem x of the post-test / where x is 1, 2, 3, 4, 5 and 6

WPx_I – Word Problem x of the intervention / where x is 1, 2, 3, 4, 7 and 8

WPx_H – Word Problem x of the homework / where x is 1 and 2

1. INTRODUCTION

Word problems (WPs), as a crucial component of mathematics education, remain a vital subject of discourse in pedagogical and research endeavours. They represent specific written mathematical tasks set in real-world contexts that require the application of mathematical concepts to answer the questions posed. They form an integral part of school mathematics education, facilitating the development of essential skills such as reading comprehension, problem-solving, and critical thinking among pupils (Blum & Niss, 1991; Englert et al., 1987; Pongsakdi et al., 2020; Verschaffel et al., 2020). WPs require pupils to bridge the gap between abstract mathematical principles and real-world scenarios, thereby preparing them for challenges beyond the classroom (Verschaffel et al., 2020). As they are regarded as an important element of teaching and studying mathematics, they have driven curricula worldwide to focus on and learn mathematics through them (Liljedahl et al., 2016). As Verschaffel et al. (2020, p. 2) assert, “...word problems, if appropriately designed and handled in a mathematics educational context, can act as accurate and valuable ‘simulacra’ of authentic mathematical modelling problems one could encounter in real life.” Furthermore, Verschaffel et al. (2000, as cited in Verschaffel et al., 2020) argue that WPs enhance pupils’ motivation to learn mathematics and foster the development of creativity and critical thinking. Therefore, WPs serve not merely as instruments for learning mathematics but also as foundational exercises that equip pupils with essential life skills in reasoning and analytical thinking.

WPs, as important as they are, often present challenges for pupils. Research indicates that these challenges are based on various aspects. These aspects include comprehension of linguistic and contextual information (Jaffe & Bolger, 2023; Verschaffel et al., 2020), translating the problem into mathematical symbols (Pongsakdi et al., 2020; Verschaffel et al., 2020), and self-monitoring the process of solving WPs (Verschaffel et al., 2020). Comprehension-related challenges can impact pupils’ success. If pupils lack comprehension, they find it difficult to decode vocabulary used in WPs and understand their context. This can be attributed to various factors, including cultural differences such as variations in language usage, daily life experience or societal norms, which may make the wording or context of WPs unclear (Verschaffel et al., 2020). In contrast, the translation-related challenges are attributed to pupils’ failure to identify and apply correct mathematical expressions and operations when solving WPs (Verschaffel et al., 2020). On the

other hand, a self-monitoring challenge is when pupils cannot reflect on their process as they engage with WPs to see if they took the right approach. Pongsakdi et al. (2020) reported that WPs that demanded the use of realistic considerations were difficult for many pupils. Boonen et al. (2014) observed that Dutch sixth-grade pupils with low performance in complex WPs had low semantic comprehension. This implies the complexities of teaching and learning WPs, which require research evidence to inform teachers about successful practices.

In response to the challenges associated with WPs, the authors proposed various ways to facilitate pupils' solutions to WPs. The proposed solutions focus on heuristics, metacognitive strategies, and visualisation (Verschaffel et al., 2020). Heuristics are cognitive strategies that guide pupils in problem-solving by using systematic steps to analyse and break down the problem. These strategies increase the chance of finding correct solutions by encouraging pupils to actively engage with and think through the problem (Depaepe et al., 2010; Schoenfeld, 1985; Verschaffel et al., 2020). In other words, heuristics are guidelines or principles that enable pupils to approach the learning task more systematically. Examples of heuristics include decomposing the problem into small patterns, guessing and checking, working backwards, acting it out, etc. (De Corte et al., 2004; Verschaffel et al., 2020). On the other hand, metacognitive strategies like self-regulatory or monitoring strategies are those that the pupils use to control their thinking and learning before, during, and after working on the task (Mayer, 1998; Verschaffel et al., 2020). Planning one's own solution, self-monitoring learning progress or evaluating the response are examples of such metacognitive strategies (Verschaffel et al., 2020).

Visualisation is regarded as part of the heuristic in which pupils create or use a schematic or pictorial representation of a WP. Compared to pictorial representation, schematic representation proved to be a better approach to help pupils transform abstract concepts into concrete ones (Verschaffel et al., 2020). Studies have reported the positive influence of interventions using various heuristics and metacognitive strategies on pupils' learning WPs. For instance, visual representations such as diagrams (Cooper et al., 2018) or bar models (Kho, 1987) have been observed to improve spatial skills, which are important for understanding the structure of WPs and translating them into mathematical models (Jitendra & Woodward, 2019; Strohmaier et al., 2022). Furthermore, Verschaffel et al. (2020) conclude that interventions using visualisation strategies positively improved pupils' competence in WPs. Nevertheless, they caution that most of the studies with positive outcomes are from experimental contexts, which may differ from real

classroom contexts. However, there are real contexts, such as in Singapore, where the bar models have been integrated into the curriculum for a long time, and the approach seems to be successful in developing pupils' competence in WPs (Kaur, 2018). Therefore, it can be argued that heuristics, metacognitive strategies, and visual representation could improve WP learning.

Kosovo is among the countries whose mathematics curriculum for the 6th to 9th grades includes WPs. One of the aims of this level of education is to enhance pupils' competence in problem-solving. This makes WPs even more relevant in Kosovo's curriculum as they have the potential to improve pupils' problem-solving skills. However, studies (Berisha & Bytyqi, 2020; Morina & Vondrová, 2021; Vula & Berdynaj, 2011) show that in Kosovo, teaching-learning practices for WPs rarely employ strategies that foster developing pupils' problem-solving skills. For instance, Berisha and Bytyqi (2020) observed a lack of explicit heuristic strategies in Kosovo's mathematics textbooks. Additionally, evidence derived from the PISA report shows that pupils in Kosovo do not have sufficient ability to solve WPs, as reflected by a mean mathematics performance score ranking among the lowest of the PISA-participating countries (OECD, 2019). Furthermore, drawing upon the author's extensive 17-year experience as a student and an additional eight years as a teacher in Kosovo, it is evident that WPs are often neglected in mathematics lessons. Pupils have little to no experience with visual representations of WPs, and the development of metacognitive skills is usually not the focus of teaching mathematics (Berisha & Bytyqi, 2020; Morina & Vondrová, 2021). Consequently, one may argue that the current teaching-learning practices for WPs in Kosovo compromise pupils' development of problem-solving skills.

A systematic literature review was undertaken to identify studies focused on WPs. This process led to the identification of multiple articles pertinent to the current study, addressing the challenges pupils encounter in solving WPs, the stages involved in WP solving, and the interventions implemented to overcome these challenges. Based on these three critical aspects, a specific teaching approach called the SIMPLE Approach was designed to fit the educational context in Kosovo. This approach combines factors such as schematic visual representation (the bar model method) and metacognition alongside constructivist teaching principles to help teachers and pupils deal with the solution of WPs. By involving teachers and pupils in the research process, this study seeks to provide them with a new approach that addresses the specific needs of teaching and learning WP solving in Kosovo. The designed approach aims to cater to pupils' needs at all stages of working on WPs, such as understanding, planning, solving, and reflecting on the solution. The

primary goal of this study is to determine whether this approach is suitable for both parties, teachers and pupils and whether it enhances pupils' ability to solve WPs. This is done through the design-based research methodology consisting of three cycles. The findings and the designed approach aim to contribute to educational practices and policies, providing a practical solution to a common challenge in mathematics education. Evidence from this study will promote teaching and learning in several ways. From the research perspective, the study will contribute to new research knowledge on facilitating the teaching and learning of WP solving. On the practical level, the study will provide teachers with a sound teaching approach that will help them enhance their pupils' ability to solve WPs. On the theoretical level, the study will provide insight into the processes through which pupils master visual representations as an effective means of WP solving and how the focus on their metacognitive skills helps them in solving WPs.

The thesis consists of six chapters divided into sections.

Chapter 2 reviews the current and relevant literature that forms the basis for the study. It starts by presenting the importance of WPs in the mathematics classroom. Then, it describes variables that influence the WPs' difficulties, presents results of intervention studies and focuses on metacognition and visual representation. The theoretical framework that guided our study and Kosovo's local context regarding pupils' ability to solve WPs is detailed. The chapter ends by emphasising the research's goal and the research questions.

Chapter 3 provides an overview of the study's design, methods, and sample and describes procedures, including data collection, data analysis, and the three study cycles.

Chapter 4 covers the results of three iterative cycles with the suggestions and changes made during each cycle. It also compares their results to see the impact of the SIMPLE Approach after the interventions' refinement. Furthermore, this chapter provides insights from the teacher's diary and teacher and pupil interviews to bring insight into the conditions of and barriers to the SIMPLE Approach implementation.

Chapter 5 restates the purpose and research question, discusses and interprets the findings, and integrates them with the findings from the literature. It also presents the study's implications and limitations.

Chapter 6 is the last chapter, which shortly describes the aim of the study, presents its key findings, explains its contribution to the existing body of research in mathematics education and provides concluding remarks on the impact of the new teaching approach in solving WPs.

The thesis is concluded by a list of references. To provide enough detail about the research process, the thesis is complemented by five appendices: Teacher Guide, teachers' semi-structured interview questions, pupils' semi-structured interview questions, teachers' diary format and all WPs used in the study with the solution using arithmetic, algebraic and bar model method.

2. THEORETICAL BACKGROUND

This chapter begins by describing the importance of WPs in mathematics lessons. Subsequently, various studies related to the challenges pupils face concerning WPs and the factors influencing these challenges are examined. Interventions that influenced pupils' problem-solving skills are presented, emphasising two critical elements: visual schematic representations and metacognitive skills. Furthermore, the chapter delineates the various stages or steps of WP solving that researchers propose to facilitate WP solutions. The theoretical framework underpinning our study is subsequently detailed. Lastly, a contextual overview of the education system in Kosovo is provided, detailing the presentation of WPs within schools, curricula, and textbooks. Overall, this review gave us a comprehensive understanding of what has been explored in the realm of WPs in education.

2.1 The Importance of WPs in Mathematics Classrooms

WPs are important in mathematics classrooms because they show pupils how abstract mathematics can be applied to real-world contexts. They challenge pupils to think critically and creatively, thus making mathematics more engaging and relevant for them (Verschaffel et al., 2020). A fundamental advantage of WPs in the classroom lies in their ability to enhance mathematical reasoning. When addressing such problems, pupils transcend rote learning and mechanical exercises, applying their knowledge to realistic problem situations where mathematical reasoning serves as a tool for arriving at solutions (Wyndhamn & Säljö, 1997). WPs guide pupils to analyse situations, identify pertinent details, and mathematically model solutions, thus fostering critical thinking and analytical skills (Reusser & Stebler, 1997). This makes them effective in preparing pupils to handle complex mathematical challenges in academic and real-life situations. In addition, WPs help pupils enhance their reading comprehension skills. By solving these problems, pupils deal with language-based issues, identify important information, and translate them into mathematical expressions (Powell et al., 2019). This combination of literacy and numeracy skills strengthens overall academic success. Additionally, WPs help pupils apply learned concepts, going beyond simple memorisation.

Incorporating WPs into the classroom aligns with constructivist educational principles by promoting pupils' active engagement and exploration. By presenting realistic situations, WPs

enhance pupils' interest in and motivation for learning mathematics, which Verschaffel et al. (2020) claim as crucial for developing higher-level cognitive skills. Finally, Jitendra and Xin (1997) argue that interventions that used visual strategies for solving WPs (e.g., diagramming) and strategy-training procedures (e.g., cognitive and metacognitive ones) effectively met the needs of pupils with diverse learning abilities. This flexibility underscores WPs' vital importance in inclusive mathematics classrooms, supporting differentiated instructional strategies.

In summary, WPs are crucial in mathematics education because they foster critical thinking, enhance literacy skills, and support pupils' engagement with mathematical lessons. Incorporating WPs into daily practices enriches learning experiences and prepares pupils for real-world problem-solving.

2.2 Difficulties Faced by Pupils in Achieving Success in WP Solutions

Various variables that contribute to WP difficulty have been explored in previous research. Researchers began exploring the challenges pupils face while solving WPs, starting with simple arithmetic WPs (Cummins et al., 1988; De Corte & Verschaffel, 1987) and proceeding to more complicated ones (Verschaffel & De Corte, 1997; Strohmaier et al., 2022). Daroczy et al. (2015) show that many linguistic and mathematical variables and their combination affect pupils' strategies, mistakes, and problem-solving success.

Unsurprisingly, pupils' insufficient ability to read and comprehend the problem is considered one of the most common sources of difficulties in WP solutions (Pearce et al., 2013). Cummins et al. (1988) report that one of the difficulties pupils have with WPs is due to their inability to comprehend abstract or unclear language. Pongsakdi et al. (2020) reveal that pupils who were weak in text comprehension but good in arithmetic outscored those who were strong in text comprehension but weak in arithmetic even on easy problems. This highlights the dual demands of linguistic and numerical skills in solving WPs, demonstrating that comprehension skills are as crucial as computational abilities.

2.2.1 Factors Contributing to WP Difficulty

In this section, selected factors which influence WP difficulty for pupils will be presented.

Linguistic Complexity: The complexity of language components significantly affects the difficulty of WPs. According to Daroczy et al. (2015), linguistic complexity may be analysed in terms of all

components of the language system, including words and their lexical and morphological characteristics, how these words generate sentences, text structure, and verbal expressions. For example, Lepik (1990) shows that structural complexity in terms of WP text features, such as the number of letters, words, sentences, sentence length, or the proportion of complex (long) words, is highly correlated with the problem-solving time and WP solution in general. Similarly, Abedi and Lord's (2001) study demonstrates the influence of pupils' language backgrounds on their performance on arithmetic WPs. Their results revealed that English language proficiency and the ability to understand the language used in mathematical WP influenced pupils' performance.

Inconsistent Language: One of the linguistic factors that make WPs difficult is the use of inconsistent language. Inconsistent language refers to situations where a term (or terms) in the problem text does not align with the operation required for its solution (e.g., the word “fewer” requires addition rather than subtraction in inconsistent language WPs). Verschaffel et al. (1992) and Vondrová (2020) highlight pupils' challenges when solving WPs with inconsistent language. A possible cause of these problems might be teaching practices where pupils are taught to associate specific terms like “more,” “less,” “together,” and “divide” with particular operations. However, instructing pupils to approach WPs this way discourages mathematical reasoning and often results in wrong solutions (Powell & Fuchs, 2018) and leads to the use of a wrong operation (Hegarty et al., 1992).

Numerical Complexity: Numerical factors such as the type of numbers, the operations involved, and the number of solution steps also significantly impact WP difficulty. Daroczy et al. (2015) note that WPs with complex mathematical structures are more challenging to solve, even when linguistic complexity is low. Additionally, multi-digit numbers are harder to comprehend than single-digit numbers because the complexity of arithmetic grows as the number of digits increases. The influence of decimal numbers in the WP statement on pupils' WP-solving abilities was studied by Koedinger and Nathan (2004), who found that whole-number problems are substantially simpler than decimal-number problems. Englert et al. (1987) found that including irrelevant numerical information in the WP text negatively impacted the success of pupils of various abilities as they struggled to incorporate irrelevant numbers into their solutions.

Combination of Factors: Research has shown that if a WP includes several complicating factors, such as irrelevant numbers and inconsistent language, it becomes even more difficult to solve.

Vondrová (2020) and Verschaffel et al. (1992) emphasise that the interaction of these factors creates an additional layer of complexity, further challenging pupils' problem-solving abilities.

2.2.2 Possible Didactic Causes of Problems

The difficulties pupils face are not only inherent to WPs but are also influenced by instructional practices. As already stated, teachers often rely on keyword strategies, associating specific terms with arithmetic operations, which can lead to misinterpretations, especially in problems with inconsistent language (Powell & Fuchs, 2018). While this approach may simplify initial understanding, it discourages mathematical reasoning and leads to errors, particularly in problems with inconsistent language and more complex problems. Hegarty et al. (1992) emphasise that teaching strategies should focus on fostering reasoning and understanding rather than relying on surface-level keyword associations. This shift in instructional focus is crucial for addressing pupils' difficulties in solving WPs.

Additionally, instructional design plays a role in the challenges pupils face. The inclusion of irrelevant or overly complex numerical information in WPs can increase the cognitive load¹, making it harder for pupils to solve them effectively (Jaffe & Bolger, 2023). Similarly, the teachers' emphasis on only one way of recording the relevant information from the WP text, which Novotná (2010) calls a legend, can negatively impact pupils' solutions. On the other hand, if the structure of the WP is harder to grasp (for example, in a WP with a mixed order of data), the creation of a legend may help the pupil gain insight into the structure of the problem (Novotná & Chvál, 2018).

2.3 Overview of the Interventions Aiming to Improve WP Solving

Following the difficulties identified in the literature above, researchers conducted interventions aiming at a better understanding of WP. For example, some interventions focused on reading comprehension (De Corte et al., 1985; Fuchs et al., 2020; Wong & Ho, 2017), and others looked at the influence of text comprehension (e.g., Dröse & Prediger, 2021; Fuchs et al., 2015; Lafay et al., 2021; Powell et al., 2019). The primary focus of this study is the development of pupils' metacognition and visual representation, as these two aspects have been found beneficial for WP solving by the meta-analysis of studies (Hembree, 1992; Perry et al., 2018).

¹ Cognitive load refers to the amount of mental effort required to process and solve a task (Sweller, 2011).

2.3.1 Metacognition as an Important Factor in Solving WPs

One of the main factors that play an important role in WP interventions is the development of pupils' metacognitive skills (e.g., Perry et al., 2018; Teong, 2003). Perry et al. (2018) describe metacognition as a higher-order thinking process needed for successful learning and problem-solving. It is also known as “thinking about thinking,” “self-regulated learning,” “thinking skills,” and “learning to learn”. It includes knowing one's thoughts and exercising control over one's cognitive processes. Metacognitive skills are an important part of mathematics education because they help pupils comprehend when, why, where, and how to apply their knowledge to solve problems successfully (Carr & Jessup, 1995). Veenman et al. (2004) refer to metacognitive skills as procedural knowledge necessary for regulating and controlling one's learning activities. Such abilities include, among others, the analysis of the problem, planning, monitoring, checking, and recapitulation.

Positive effects of metacognitive strategy training on mathematical problem-solving achievement were reported by Özsoy and Ataman (2009). It was accomplished through intervention research in which pupils were guided through stages that included understanding the problem, planning a solution, executing the plan, and reflecting on the process. During the intervention, teachers facilitated the learning process by asking reflective questions like, “What will you do now?” or “How do you know this works?” to encourage pupils' metacognitive thinking and self-monitoring. The results revealed that pupils who were taught metacognitive strategies had improved problem-solving skills. A metacognitively skilled pupil can better focus on the WP's important information, which is required for constructing an appropriate problem representation. As a result, they can create a clear plan for solving a problem. Such an elaborate plan, containing goals and directions for activities, entails the possibility of a better solution to a problem (Veenman et al., 2004).

Various models of problem-solving stages have inspired interventions that emphasise metacognition. Beginning with Polya (1945), researchers have analysed pupils' steps when solving WPs and used these insights to develop teaching frameworks. Polya's four-step model involves understanding the problem, devising a plan, carrying out the plan, and reflecting on the solution. Mason et al. (1982) simplified this into three phases: Entry, Attack, and Review. Schoenfeld (1985) extended Polya's framework by dividing the planning phase into “design” and “exploration”, highlighting exploration as the “heuristic heart” of WP-solving. Yimer

and Ellerton (2010) developed a five-phase cognitive/metacognitive model that includes engagement, transformation, implementation, evaluation, and internalisation. Lucangeli et al.'s (1998) model includes six skills that, according to the authors, gained the most empirical support: text comprehension, problem representation, problem categorisation, a solution plan, result estimation, and self-evaluation. Text comprehension helps pupils to determine what information is necessary and appropriate for solving a WP. Problem representation aids the organisation of textual information and comprehension of their interactions, which leads to the formulation of the solution plan. The ability to categorise a problem or understand its deep structure is known as problem categorisation. A solution plan is needed to obtain the appropriate solution. The four skills mentioned above belong to cognitive skills. The estimation of the result, the process's self-evaluation, and the calculation's self-evaluation are derived from metacognitive abilities. The estimation of the result includes recollections of past personal experiences with calculating numbers and similar problems. Self-evaluation of the process and calculation, on the other hand, is the capacity to assess our own performance concerning our abilities to solve mathematical WPs in general.

The intervention studies focusing on developing WP solving based on metacognitive learning have done so in different ways. For example, Teong (2003) developed a metacognitive strategy called CRIME (the acronym stands for Careful Reading; Recall Possible Strategies; Implement Possible Strategies; Monitor and Evaluation) and provided evidence that even low achievers improved their performance in WP solving as they became more aware of their cognitive processes when solving WPs.

The SOLVE Strategy (Study the problem, Organize the facts, Line up a plan, Verify your plan with action and Evaluate your answer) is another mnemonic-based learning strategy developed to help pupils solve WPs (Freeman-Green et al., 2015). The authors documented a positive impact of this strategy on the problem-solving performance of pupils with learning difficulties. Another strategy that positively affected pupils' ability to solve WPs is the Hybrid Strategy (Rosli et al., 2020), which consisted of drawing and perceiving WPs as stories. The pupils were asked to use graphic representations and interrogative terms (who, what, where, when, and how) with the mnemonic "Mr How and his 4 Warriors" to improve their metacognitive ability.

Another strategy that helps pupils realise problem-solving stages is the STAR strategy, with each symbol representing a cognitive strategy step (Search the problem; Translate the words into an equation in a picture form; Answer the problem; Review the solution). Findings revealed that pupils who used the STAR strategy successfully solved the change problems involving one-stage addition and subtraction (Maccini & Hughes, 2000; Özkubat et al., 2021).

2.3.2 Visualisation and the Bar Model Method

The second factor shown to be effective in enhancing WP solving ability is based on the assumption confirmed by research (e.g., Csikos et al., 2012; Hembree, 1992) that visual representations help pupils solve WPs as they bridge the challenging step from reading a WP to creating a mathematical model (Gani et al., 2019). A graphical representation of a WP is a heuristic method that can assist in the early stages of a WP's solution process by providing a variety of relevant sources of information (Verschaffel et al., 2020). Presmeg (1986) defines the visual method of solving a mathematical WP as one that incorporates visual imagery, with or without a diagram, as an important aspect of the solution process. According to Van Garderen and Montague (2003), a strong positive relationship exists between the total number of visual-spatial representations and pupils' general mathematical problem-solving skills. Hegarty and Kozhevnikov (1999) further demonstrate that schematic spatial representations in mathematical WPs are highly beneficial and associated with problem-solving success.

Kribbs and Rogowsky (2016) confirm the effectiveness of visual-spatial schematic representations on WP solving; nevertheless, they emphasise that the presentations, whether visual or schematic, are difficult to categorise and, even when designed correctly, might lead pupils off track in the solution due to misreading the diagram. Boonen et al. (2014) warn that it matters how the visual representation looks. They found out that if the representations were inaccurate, pupils had a lower chance of completing a WP correctly by nearly six times compared to pupils with accurate representation. Fig. 2.1 shows an example from their study demonstrating how pupils give an accurate/inaccurate visual-schematic representation.

Some studies used prompts for pupils such as “organise information using a diagram,” “draw a picture to tell what is happening,” or “visualise the problem in a picture or diagram” (Jitendra & Star, 2011; Montague, 2008; Pantziara et al., 2009), leaving it to the pupils to create a picture. It has been shown that when pupils use schematic drawings instead of pictorial ones, they are more

successful in solving WPs (Boonen et al., 2014; Hegarty & Kozhevnikov, 1999; Rogowsky et al., 2016). Thus, learning a schematic representation may be more beneficial for pupils.

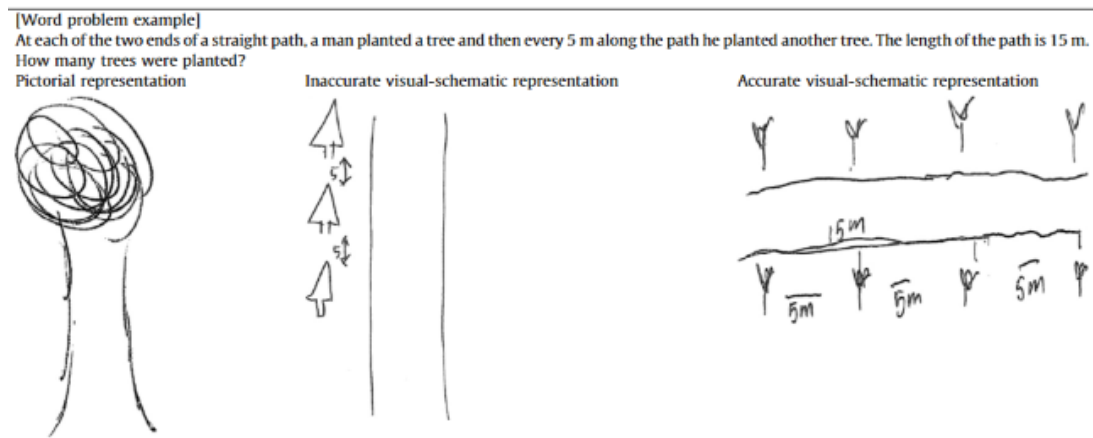


Fig. 2.1: Inaccurate/Accurate visual-schematic representation (Boonen et al., 2014, p.17)

The above shows that representations positively influence pupils if used/chosen/designed properly, but research lacks information on what type of visual representations is the most beneficial in WPs. Nevertheless, at least indirectly, we may infer that a schematic representation such as a bar model is appropriate. For example, Gani et al. (2019) point to the fact that Singapore pupils regularly rank in the top countries in mathematics, which may also be attributed to their successful adoption of the bar model (BM) method. Bar models are rectangular-shaped representations that visually illustrate relationships among quantities. They were developed by the Singaporean Primary Mathematics Project team led by Kho Tek Hong in the 1980s and became a distinguishing feature of the curriculum and mathematics textbook (Kho, 1987). This method provides a visual representation of the quantities given in a WP, allowing pupils to link the known and unknown quantities to solve the problem with the proper operation.

There are three basic types of BMs: the part-whole model, the comparative model, and the change model (Kaur, 2018). The part-whole model describes the relationship between the whole and its constituent parts, and it is based on the fact that the whole is made up of two or more parts. The length of each rectangle indicates the (known) quantity it represents and reflects the relationship between these quantities. In problem situations involving comparison models, a quantity is compared to another, showing how much one part is greater/smaller than the other. In this case, the length of each rectangle indicates the (unknown) quantity that the rectangle represents. Finally, problems that involve change situations can be solved using the before-after model. A change

model depicts the relationship between a quantity's new value and its original value after an increase or decrease (Fig. 2.2).

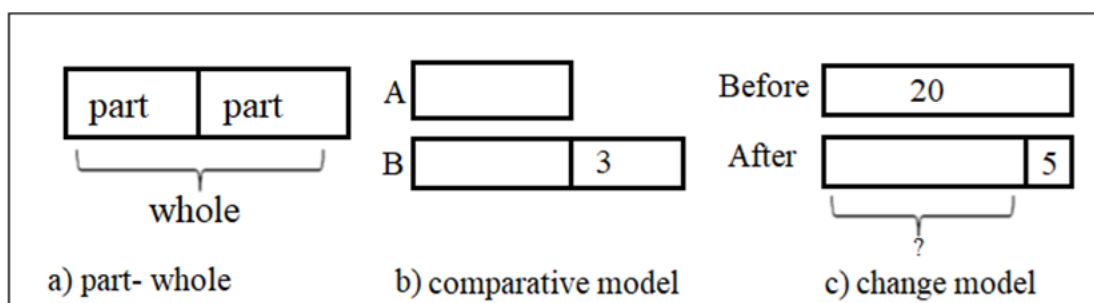


Fig. 2.2: Types of bar models

According to Ng and Lee (2005), a BM method is valuable for addressing WPs when pupils lack formal algebra knowledge. Furthermore, it is a good teaching aid for mathematics teachers because the information provided by the pupils' model can help teachers recognise the cause of pupils' difficulties when solving WPs. The BM method was found to be appropriate for lower secondary levels and can also be used for complex WPs with high processing needs (Ho & Lowrie, 2014). For example, Ho and Lowrie (2014) used the problem in Fig. 2.3 in their study and found that pupils who drew the BM correctly always arrived at the correct answer.

Postcard Problem: *Gilbert and Hazel have some postcards. After Gilbert gives 18 postcards to Hazel, he has 20 postcards more than her. How many more postcards than Hazel does Gilbert have at first?*

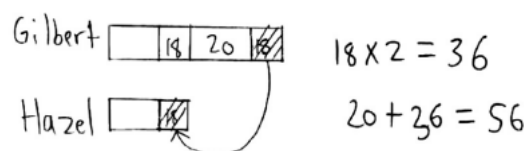


Fig. 2.3: Pupil solution to the Postcard Problem (Ho & Lowrie, 2014, p. 92)

There is evidence from countries like Japan, Singapore (Kaur, 2018; Murata, 2008; Ng, 2003), and the Netherlands (de Koning et al., 2022), where visual representations are consistently used, that the BM method effectively enhances mathematics learning. Early on, this method aids in solving arithmetic problems, and later, it helps represent known and unknown quantities or variables.

Successful attempts to introduce the BM method at the primary level have been made globally (Bao, 2016; Baysal & Sevinc, 2021; Gani et al., 2019; Ho & Lowrie, 2014; Kaur, 2018), showing learning gains even for pupils with specific learning disabilities (Morin et al., 2017; Xin, 2019).

Regarding Kosovo's context, two case studies with three 4th-grade pupils (Morina & Vondrová, 2021) and three 8th-grade pupils (Morina, 2023) reported positive impacts of the BM method.

2.4 Active Learning in Mathematics

Research in mathematics education supports pupils' active role in learning (Gani et al., 2019; Kaur, 2018; Özsoy & Ataman, 2009; Teong, 2003). Intervention studies on WP solving also include characteristics of active learning, such as studies by Özsoy and Ataman (2009) or Teong (2003), who use metacognitive stages (planning a solution, evaluating the correctness of solutions, or reflecting on the process), to engage pupils in reflective thinking and require them to take an active role in their learning rather than passively follow instructions. Similarly, interventions using BMs for WP solutions require pupils to actively engage with the problem by organising the data, visually presenting it, and connecting the representation to the mathematical operation, thus encouraging active participation and deeper understanding (Gani et al., 2019; Kaur, 2018). A pupil's active role in acquiring knowledge is a basic principle of constructivism (Chand, 2023; Noddings, 1990; Shah, 2019). According to the constructivist philosophy of education, knowledge and critical thinking are actively constructed by involving the learners in the learning process and engaging them in classroom activities rather than having them passively absorb information from the teacher (Zajda, 2021). Mcnamara et al. (2002) emphasise that teachers must create opportunities for pupils to engage with mathematical concepts actively and collaboratively, thus mirroring the constructivist belief that understanding is co-constructed in social and educational settings.

The theory of constructivism originates from the work of Jean Piaget and Lev Vygotsky, who initiated the idea that traditional learning is passive learning (Chand, 2023; Fosnot & Perry, 1996; Piaget, 1973). Piaget emphasises that knowledge about the world is constructed rather than received. According to Piaget's work, humans assimilate and adapt to new information based on prior knowledge, which calls for an educational approach that encourages problem-solving and critical thinking (Von Glasersfeld, 1982). Knowledge construction involves various processes such as assimilation, accommodation and equilibration. Assimilation is when new information is integrated to fit the existing ones (Hergenhahn & Olson, 2005). Accommodation is when the existing information structure is altered to fit the new information. Equilibration is the maintenance of the status quo so that there is no conflict between new and existing ideas (Chand, 2023).

In contrast, Vygotsky brought the notion of social constructivism, emphasising the significance of sociocultural learning. According to Vygotsky, cognitive and intellectual development depends on pupils interacting with adults, peers, and more knowledgeable ones (Smagorinsky, 2007). In both cases, student-centred teaching is emphasised through interactivity, the use of past experiences, and the incorporation of pupils into the learning process.

Our intervention (see Section 3.3) is embedded in the basic tenet of social constructivism (Simon, 1995), in which pupils are active participants in the learning process, and the teacher acts as a facilitator. It involves collaboration among pupils while solving WPs, as problem-solving in groups is a rich learning experience and was found to support WP-solving skills (Ventistas et al., 2024).

In contrast to constructivist approaches to teaching, the traditional approach, also known as transmission teaching, positions the teacher as the main authority on knowledge, neglecting the pupils and their cognitive engagement. It focuses more on memorisation rather than on the development of creative thinking (Lord, 1999). The new education trends started to use constructivist teaching globally, which gives space to pupils, allowing them to construct knowledge based on their previous experiences and social negotiation (Khalid & Azeem, 2012). However, applying this theory in the classroom requires the adoption of teaching strategies that reflect how children naturally learn and construct their knowledge. As learning occurs most effectively in the classroom when pupils actively participate in the process and build on their prior knowledge and experiences (Piaget, 1973), the role of the teacher moves from being the primary source of knowledge to that of a learning facilitator who does not simply transmit information but guides, supports, and challenges pupils to construct their understanding.

Challenges often arise when teachers revert to traditional methods, even after training in constructivist approaches, due to their familiarity with the conventional way or concerns about classroom management (Zielinski, 2017). For instance, Windschitl (2002) highlights that teachers sometimes struggle to fully adopt student-centred methods, preferring teacher-led practices to ensure control and efficiency. They are reluctant to accept that they are not the primary source of education, and often, the culture of the pupils in the classroom makes it difficult to comply with the constructivist philosophy. These findings underline the importance of comprehensive training

and ongoing support for teachers to build confidence in new approaches and avoid reliance on familiar but less effective strategies.

In the lessons in which the traditional form of teaching prevails (such as in Kosovo), the focus on pupils' active learning will require substantial changes in the teacher's work. In other words, it will likely lead to re-negotiating the didactic contract. The didactic contract consists of implicit agreements between teachers and pupils about their roles, responsibilities and expectations in learning (Brousseau, 1997). The didactic contract in a constructivist classroom posits that pupils are encouraged to actively engage with the new material, to cooperate and interact with their peers, while the teacher's role includes providing scaffolding, monitoring pupils' progress, setting up discussions and encouraging collaboration.

2.5 Kosovo Context

Kosovo, a small, recently independent country in the Western Balkans with a population of about 1.7 million, has had an unstable recent history, particularly during its repressive measures in the 1990s, which led to a war from 1997 to 1999. After the war, the United Nations (UN) administered Kosovo for most of the 2000s before Kosovo declared its independence in 2008.

The United Nations Children's Fund (UNICEF), part of the UN, was one of the numerous international organisations that arrived in Kosovo after the war to assist in various areas, including education. They contributed to creating the new Kosovo Curriculum Framework, which was adopted in 2001 and represents the first educational reform in Kosovo. The 2001 Curriculum Framework (Georgescu et al., 2001) led to the production of new textbooks created in accordance with the curriculum requirements. The main aim of this curriculum was to break the traditional teaching method with teacher-centred practice toward a pupil-centred approach to teaching and learning (Tahirsylaj & Fazliu, 2021).

In 2011, the authorities introduced Kosovo's New Curriculum Framework (MEST, 2011), resulting from years of educational reform work. This was a significant milestone in Kosovo's education system, orientating it to competency-based learning outcomes (Beka, 2017). After some efforts, in 2016, the curriculum framework was revised due to several pilot classes (MEST, 2016), and this revision is now being used in Kosovo's schools. The revised curricular framework aims

to prepare pupils to face the challenges of the 21st century and to generate new competitive knowledge for the labour market.

The pre-university education system in Kosovo consists of nine years of compulsory education and three years of secondary education. Primary education includes grades 1–5 (primary level) and grades 6–9 (lower secondary level), while secondary education includes grades 10–12 (upper secondary level). Our study is situated in the lower secondary level, namely 8th-grade pupils.

In Kosovo, WPs are rather rarely used in mathematics lessons, and similarly to other countries, pupils face difficulties solving them. Although the Kosovo State Curriculum specifies that primary school pupils' learning aim is to use knowledge in problem-solving (MEST, 2011, 2016), this is not realised to an adequate extent. Berisha et al. (2013) found that in Kosovo's mathematics textbooks, the general problem-solving strategies are not employed; teachers and pupils seldom use problem-solving heuristics, and the heuristics are only presented implicitly and indirectly. Furthermore, Berisha and Bytyqi (2020) revealed that most of the tasks in Kosovo's mathematics textbooks pertain to categories that mainly support lower-order learning outcomes. The textbooks do not present visual representations for WP solving, and teachers rarely, if at all, present visual representations when dealing with WPs (Morina & Vondrová, 2021). This is further confirmed by Vula and Berdynaj (2011), who state that Kosovo teachers continue to use traditional teaching with computations to complete arithmetic operations, depending exclusively on textbooks. Furthermore, metacognitive skills are not mentioned anywhere in the Kosovo curriculum, and based on the author's experience as a teacher, they are rarely promoted in the classroom.

2.6 Goal of the Study and Research Questions

The literature presented in the previous sections highlights a conception of teaching WP solving that integrates the BM method and the development of metacognitive strategies grounded in constructivist teaching principles. This conception will make the basis for a teaching intervention that will be implemented in grade 8 teaching in Kosovo in three cycles of design-based research. The study will augment international research on teaching interventions on WP solving in several ways. Although various approaches and models are proposed to support pupils in WPs, a gap remains regarding their application in the classroom worldwide. Considering Kosovo's specific context, the question remains whether such approaches could be helpful in teaching and learning WPs in the cultural context where the transmission way of teaching prevails, and visualisation and

metacognition are not promoted. A Teacher Guide prepared within the study will aid this process and will help teachers strengthen and support pupils' participation in the learning process. The study will add methodological results to the body of research. Unlike the studies presented above that evaluated the effectiveness of the intervention without ongoing development, design-based research with iterative cycles will be used to develop the new teaching approach within the study.

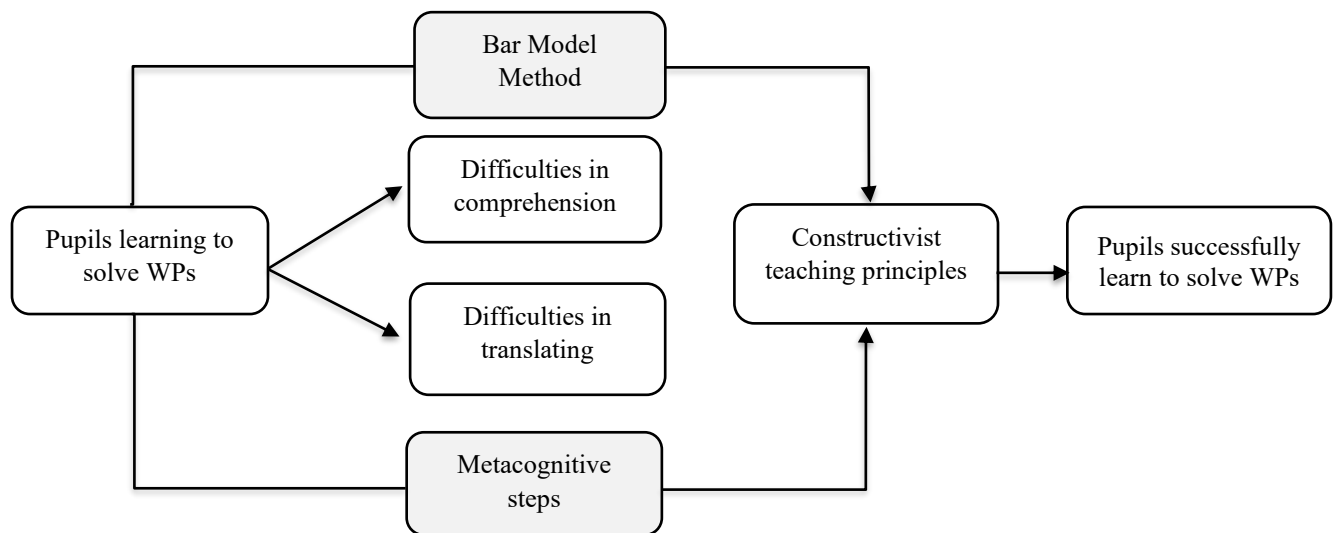


Fig. 2.4: Conceptual framework

Within the study, an intervention (called the SIMPLE Approach) aiming to develop grade 8 pupils' ability to solve WPs will be developed and tailored to the needs of Kosovo teachers. The intervention will be based on the presence of the BM model and the development of metacognitive skills embedded within the social constructivist approaches to teaching. The implementation of this intervention will be investigated to answer the following research questions:

- RQ1. How do pupils and teachers perceive the SIMPLE Approach's usefulness, and how confident are they with adopting it in WP solutions after the intervention?
- RQ2. What are possible conditions of and barriers to implementing the proposed intervention?
- RQ3. What impact does the intervention have on pupils' success when solving WPs? To what extent can they use the BM method and the SIMPLE steps in their solutions?

3. METHODOLOGY

This chapter discusses the research methodology, which utilises a design-based research approach. Van den Akker (1999) refers to such an approach as “development research,” Hoadley (2002) as “design-based research,” Cobb et al. (2003) used the term “Design experiments,” etc. Cobb et al. describe design experiments as theoretical and pragmatic in nature, focusing on how the design works and how it affects learning. This work will use the term Design-Based Research (DBR).

The chapter begins by describing the characteristics and significance of DBR in education. Next, a description of the teaching approach and WPs used during the intervention are presented, followed by case study results. The chapter continues with the data collection methods, justifying their usage in all the study phases. An overview of DBR, as used in our research, including sample selection, ethical consideration, and intervention implementation, is provided. Lastly, a description of the data collection and analysis procedures is provided, highlighting the systematic approach employed to extract insights from the gathered data.

3.1 Design-Based Research

DBR is realised through iterative cycles (DBRC, 2003; Gravemeijer & Cobb, 2006) and is interventionist in nature. Barab and Squire (2004) describe DBR as a collection of approaches aimed at developing new ideas and practices that influence learning and teaching in realistic environments.

DBR is vital for understanding how educational innovations work in classroom practice. It shows the connection between theory, practical tools, and teaching methods (DBRC, 2003). Educators use this methodology to increase education research’s impact, transfer, and translation into practice. The DBR methodology underlines how important it is to build theories and design principles that may guide, inform, and improve both practice and research in the field of education (Anderson & Shattuck, 2012). Anderson and Shattuck further highlight the importance of collaboration between teachers and researchers when conducting a DBR in a regular classroom, as their interaction provides valuable information for the research process and findings. Similarly, Sterner (2019) argues that the participation of teachers in all phases of the design process is central to their learning. The benefit of including teachers in curriculum development lies in increasing their engagement and confidence when applying novel methods, fostering a deeper understanding

of instructional strategies and increasing the likelihood of sustainable implementation (Brown et al., 2020). However, in our study, teachers were not involved in the design process because they were unfamiliar with the bar model method and were lowly proficient with teaching WPs. In DBR, maintaining fidelity of implementation by the teachers is crucial to ensure that the intervention is delivered as intended, with teachers as deliverers of the teaching being essential for such fidelity. Fidelity in educational studies refers to the extent to which an intervention is delivered as planned and conveyed to pupils based on the instructions given in a training session or a written guide (O'Donnell, 2008; Smith et al., 2007). There is usually a difference between intervention-as-designed and intervention-as-implemented (Nelson et al., 2012). High fidelity is all the more difficult to achieve in teacher-led interventions (as opposed to researcher-led interventions), as their commitment and adaptability greatly influence their success. Thus, intervention studies involve the phase of teacher training. On the other hand, while strict adherence to the original design of an intervention is important, teachers often need to adapt their teaching to factors such as concrete class and pupils' needs, classroom dynamics, the availability of resources, pupils' misbehaviour (Combs et al., 2022), etc. Nevertheless, these adaptations should not stray from the core components of the intervention to ensure that the intended outcomes are achieved (Carroll et al., 2007). In our study, fidelity includes the alignment between the proposed implementation with its three characteristics (metacognitive scaffolding, bar models, and constructivist underpinning), and its actual practice in the classroom.

The DBR process usually starts with identifying an educational challenge that needs investigation (Step 1 in Fig. 3.1). Step 2 involves deriving insights from existing literature. The proposed initial design, outlined in Step 2, undergoes refinement through iterative cycles of analysis, design, implementation, and redesign in Step 3. After the result analysis and discussion with teachers and pupils, this process may result in new design principles and implementations in Step 4. This design can potentially provide a theory that supports teacher needs and educational reforms in a classroom setting.

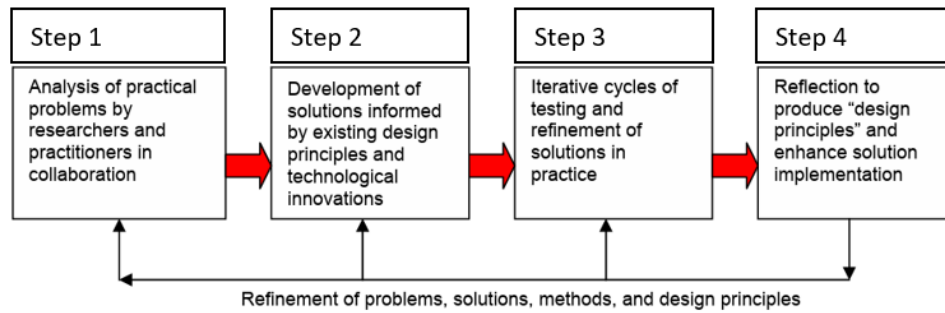


Fig. 3.1: The iterative DBR process (Amiel & Reeves, 2008, p. 34)

Therefore, the DBR was chosen as it works well for developing and refining educational interventions in practical contexts. My main goal is to develop an intervention with an approach that aids pupils in solving WPs by combining the BM method with metacognitive steps embedded in teaching based on constructivism. This challenging educational aim requires an iterative, flexible, and context-sensitive research approach, which DBR offers. According to DBR's iterative nature, the teaching approach can be continuously improved and refined. In the present study, the DBR methodology allowed me to evaluate and improve the intervention's effectiveness systematically, analysing the data and redesigning the Teacher Guide during three cycles of iterations. Additionally, it provided valuable feedback on pupils' and teachers' viewpoints regarding the teaching approach through direct collaboration with them. Therefore, I chose DBR as it ensures that the intervention evolves in response to practical challenges and insights acquired from actual classroom settings, making it more elaborate and adaptable. I reviewed the current literature in the problem definition phase (see Fig. 3.1). I gathered information about interventions in WP solving (Section 2.3), determining that the difficulties in solving WPs among pupils are related to challenges in comprehending the problem and translating it into mathematical symbols. This implies that any teaching approach for facilitating pupils' learning to solve WPs should cater to both factors. The literature review led to the decision to focus on the visualisation and development of metacognition in line with the constructivist approach (see Sections 2.3.1 and 2.3.2).

In the designing phase, I developed a teaching approach based on the literature review, considering the Kosovo context. The approach is based on the presence of the BM method, the development of metacognition and principles of constructivism via the so-called SIMPLE Approach (see Section 3.3).

The evaluation phase started by implementing the SIMPLE Approach in a case study (Section 3.5). This helped me adjust the methodology and prepare a Teacher Guide. Next, three intervention cycles were organised with three mathematics teachers and three 8th-grade classes, the results of each informing the next cycle (see Section 3.7).

Throughout this procedure, I collected diverse types of data to evaluate the effectiveness of the intervention and pupils' and teachers' perceptions of it (see Section 3.8).

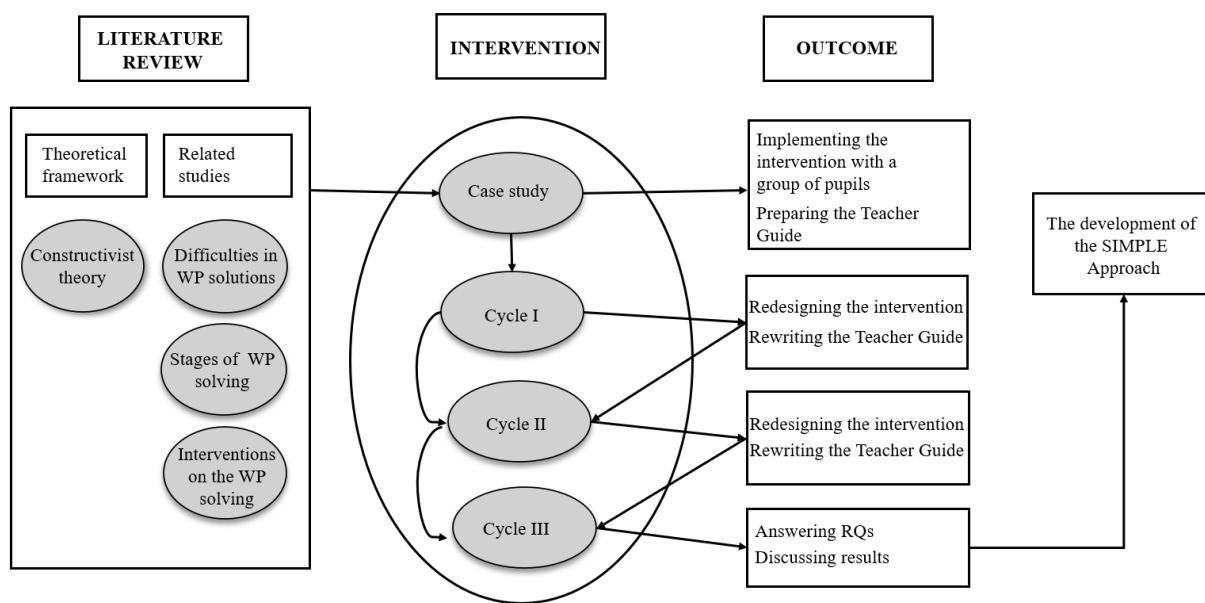


Fig. 3.2: An overview of DBR in the thesis

3.2 Case Study

A case study design involves an in-depth investigation of a specific subject, such as a person, institution, or place (Priya, 2021). This research design is helpful in a context where the research goal is to provide concrete and contextualised evidence for an in-depth understanding of the subject (Yin, 2014). The case study design is regarded to be useful for addressing complex issues, such as proposing or examining practical solutions in real-life contexts (Ebneyamini & Moghadam, 2018). In most cases, the design employs qualitative data collection methods such as interviews, observation and document analysis. Using these data collection methods helps the researcher gain an in-depth understanding of information as they allow the researcher to probe more if they need more data.

Based on the characteristics of this research design, I found the case study method relevant to my work because I am interested in gaining in-depth knowledge of how pupils might perceive the developed approach and acquire the skills promoted in it. The case study in my research aimed to trial and adjust the teaching plan before its use in real classroom settings in Cycle 1 of DBR.

3.3 Description of Teaching Approach

The study aims to examine the pupils' use of the SIMPLE Approach combining the BM method, the SIMPLE steps, and teaching based on constructivism (see Sections 2.3.1, 2.3.2 and 2.4) after a brief instructional intervention and determine whether it was linked to their problem-solving success. The sections below will present a brief overview of these three aspects only, as the content and course of the intervention will be described in more detail when DBR cycles are presented.

3.3.1 Bar Model Method

As shown in Section 2.3.2, the BM method makes it easier for pupils to break down and comprehend the relationships between elements of WPs. Through the visual representation of abstract numerical relationships, the BM method facilitates pupils' comprehension of complex concepts helping them link the known and unknown quantities to solve the problem with the proper operation. As the BM method was to be used with Kosovo pupils with no experience with visual representations for solving WPs, they were to be introduced to the three basic types of BMs on examples of several WPs and then practise making such models for some more WPs. More details on how the BM method was introduced in the intervention lessons and how it was applied in a WP solution are presented in Section 3.7.2.

3.3.2 SIMPLE Steps

The SIMPLE steps are metacognitive steps grounded in the frameworks which divide solutions to WPs into steps and which were shown to be beneficial for developing pupils' WP solving (Ho & Lowrie, 2014; Polya, 1957; Powell & Fuchs, 2018; Teong, 2003; Xin, 2019). None of the frameworks from the literature above includes an explicit step devoted to the visual component (BM) and thus, they were modified and what I called the SIMPLE steps originated.

The SIMPLE steps address factors that influence pupils' difficulties in solving mathematical WPs and facilitate WP solutions from the beginning till the end. The acronym stands for "Study the problem", "Involve the BM method", "Monitor the process", "Prepare/Present the solution",

“Look over the solution again”, and “Evaluate your answer”. Table 3.1 shows how comprehensive the steps are and what they require pupils to do to solve mathematical WPs successfully. Section 3.7.2 illustrates the application of these steps in the solution of two WPs and presents the intervention lessons in greater detail.

In Albanian the mnemonic is “LEHTE”– Lexo problemin, Eksploro/Monitoro qasjen blok model, Harto/Vëzhgo një plan, Trego zgjidhjet, Ekzamino përgjigjet.

Table 3.1: The SIMPLE steps

Study the problem	Read the problem and rephrase it. “Have I understood what I am supposed to find?”
Involve the Bar Model method	Draw a diagram (create a visual representation of the word problem). How are the pieces of information in the word problem related? How can I represent these relations by rectangles?
Monitor the process	Go back to the beginning, including the problem formulation. Ask yourself: “Is the bar model method helping me to understand how to reach the solution?” / “Am I getting any closer to my objective?”/ “Am I still implementing the solving strategy I picked?”
Prepare / Present the solution	Make the expression/calculation/equation, write the solution, and present it.
Look over the solution again	Check the whole solution, solving steps and the calculations.
Evaluate your answer	Evaluate whether your solution makes sense and if there is a better way to solve the problem. Write up the answer.

3.3.3 Constructivist Teaching

As shown in Section 2.4, constructivist teaching is an approach that focuses on pupils’ learning by encouraging them to explore, ask questions, and construct their own understanding through interactions with the teacher and their peers. This approach is intended to promote a deeper comprehension of WPs. The SIMPLE steps employed in the study encourage pupils to reflect on their problem-solving processes, organise their solutions, monitor the process, and evaluate their answers, which is an important aspect of cognitive development.

Teachers are critical in implementing the constructivist teaching approach in our study as they guide pupils through the learning process. The teacher implements lessons by incorporating the BM method and the model of solving steps fostering metacognitive skills. Consistent with constructivist principles, the lessons promote active involvement, problem-solving, and reflection

through group work and discussions with and among pupils. The teacher provides scaffolding to aid pupils' learning during the intervention phase. To support the learning of the SIMPLE Approach, the teacher explains basic concepts, guides pupil practice, sets up discussions, encourages collaboration, lets pupils construct their own solutions and provides feedback as pupils work through WPs. The teacher gradually lowers the support as pupils gain proficiency to encourage independent learning. Therefore, the SIMPLE steps and the BM method can be seen as scaffolding techniques which should provide strong support for pupils to develop skills necessary to learn WP solving (BMs as a bridge towards forming equations and SIMPLE steps as a support for moving through the solving process) so that they could do without it later when the skills become internalised.

3.3.4 Teacher Guide

The Teacher Guide is designed for primary school mathematics teachers. It has been developed to support them in their efforts to teach WPs with the SIMPLE Approach. It outlines detailed instructions and recommendations on how to implement the Approach to make use of its potential in their teaching. The Guide introduces the BM method, describes its types, and then provides some WPs for practice that include the BM method in their solutions. The SIMPLE steps are then explained, along with their indicators, significance, and their application in some WPs' solution processes. In addition, the Teacher Guide presents teachers with principles of a constructivist approach to teaching, emphasising principles that promote pupils' active involvement, problem-solving, and reflection through group work and discussions with and among pupils.

The Teacher Guide is designed to be user-friendly, enabling teachers to find relevant information quickly and easily. The teachers were provided with the Guide in their mother tongue (Albanian). Its English translation is in Appendix 1.

3.4 Data Collection Methods

The data for the study were collected through multiple methods, namely tests, interviews, audio recordings, observation, and teacher diaries. Using multiple methods enriches the data as these methods complement each other (James et al., 2008; Koshy, 2005) and helps researchers support the trustworthiness of their findings through data corroboration (Flick, 2009). Using multiple data collection in the study was also motivated by the need to get an insight into the complex issues

connected to classroom teaching, the developed teaching approach and its implementation with teachers and pupils.

3.4.1 Pre- and Post-Tests

A pre-test was utilised to determine the pupils' baseline WP solving skills before the intervention, and a post-test was employed to evaluate a possible change following the intervention. The selection of WPs in the pre-test and post-test was guided by the Kosovo curriculum, 8th-grade mathematics textbooks, and advice from the mathematics teachers of the experimental classes. This selection ensures that the WPs chosen are age-appropriate and relevant to the pupils' current learning stage. Thus, they are valid measures of pupils' skills within their educational context.

The pre-test and post-test were designed with different WPs to minimise potential learning effects from a short intervention. However, both tests were carefully elaborated to ensure equal difficulty levels.

Both the pre-test and the post-test include six WPs. All were formulated in a context familiar to the pupils: buying or selling, having money or books, school, going to a game park, reading books, etc.

Five of the problems in each test are formulated within the domain of whole numbers to avoid the additional cognitive load that might arise from working with fractions and decimals. One problem in each test requires the use of decimal numbers or fractions but the calculations are not complicated. The numbers are in the role of quantities (three in the pre-test, four in the post-test), measures (one in the pre-test) and currency (two in the pre-test, two in the post-test).

In terms of the operation required for their solution, the problems can be divided into three groups: additive (only additive operations are needed; four in the pre-test and two in the post-test), multiplicative (only multiplicative operations are needed; one problem in the post-test) and additive-multiplicative (two in the pre-test and three in the post-test). Thus, in terms of the operations, the problems in the post-test were meant to be slightly more difficult which was considered appropriate considering that pupils would get much practice in solving WPs during their teaching.

All the WPs are two-step problems and could be solved by arithmetic and algebraic strategies and the use of BMs (see Appendix 5). Most of the problems contain comparison terms, such as “more

than,” “less than,” and “ n times as many,” so that the basic types of BMs could be used (Ng & Lee, 2009).

There are three consistent and three inconsistent problems in each test. Recall that consistent WPs use straightforward language and do not contain contradictions or misleading cues (de Koning et al., 2022), while inconsistent WPs include words, phrases, or cues that refer to an operation inverse to the one required for the solution. Pupils are usually less successful with inconsistent WPs (see Section 2.2.2).

Tables 3.2 and 3.3 present problems from the pre-test and the post-test.

Table 3.2: WPs used in the pre-test (E stands for the pre-test)

WP1_E (Inconsistent, Additive)	WP2_E (Consistent, Additive)	WP3_E (Inconsistent, Additive, Multiplicative)
Of the 450 books by an author, 124 were sold to one publishing house on the first day, which is 24 more books sold than on the second day. How many of the author’s books have not been sold yet?	During the running competitions, on the first day, the pupils ran 0.7 km; on the second day, 0.4 km more than the first day; and on the third day, 0.3 km less than the second day. How many km did they run in total?	Three groups of pupils in one class bought 35 pizzas to celebrate their success. The first group got 16 pizzas, which is five pizzas more than the second group and two times more than the third group. How many pizzas did the second and third groups get?
WP4_E (Consistent, Additive, Multiplicative)	WP5_E (Consistent, Additive)	WP6_E (Inconsistent, Additive)
On the second day of the trip, a family decides to go to a game park. They buy four adult tickets for \$49 each and eight student tickets for \$31 each. How much money do they get back if they pay with five \$100 bills?	Andi, Era and Ari wanted to buy a bicycle that cost 101 euros. Andi had 20 euros more than Era. Era had six less than Ari. How much money did Ari have?	Emma bought 22 children's toys, which is five more toys than Andra bought. They donated 22 of their toys. How many toys are left?

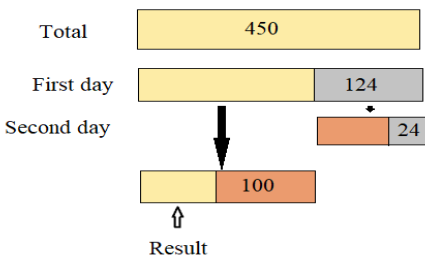
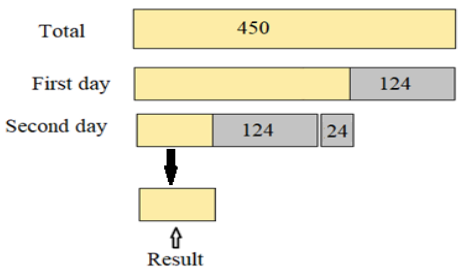
Table 3.3: WPs used in the post-test (O stands for the post-test)

WP1_O (Consistent, Additive, Multiplicative)	WP2_O (Inconsistent, Additive, Multiplicative)	WP3_O (Consistent, Additive, Multiplicative)
Roni had three times as much money as Kiara. Greta had two times as much money as Kiara. If they have a total of 294\$, how much money does each have?	Era, Hana, and Andy had 64 books to read this summer. Era read 16 books, which is 10 books more than Ana and two times less than Andy. How many books are left unread?	Four owners of a sports field must contribute to the purchase of new equipment. Its price is 550 dollars. If the first person gives \$35, the second person twice as much as the first, the third \$30 more than the second, and the fourth the total of the first three, how much money has not been given?

WP4_O (Inconsistent, Additive)	WP5_O (Inconsistent, Additive)	WP6_O (Consistent, Multiplicative)
Era had read 25 books this year, which is five books more than Hana and 12 books less than Andi. How many books did they read together?	Drena Primary School has 280 pupils, which is 89 fewer pupils than Dielli Primary School and 62 fewer pupils than Hana School. How many pupils are there in total?	A shepherd has 90 sheep. He has 3.5 times less black sheep than white ones. How many white and black sheep does he have?

A priori analysis of all the problems was conducted to anticipate possible correct and incorrect solving strategies, pupils' mistakes, and the use of visual representations. WP1_E of the pre-test is presented in Table 3.4 as an example of the results of such analysis with possible correct and incorrect solutions, using a strategy with BMs, an arithmetic (using calculations with numbers) and an algebraic strategy (using algebraic equations).

Table 3.4: Examples of possible correct and incorrect solutions to WP1_E

A possible correct solution to a WP1_E with the BM method	A possible incorrect solution to WP1_E with the BM method
This way of solving uses the bar model method to identify known and unknown quantities in the solution. The pupils can easily determine which mathematical operation to apply in each step using bars. After drawing, the pupil can calculate the missing data and find the solution.  First day: 3 Second day: $124 - 24 = 100$ Total unsold: 226	An incorrect solution can result from inaccurate bar drawings. In the figure below, the pupil made a mistake in the mathematical operation to find the number of books sold on the second day. Instead of reducing the bar, the pupil can enlarge it based on the sentence, "which is 24 more books sold than on the second day."  First day: 326 Second day: $124 + 24 = 148$ Total unsold: 178
A possible correct solution with the arithmetic method	A possible incorrect solution with the arithmetic method
The solution can be reached easily if the pupil can correctly use mathematical operations and brackets in the proper place.	The probability of arriving at an incorrect solution is higher when the calculation is done directly. This error occurs more often when the WP is inconsistent (Boonen et al., 2016). In the

$450 - 124 = 326$ $326 - (124 - 24) = 326 - 100 = 226$	case below, the pupil may make a mistake, as not placing the brackets creates confusion in calculating the books sold on the second day. $450 - 124 = 326$ $326 - 124 - 24 = 178$
A possible correct solution with the algebraic method By using letters to represent quantities, we can reach the solution. We can formulate the equation using these unknowns: m — the number of unsold books (unknown). t — the total number of books at the beginning (known). $f1$ — the number of books sold on the first day (known). $f2$ — the number of books sold on the second day (unknown). $m = t - f1 - f2 ; t = 450; f1 = 124$ $f2 = (124 - 24) = 100$ $m = 450 - 124 - 100 = 226$	A possible incorrect solution with the algebraic method Pupils can make mistakes if they do not represent the data in the correct form. For example, let x be the number of books sold on the second day, and y be the total number of unsold books after two days. We need to find the x value first, then the y value. The mistake here may be in the calculation of the book sold on the second day because of the inconsistency of the word “more”. The pupil may add instead of subtracting 24 from the 124. $x = 124 + 24 = 148$ $y = 450 - x = 450 - 148 = 302$

3.4.2 Interviews

Interviews were chosen due to their potential to collect detailed information (James et al., 2008; Koshy, 2005). Moreover, interviews allow participants to express their views honestly and in a less limited manner than structured surveys or questionnaires (James et al., 2008; Koshy, 2005). Nevertheless, the method is criticised for involving small, non-random samples, which limits the generalisation of findings. In cases where the research involves many participants, interviews can be time-consuming (James et al., 2008; Koshy, 2005). In the current study, these two factors were irrelevant. The number of participants was manageable; the interviews were conducted with three mathematics teachers in total and three pupils in each cycle. To cater to the limited generalisation of the findings, other data collection methods were used, such as observation, audio recordings, and teachers’ diaries, which added more credibility to the results.

A semi-structured interview was used because it allows for flexibility in the interview process. This type of interview is exploratory and combines a set of predetermined questions with the flexibility to incorporate new questions based on the interviewee’s responses, making the process more dynamic and responsive (Zazkis & Hazzan, 1998). This type allowed me to prepare specific

questions in advance while remaining open to teachers' and pupils' thoughts and beliefs about the SIMPLE Approach. The interviews were designed with the primary goal of gathering information on the participants' thoughts about the implemented teaching approach, their perceptions of its usefulness for solving WPs, the aspects of the teaching approach they found beneficial or not, and their potential future use of it in problem-solving. Suggestions to refine the intervention were also sought. See Appendix 2 for the teacher's interview questions and Appendix 3 for pupils' interview questions. Here, only the questions instrumental in eliciting responses to the main points above will be presented.:

Questions for teachers: *What do you think about the teaching approach, and how do you perceive its usefulness? Did the teaching approach help the pupils to solve WPs? How confident are you with adopting it in your teaching? Do you want to add any suggestions to improve the intervention?*

Questions for pupils: *Do you think this approach is suitable for solving WPs? Did the BM method and the SIMPLE steps help you solve WPs? Which of the SIMPLE steps seems the most beneficial to you? Are any steps hindering you or not helping you solve WPs? Would you use it in future WP solutions? If not, why not?*

The interviews were audio recorded. Audio recordings capture participants' exact words, tones, and pauses, which is crucial for understanding context and meaning and for the reliability of the analysis (Rutakumwa et al., 2020). The data consisted of transcriptions of interviews and my field notes taken during the interviews.

3.4.3 Observation of Mathematics Lessons

The observation method involves the researcher's presence in the setting of the subject of the research and collecting data from what they see (Flick, 2009; Koshy, 2005). The researcher may observe as an insider by playing an active role in the ongoing event or as an outsider by having a passive role in the event (Creswell et al., 2007). One of the advantages of this method is that it allows researchers to collect more data (e.g., through writing field notes). On the other hand, the method is criticised for being less informative about the potential motivation for the participants' behaviour in action. This makes it important to use observation alongside other methods that could complement its weaknesses (e.g., post-lesson interviews exploring the less observable issues).

In this study, the observer had a passive role in observations. The lessons were observed without interfering and were audio-recorded. The main goal was to discern situations in which the implementation fidelity or its breach was apparent. These aspects were followed during the lessons:

- Adherence to the Guide: Observing whether the teacher followed the instructions given in the Teacher Guide regarding the core design characteristics of the intervention (the SIMPLE steps, bar models).
- Adapting the constructivist principles: Evaluating how well the teacher presented the materials and engaged pupils during the lessons, ensuring that constructivist principles were applied.
- Pupil engagement: Monitoring pupil participation and interaction during lessons to evaluate if the approach facilitated active learning as intended.
- Adaptation and flexibility: Noting any modifications the teacher made while presenting the SIMPLE Approach and considering their suggestions for further refinement of the intervention.

This information was collected both onsite while observing in the form of field notes and later when replaying the audio recordings of the lessons. Audio recordings were made to capture moments that may have been lost during the observation and to verify the field notes later. Thus, the data consists of transcripts and audio recordings of lessons. Audio recordings were employed in lieu of video recordings due to concerns expressed by both pupils and teachers that the presence of video recording equipment might induce discomfort. Consequently, there existed a potential risk of altering their natural behaviour.

3.4.4 Teacher Diary

Teacher diaries are useful tools in education because they allow participants to record information in written or spoken form about themselves, their actions, or their thoughts (Arndt & Rose, 2023). Although teacher diaries were originally perceived as a means of reflective practices in teacher education, they can be used for many forms of data analysis and are, therefore, useful as research tools in education research (Yi, 2008). In our study, teachers' diaries were used to obtain their reflections on the mathematics lessons they taught during the intervention and on the individual aspects of the teaching approach they were asked to implement (see Appendix 4). The teachers

were asked to write the diary immediately following each lesson for their reflections to be current. They were instructed to start by presenting the lesson, describing how the intervention went, and offering recommendations, critiques, or new ideas regarding future lesson implementation.

3.4.5 Pupils' Artefacts

During the teaching intervention, pupils worked individually or in a whole group under the teacher's guidance on 16 WPs. This made it possible to collect all pupil work samples completed during the teaching intervention and their homework, which supported the qualitative data and provided context to the learning progress. Solutions to six of the WPs were selected from the lessons on the BM method, solutions to six more from the lessons on the SIMPLE steps and solutions to four other WPs were taken from pupils' homework. Together with the solutions of WPs from the pre- and post-tests, the solutions to WPs from the intervention provided information about pupils' progress and the effectiveness of the intervention in improving their problem-solving skills.

3.5 Pilot Stage – Case Study

The pilot study is an indispensable part of any intervention study since it offers specific, contextualised evidence that helps researchers gain an in-depth understanding of the intervention's process (Yin, 2014). It helps to explore and adjust the intervention at an early stage before the main study regarding methodology, process, data collection and analysis (Forgasz & Kaur, 1997).

In the current study, the pilot stage consisted of a case study with three 8th-grade pupils, two girls and one boy, from Bota's Primary School in Gjilan, Kosovo. I intentionally worked with a small number of pupils to get an insight into their perceptions of the two main features of the developed teaching approach (the Bar Model method and SIMPLE steps) and how they could grasp and use them for solving WPs. The study was conducted by me and consisted of three phases, mimicking the three phases of the planned main intervention within three weeks (two days in the first week, two days in the second week, and one day in the third week.) The case study was described in detail in Morina (2023). Here, only the main results will be summarised.

Pre-intervention: Individually, the pupils were assigned one inconsistent WP to solve in any way they had been taught in their mathematics lessons. All three pupils had difficulty reading and

comprehending the problem accurately. They mostly tried to solve it directly using keywords, and none attempted to visualise their solution. All three pupils gave incorrect responses to the WP.

Intervention: First, I introduced the pupils to the BM method, and they practised it in four WPs (two consistent and two inconsistent, e.g., WP1, Fig. 3.3). The pupils were asked to participate in the solution process and encouraged to make the BMs independently for all the problems, to ask questions, and to engage in discussion with each other and with me. They were given two more problems to complete at home. In their homework, the pupils were instructed to use BMs and to write down how they connected them with the mathematical operation required in the solution to the WP.

Second, I introduced the pupils to the SIMPLE steps and their benefits in problem-solving. I explained the meaning of each letter and gave the pupils an instruction sheet with an explanation. Third, I demonstrated how to execute each step by solving WP1 as an example. The pupils participated in the solution by discussing together, asking questions, and providing their opinions. After that, one by one, I assigned the pupils six WPs (three consistent and three inconsistent) and asked them to solve them individually with the help of the BM method and SIMPLE steps. Since the pupils had just started to learn the SIMPLE steps, I assisted them when they were confused. After I noticed that the pupils could easily name, explain, and list each step, they moved to the post-intervention phase. They were given two more WPs to solve at home.

WP1: Andi, Era, and Ari scored 106 points in a volleyball match. Andi scored 15 more points than Era. Era scored 7 fewer than Ari. How many points did Ari score?

A	x	15	$x + 15 + x + x + 7 = 106$ $3x + 22 = 106$ $3x = 106 - 22$ $3x = 84$ $x = 28 \quad Ar = 28 + 7 = 35$
E	x		
Ar	x	7	

Fig. 3.3: The BM method of WP1 of the intervention lesson

Post-intervention: The pupils solved two more WPs, a consistent and an inconsistent one. Finally, they were asked how they perceived the usefulness of what they had learned and their confidence in adopting it in the future.

When analysing the pupils' solutions in the post-test, I observed that the intervention apparently helped the pupils solve WPs. They produced correct visual, symbolic, and verbal representations using each SIMPLE step and could use the BM method in a brief intervention period. Additionally, their excitement throughout the study showed how beneficial they found the newly introduced method. Therefore, this case study supported the assumption that the proposed teaching approach could be helpful for Kosovo 8-grade pupils. It remained to be seen how such an intervention could be implemented in classroom settings with the whole class and by their mathematics teacher.

3.6 Sample for the Main Study and Ethical Considerations

An appropriate sample was sought after conducting the case study and preparing the Teacher Guide. Convenient sampling technique involves selecting participants based on their accessibility. It is considered cost- and time-effective as the researcher has access to participants without difficulties (Creswell & Creswell, 2023); however, it may be vulnerable to biased samples as the researcher has no control over the characteristics of the involved participants (Creswell et al., 2007). The purposive sampling technique involves selecting participants based on specific criteria set by the researcher.

In my research, I combined both types of sampling. First, a convenient sample of schools was found. Schools were located in Gjilan, in my vicinity, as I planned to be present for all the lessons. As I intended to develop a teaching approach that is widely applicable in Kosovo, these schools did not stand out; they were considered "typical." The three selected public schools have approximately 1,000 students each. These schools consist of three levels: the pre-primary (children aged 5–6), primary (children aged 6–11) and lower secondary levels (children aged 11–15). The lower secondary level was the focus of the study, specifically 8th-grade pupils, and thus, the mathematics teachers were purposefully selected: they had to teach grade 8 in the selected schools and were to have more than ten years of teaching experience (as a beginning teacher might find it difficult to learn a new approach before acquiring teaching skills). Grade 8 was chosen because Kosovo's curriculum includes more WPs in that grade, and pupils are familiar with introductory algebra and can solve linear equations. Furthermore, pupils of that age undergo several mathematical standardisation tests, most of which include the solution of WPs. The intervention was implemented through three mathematics teachers and delivered to pupils from three classes.

When conducting the study, ethical considerations were considered to protect participants from physical and psychological harm. Obtaining consent, keeping participants' identities hidden, and optimising participant benefits are ways to ensure collaboration (Flick, 2009; James et al., 2008).

After explaining the purpose of my study, I obtained permission from the principal of each participating school and the three mathematics teachers. The teachers notified the pupils' parents and agreed to be part of this study. The teachers were informed about the study's aim both verbally and in writing using the Teacher Guide. Anyone was welcome to request clarification if they believed any information, spoken or written, was unclear. This was done to ensure that before participants agreed to the study, they were fully informed about the potential risks and benefits of participating in it. The potential risk can be an increased teacher workload, potential stress due to the learning and teaching of a new method, or the need to pause what they were supposed to do in their mathematics lessons to implement the intervention. The benefit consists of providing teachers with a new methodology of teaching that would help them while teaching WPs. The benefit for pupils lies in acquiring new methods for solving WPs.

To protect the participants and lower the risk of their identification, I changed the names of the participating schools and addressed teachers and pupils based on numbers only. Teacher 1 is the teacher from the first cycle, Teacher 2 is from the second cycle, and Teacher 3 is from the third cycle. Pupils were labelled with numbers from 01 to 62.

3.7 Overview of Three DBR Cycles

Three cycles of the intervention were conducted with three 8th-grade classes. Each cycle started with a pre-test and ended with a post-test. A teaching intervention was implemented between the two tests by the cooperating teacher. After each cycle, the gathered data (see Section 3.8) were evaluated, and the results were used to inform the content and organisation of the next cycle. I was present in all mathematics lessons taught within the intervention. The teachers introduced me to the pupils and explained the purpose of the study before the intervention began. The timeline for carrying out the cycles is presented in Fig. 3.4.

The first cycle included six lessons spread over two weeks: one for the pre-test, two for the BM method, two for the SIMPLE steps, and one for the post-test. The intervention was conducted by a male teacher with 25 years of experience teaching mathematics and the third year teaching in the class in question.

Based on the experience from the first cycle, one more lesson on the BM method was included so that pupils got more experience with it. Thus, the second cycle included seven lessons. The intervention was conducted by a female teacher with 20 years of experience teaching mathematics and her third year teaching in the class in question.

The third cycle consisted of seven lessons organised in the same way as the second cycle. The class was taught by a female teacher with 12 years of experience in teaching mathematics and informatics and her first year as a teacher in that class.

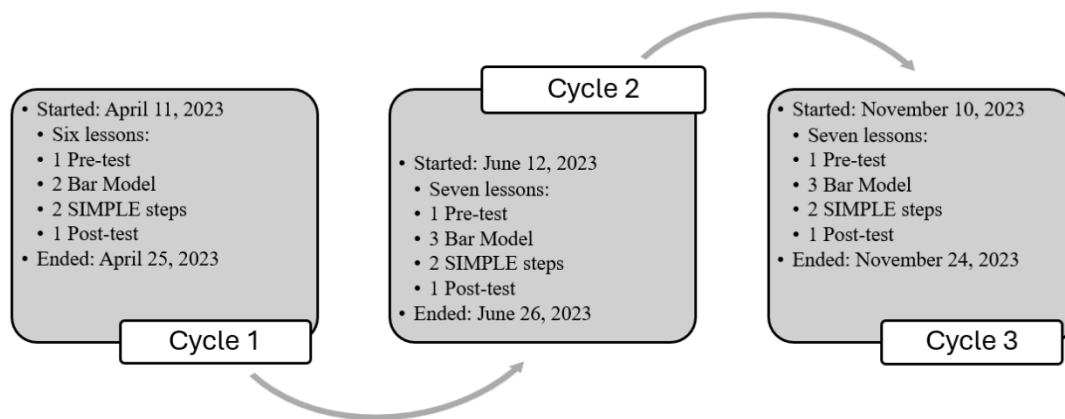


Fig. 3.4: Timeline of carrying out the cycles

3.7.1 Pre- and Post-Test

The content of the pre-test and post-test was the same for all the cycles. The WPs used in them are given in Section 3.4.1. The tests were assigned in a classroom setting during regular class time to reduce outside distractions and ensure consistent testing settings. Each test was allocated a 45-minute lesson. The cooperating teachers assigned both tests and I was present to ensure that the pupils in all the classes had the same conditions. The teacher clearly instructed the pupils regarding the purpose of the tests before administering them. The intention was to support their efforts and minimise any possible test anxiety. The tests were presented to the pupils on a paper sheet. The pupils worked individually to ensure the integrity of their responses. The teacher closely monitored the process to address pupils' questions or concerns, like "Should I write every SIMPLE step for all the WPs?", "Do we have enough time to complete all WPs?", etc. The teacher tried to answer and support pupils while ensuring they worked individually on the solution.

For the pre-test, pupils did not receive any instruction, allowing them to solve WPs as they would normally do. After receiving the intervention and learning the new approach, in the post-test WPs, pupils were instructed to use the BM method and the SIMPLE steps within their solution.

3.7.2 Teaching Intervention

As high fidelity of the intervention was sought, that is, the teachers were to implement the intervention in the way it was planned, a Teacher Guide was prepared. It was written in Albanian and included a description of all the lessons regarding which WPs should be used and how they should be implemented. This Guide was given to the teachers in writing prior to the intervention so they could refer to it when executing the lessons with the pupils. In addition to the Teacher Guide, I introduced the teachers to the three core aspects of the intervention (the BMs, the SIMPLE steps and a constructivist way of implementing WPs) during an in-person 45-minute initial session and answered any questions they had.

The course of the intervention varied in each cycle based on the teacher's adherence to the Guide and their regular teaching style. This variation will become clear when each cycle is presented in Chapter 4. Here, only the intended implementation of the SIMPLE Approach will be briefly described.

Constructivist Principles

During the intervention phase, the teacher is responsible for outlining the basic concepts of each lesson plan, guiding pupil practice, holding discussions, setting up individual and group work, and letting pupils construct their solutions while working through WPs.

To do this, four teaching lessons are planned. There will be two periods of expositions of new concepts to the pupils followed by the teacher gradually providing more space for pupils' own work. The exposition will consist of the teacher presenting the basics of the BM method and the SIMPLE steps, demonstrating their application to the pupils on at least one WP for each. Afterwards, the teacher will engage the pupils in interactive practice, reinforcing their understanding through collaborative solutions. This can be achieved by assigning pupils to work in pairs and larger groups, where they can discuss and solve problems together before moving on to individual practice. Teachers should encourage pupils to ask questions at all times, develop discussion in class, and create a friendly environment so that pupils feel they have the necessary

support to succeed. After a while, the support needs to be lifted in order to promote a pupil-centred learning environment where pupils can have significant control over their learning process. After the initial exposition of the teacher, this way of teaching adapts to the principles of constructivism intended for our study.

Bar Model Method

One of the intervention's core characteristics is using visual representations in WP solving. The intervention will start by presenting pupils with the BM method in two lessons.

In the first lesson, the teacher will introduce pupils to the BM method and its three types, using simple examples to clarify its application for different types of WPs. The teacher will draw all the types of BMs on the blackboard and ask pupils to take notes. After that, discussions about the BM application are encouraged to help pupils clarify their understanding and address possible misconceptions. Following the introduction, the teacher will present and explain the solution of WP1_I (see below; I stands for the implementation of the intervention) to the pupils, ensuring they understand the process. Next, the teacher will guide pupils through solving WP2_I, emphasising the application of the BM method. Pupils are encouraged to work independently or in small groups, with the teacher facilitating discussions and addressing questions as needed. The focus remains on reinforcing the correct presence of the BM method for different types of WPs.

In the second lesson, pupils will practise four additional WPs (see Appendix 3 for intervention WPs). Discussions about applying the BM method and pupils' problems are encouraged to help pupils reflect on their learning process. Pupils are encouraged to work individually and in small groups to foster both independent and group learning. The teacher works as a facilitator, leaving the solution to the pupils. At the end, the teacher will assign two additional WPs as homework (see Appendix 3 for homework WPs) to further reinforce the lesson learned.

The selection of WPs in the intervention phase was done similarly as in the pre- and post-test phases. They were formulated within the domain of whole numbers and most of them contain comparison terms, such as "more than," "less than," and " n times as many". The teachers were recommended to start from the simplest ones with the purpose of including such WPs for which BMs were not difficult to make. To illustrate, four WPs from the intervention phase and one from the homework with possible BMs are presented below. The rest is in Appendix 5.

WP1_I. There were 120 visitors to Clara's art exhibit on the first day, which is 20 more than on the second day. How many visitors came to the exhibition on the two days?

I

100	20
-----	----

$I: 120$

II

--

$II: x = 120 - 20 = 100$

$T = I + II = 120 + 100 = 220$

To find the number of exhibition visitors for two days, it is needed to find the number of visitors on the second day (by subtracting rather than adding 20 even though the word more is used) and then add them together with the first one. Therefore, the problem is additive and inconsistent.

WP2_I. Klara gives half of her paintings to her sister, Suzanne, and then buys herself four more. If she now has 12 paintings, how many did Klara have at the beginning?

I

x

$\frac{1}{2}x + 4 = 12$

II

$\frac{1}{2}x$

$\frac{1}{2}x = 8$

III

$\frac{1}{2}x$	4
----------------	---

$x = 16$

Both addition and multiplication are needed in the solution: addition to add four bought paintings, and multiplication to multiply the number with painting by 2 as half were given to the sister. Therefore, the problem is consistent and requires both additive and multiplicative operations.

WP3_I. Xhamie picked 17 flowers, which is 5 more flowers than Linda picked. They donated 20 of the flowers. How many flowers were left?

XH	12	+5	}	17	12
L	x			20	9

$L = x$

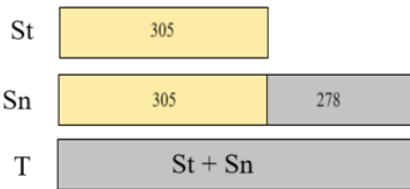
$L + Xh = x + x + 5 = 12 + 17 = 29$

$Xh = x + 5 = 17 \rightarrow x = 12$

$Left: 29 - 20 = 9$

The problem requires finding the number of flowers left after Xhamie and Linda donated 20. To have the solution, we need to find first the number of flowers Linda has. This is achieved if we subtract 5 from the number of flowers that Xhamie has (as she has 5 more than Linda). Therefore, the problem is additive and inconsistent.

WP4_I. 305 children went to a ZOO on Saturday. The number of children who visited the ZOO on Saturday was 278 less than on the Sunday visit. How many children went to the ZOO on Sunday?

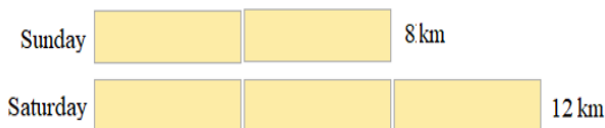


$$Sn = St + 278$$

$$Sn = 305 + 278 = 583$$

To find the number of children that visited the ZOO on Sunday, we have to add the number of children that went to the ZOO on Saturday to the known number 278. Therefore, this problem is additive and inconsistent.

WP1_H. Some friends decided to go for a run on the weekend. They run 8 kilometres on Sunday. Sunday's run was 1.5 times shorter than Saturday's. How many kilometres did they run on Saturday?



$$\frac{x}{1.5} = 8$$

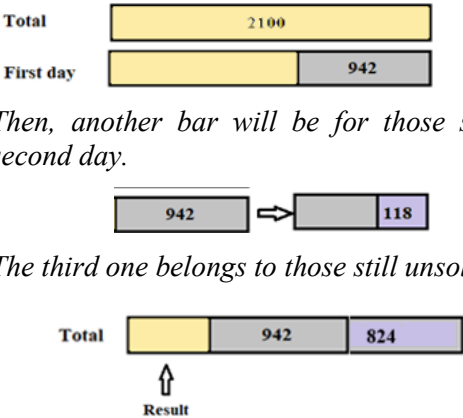
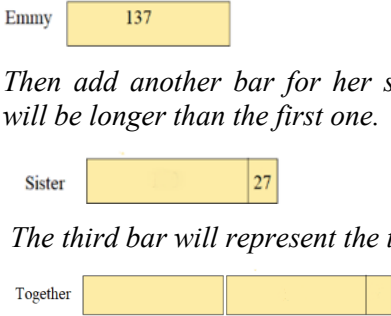
$$x = 12$$

The solution requires multiplying the number of kilometres on Sunday by 1.5 to find the number of kilometres run on Saturday. Therefore, the WP is multiplicative and consistent.

SIMPLE Steps

In the following two lessons, the teacher will introduce the SIMPLE steps to pupils and let them experience their use in the solution of WPs. The third lesson of the intervention will start with the teacher explaining the meaning of each letter (Table 3.1, Section 3.3.2) and demonstrating how to execute each step by thinking aloud while solving WP7_I. Afterwards, the teacher will assign the pupils WP8_I, and they will practise their solution by working together on the blackboard. Table 3.5 exemplifies how the SIMPLE steps can be applied in the solution of these two WPs. The statements in the table are hypothetical, not actual quotes by the teacher. The teacher should initiate discussion and ask questions after applying each step.

Table 3.5: Example of the application of the SIMPLE steps in WP7_I and WP8_I

	WP7_I: Of the 2100 kilograms of potatoes in the warehouse, 942 kilograms were sold on the first day, while on the second day, 118 kilograms less than on the first day. How many kilograms of potatoes are left in the warehouse? (Consistent, Additive)	WP8_I: Emmy collected 137 stamps, which is 27 stamps less than her sister. How many stamps did they collect together? (Inconsistent, Additive)
S	<i>I must find the kilograms of potatoes that are still unsold after two days. We know the number of potatoes sold on the first day; we need to find it for the second day and then those left in the warehouse.</i>	<i>I must find the number of stamps collected by Emmy and her sister, knowing the number of stamps collected by Emmy.</i>
I	<i>So, I will draw bars showing the kilograms of potatoes left after being sold on the first day.</i>  <i>Then, another bar will be for those sold on the second day.</i> <i>The third one belongs to those still unsold.</i>	<i>I will draw the bar showing Emmy's stamps first.</i>  <i>Then add another bar for her sister, which will be longer than the first one.</i> <i>The third bar will represent the total.</i>
M	<i>I think that I drew the bar correctly, since now I know what and how much I must subtract to find the final answer. I think I am on the right path and getting closer to the objective. I will prepare the solution to the problem based on the bars that I drew now.</i>	<i>The bar model is helping me to come to the solution because, based on the bars, I understand that Emmy's sister has 27 more stamps than her. I am on the right path, and I will continue the solution based on the bars.</i>
P	<i>Looking at the bars, I can see that first, I subtract the total number of kilograms sold on the first day.</i> Day I: $2100 - 942 = 1158$ <i>The number of kilograms sold on the second day is determined by subtracting 118 from the first day's total. After that, I also eliminate the ones from the second day,</i> Day II: $1158 - (942 - 118) = 1158 - 824$ <i>which leads to the solution.</i> Left = 334	<i>Let x be the number of stamps that Emmy's sister has.</i> Emmy $\rightarrow 137$ Emmy's sister $\rightarrow x$ <i>Then we find the number of stamps that Emmy's sister has by adding the Emmy stamps with 27.</i> $x - 27 = 137$ $x = 164$ <i>Finally, we find the number of stamps that they have together.</i> Together $\rightarrow 137 + 164 = 301$

L	<i>I will check my calculation. describe here exactly what this person says to himself and what he writes. Similarly, everywhere</i> $334 + 824 + 942 = 2100$ (left + second day + first day) = total	<i>I will check if my calculation makes sense now. If from the total we subtract the number of Emmy stamps, we get those from her sister.</i> $137 + 27 = 164$ $301 - 137 = 164$
E	<i>The final answer is 334, which makes sense since we arrived at the warehouse's total kilograms of potatoes by working backwards.</i>	<i>The result 164 makes sense. We found the number of stamps Amy's sister has by working backwards, specifically subtracting the total number of stamps with the number of Emmy's stamps.</i>

The pupils will also receive an instructional sheet explaining each letter of the SIMPLE steps (Fig. 3.5). The sheet is written in Albanian and has been translated for the sake of the thesis. The pupils can use the sheet in their further work.

S	I	M	P	L	E
Study the problem Read the problem and rephrase it. "Have I understood what I am supposed to find?"	Involve the bar model method Draw a diagram (create a visual representation of the word problem). How are the pieces of information in the word problem related? How can I represent these relations by rectangles?	Monitor the process Go back to the beginning, including the problem formulation. Ask yourself: Is the bar model method helping me to understand how to reach the solution?/ "Am I getting any closer to my objective? Am I still implementing the solving strategy I picked?"	Prepare / Present the solution Make the expression/ calculation/ equation, write the solution, and present it.	Look over the solution again Check the whole solution, solving steps and the calculations.	Evaluate your answer Evaluate whether your solution makes sense and if there is a better way to solve the problem. Write up the answer.

Fig. 3.5: The front and back of the SIMPLE steps instructional sheet

At the end of the lesson, the teacher will give the pupils two more WPs set as homework. They will be asked to use every SIMPLE step in their solution.

The fourth intervention lesson will start by reviewing the homework WPs, addressing common errors, and answering pupils' questions. Then, the teacher will assign the pupils four WPs and ask

them to work first individually, then in pairs and small groups, using both the BM and SIMPLE steps in their solution. The teacher needs to ensure pupils can name, explain, and list each step.

This is how the intervention is intended to be implemented in the first cycle. Depending on the observations and other data gathered, this can be changed in the following cycles.

3.8 Data Collection

Section 3.4 summarises which data types were collected in the three cycles. Table 3.6 provides the totals for each data type, and the text below provides some additional information about the data.

Table 3.6: The total for the data collected in the three cycles

	Cycle 1	Cycle 2	Cycle 3	Total
Notes from the initial session for giving instruction before the intervention and informal chat	1	1	1	3
Pupils' solutions to the WPs in the pre-test*	6×19 = 114	6×20 = 120	6×23 = 138	6×62 = 372
Pupils' solutions to the WPs in the Intervention*	16×19 = 304	16×20 = 320	16×23 = 368	16×62 = 992
Pupils' solution to the WPs in the post-test*	6×19 = 114	6×20 = 120	6×23 = 138	6×62 = 372
Observation and field notes	Six lessons	Seven lessons	Seven lessons	Twenty lessons
Audio recordings of lessons, length	4.5 hours	5.25 hours	5.25 hours	15 hours
Entries in the teacher' diary	4	5	5	14
Recordings and transcripts of interviews with teachers	1	1	1	3
Recordings and transcripts of interviews with pupils	3	3	3	9

*The number of solved WPs times the number of pupils

Throughout the intervention, pupils were asked to solve 12 WPs in the lessons and four as homework. During the lessons, they solved the problems individually and they were encouraged to ask questions and collaborate on the solution. Their work was collected and evaluated 'on the fly' to understand their progress in grasping the SIMPLE Approach and whether they needed further support or were ready to move to the post-test. To understand pupils' progress in the intervention, six WPs, which were solved individually, were selected for a more precise analysis.

As for the pre- and post-tests, teachers collected pupils' written work for the analysis. There were 69 eighth-grade pupils in the cycles (25 in the first, 19 in the second and 25 in the third cycle). Five pupils from the first cycle missed an intervention lesson, and two pupils in the third had special needs, which prevented them from participating fully in the lessons. Thus, their data were not analysed, even though they were present in all the lessons. Consequently, in total, 62 pupils were included in the study (20 in the first, 19 in the second and 23 in the third cycle). The pupils' solutions were scanned and labelled with numbers from 01 to 62, which corresponded to each pupil (numbers 01–20 are used for pupils from the first cycle, 21–39 for pupils in the second cycle and 40–62 for pupils in the third cycle). Since the pre and post-tests had six WPs each, 744 WP solutions were analysed.

It is worth noting that I did not observe any lessons of the teachers before the intervention. The observation was done only in the lessons during the implementation of the SIMPLE Approach. I took notes on everything that caught my attention during the intervention and the audio recording could not capture it. The notes included informal chats with the teachers in the initial session before the intervention and during the implementation of the lessons.

The initial session consists of discussions with the teachers before the intervention, asking them about their typical teaching approaches and instructing them on the key aspects of the intervention, including the implementation of constructivist principles as outlined in the Teacher Guide.

Regarding the pupils' observation, I took notes to analyse their behaviour, facial expressions, and reactions when working individually and with peers during the lessons. I also took notes if I noticed signs of fear of asking questions, nervousness when pupils did not understand something, or happiness when they successfully completed a WP that they did not express in words. The observations also include the behaviour of the teachers, the way they handled the lessons, and how they worked with the pupils.

The audio recording occurred in real-time and captured the pupils' solutions produced during the lesson and the teacher's efforts to explain the concepts.

At the end of each lesson, the teachers were asked to write a diary about how the intervention went and what could have been done differently compared to the instruction in the Guide.

Semi-structured interviews were conducted with the teachers, one with each, to explore their views on the teaching approach, their experiences with implementing the intervention, the teaching approach's efficacy, the challenges encountered, and their recommendations for improvement. These formal interviews were conducted after the end of each cycle and lasted around 20–30 minutes, depending on the teacher's responses. In addition to the formal interviews, informal discussions were held with the teachers before and after each lesson to gather immediate feedback. While these informal chats were not structured interviews, they provided valuable insights into the teachers' experiences and perspectives and were captured by field notes.

In addition to teacher interviews, a semi-structured interview was conducted with a group of three pupils from each cycle, nine pupils in total, in which their overall experiences with the WPs were addressed, their feedback on the intervention, and their understanding of the learning experiences from their perspective. The pupils were interviewed after the post-test and selected based on their willingness to participate. The whole class was asked, and when some pupils raised their hands, I selected three randomly. The goal was to gather immediate feedback, as the intervention was still fresh in their minds.

The interviews with teachers and pupils were conducted in a comfortable and private setting to facilitate open and candid communication. With participants' agreement, the interviews were audio-recorded to capture their comments accurately. Notes were taken during the interviews to document nonverbal cues and contextual information. The audio recordings were transcribed.

3.9 Data Analysis

This study adopted a comprehensive analysis approach combining content analysis and thematic analysis. Content analysis may deal with both qualitative and quantitative data and involves categorising and coding data based on predefined criteria (Kuckartz, 2019; Mayring, 2015). Thematic analysis is a qualitative method that focuses on the content and context of the data gathered, enabling researchers to examine and understand the data more flexibly and inductively (Humble & Mozelius, 2022). As shown above, a variety of data types was collected. Their analysis will be described separately.

3.9.1 Pre- and Post-Test Analysis

The scans of the pupils' solutions were uploaded to Atlas.ti. Each written solution was number-coded: the first two digits denote the number assigned to each pupil (01–62), and the next digit depicts the number assigned to each WP. In the first stage of analysis, the basic characteristics of the solutions were determined and coded as “correct answer” and “incorrect answer” (see Table 3.7).

Table 3.7: The evaluation of the solutions in the pre- and post-tests

Criteria	The incorrect solution, no solution at all	Partial understanding visible	The correct idea, but slight errors in the calculation	Entirely correct solution
Correctness	Incorrect solution		Correct solution	

Examples of solutions to one problem, namely WP5_E (see Table 3.2) are in Fig. 3.6. If the solution was incorrect or absent or shows partial understanding (this may indicate that a pupil identified some elements of the solution, such as known and unknown quantities, presented some data, or acknowledged conditions related to the problem, but the solution provided is incorrect or not finished), it was categorised as “Incorrect solution”. If the solution was correct with a slight error in the calculation or was entirely correct, we categorised it as “Correct solution”.

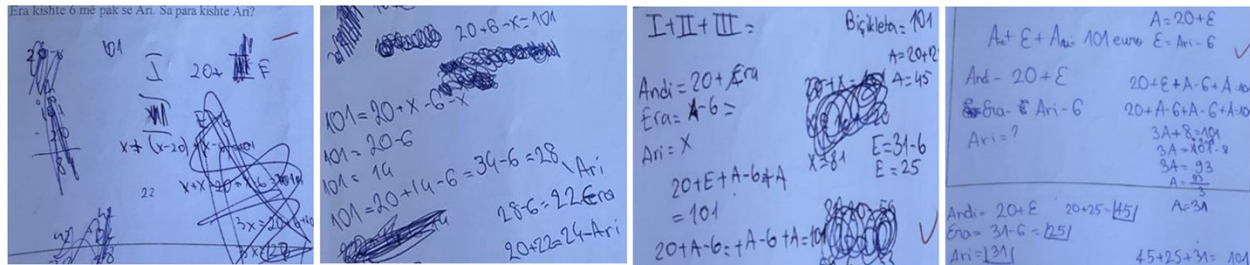


Fig. 3.6: Example of WP5_E solutions evaluated based on the four criteria above (from left to right)

We also used correlation coefficients between individual WPs and the total score to assess the relationship between the correctness of each problem and pupils' overall performance. To compute the correlation coefficients, the correctness scores for each WP (coded as 1 for correct and 0 for incorrect) were compared to the total score for each pupil using Pearson's correlation method. This analysis was performed separately for the pre-test and post-test datasets to identify how individual problems contributed to overall performance. Additionally, Cronbach's alpha was calculated to

check how consistently the six WPs measured the same skill or concept. This helps determine whether the problems work well together as a reliable set. The calculation of Cronbach's alpha considered the variability of scores for each WP and the total score. All statistical computations were conducted using Microsoft Excel.

Next, the solutions of the pupils who drew a BM and apparently used SIMPLE steps were targeted. Given that no solution in the pre-test included an attempt to draw the situation, this analysis was only done in the solutions in the post-test.

Based on the results of others (de Koning et al., 2022; Ng & Lee, 2009) and a priori analysis of the WPs, the BM method application was assessed to determine whether the shape of the BMs fit the mathematical operation required by the WP, whether all the BMs were drawn or some were missing, and whether they had correct dimensions. Five mutually exclusive codes emerged from this analysis: "All bars present and with the correct dimensions", "All bars present but not with the correct dimensions", "Some bars missing", "All bars present but not all fit the mathematical operations" and "Unclear representation". The last code was used for (for us) ambiguous drawings.

These codes were merged into three categories: "correct bars", "incorrect bars", or "partially correct bars". The bars in the "correct" category align well with the information from the WP. The "partially correct" category includes the bars that fit the required mathematical operation, but (1) their dimensions are not always precise, and/or (2) the bars for the second step are missing. The "incorrect" category consists of solutions with bars that do not match the correct mathematical operation or unclear BMs. Next, the link between the BMs' accuracy and the answers' correctness was investigated. Some solutions, even those with incorrect bars, were correct. Five categories emerged: "correct bars and correct answers", "incorrect bars and correct answers", "partially correct bars and correct answers", "incorrect bars and incorrect answers", and "partially correct bars and incorrect answers". The resulting framework is presented in Fig. 3.7.

Finally, the correlation between the presence and quality of the applied BMs (coded as 0 – no BM, 1 – incorrect BM, 2 – partially correct BM, 3 – correct BM) and the correctness of the solution was calculated.

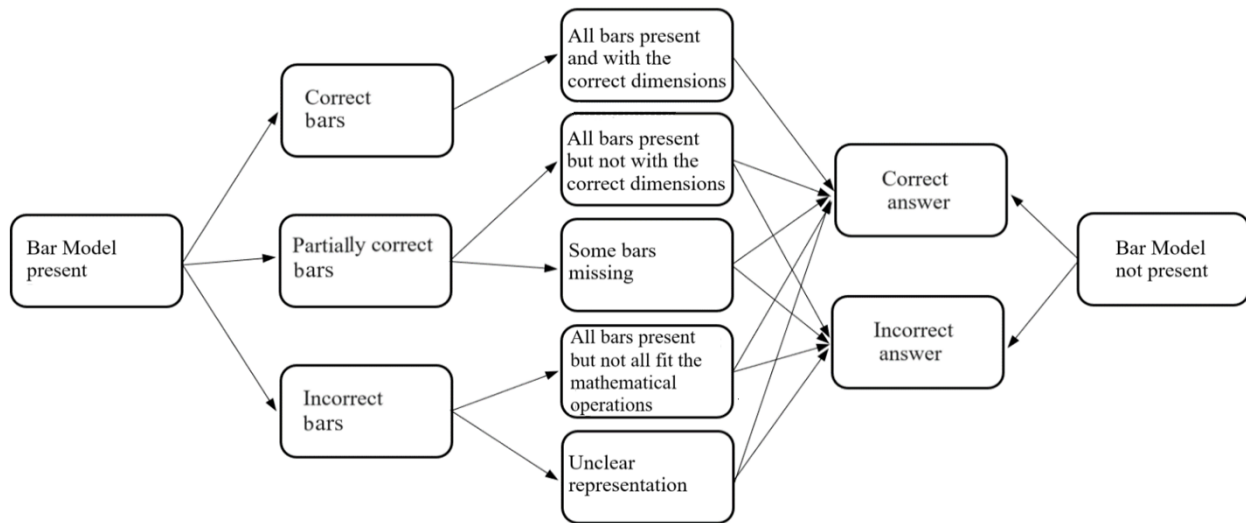


Fig. 3.7: Framework for categorising pupils' solutions in terms of BMs used

Next, the pupils' usage of the SIMPLE steps and the correctness of solutions were examined. If there was clear evidence in the solution that the pupil used the SIMPLE steps, it was coded "The SIMPLE steps present". If some elements of the SIMPLE steps could be seen in the solution, it was coded "The SIMPLE steps partially present". The other solutions were coded "The SIMPLE steps absent". Finally, the link between the three categories and the correctness of the solution was investigated, as shown in Fig. 3.8. Six categories thus emerged: "The clear usage of SIMPLE steps and the correct solution," "The clear usage of SIMPLE steps and the correct solution," "The partial usage of SIMPLE steps and correct solution," "The partial usage of SIMPLE steps and incorrect solution," "No indication of SIMPLE steps and correct solution" and "No indication of the SIMPLE steps and incorrect solution."

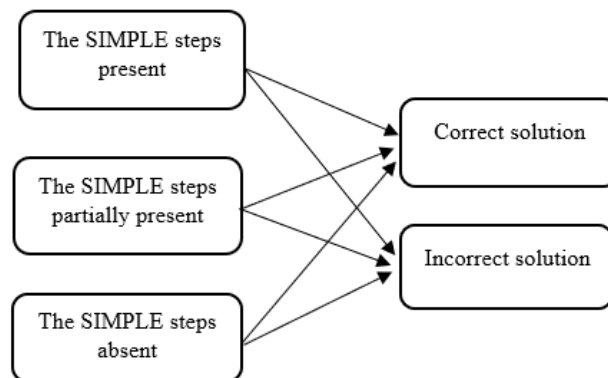


Fig. 3.8: Framework for categorising pupils' solutions in terms of the SIMPLE steps used and the correctness of the solution

Next, the pupils' work in the pre-test and post-test was compared using descriptive statistics.

Finally, the pupils were divided into three categories according to the number of correctly solved WPs (Table 3.8). The criterion was that the pupil's achievement was considered excellent if they solved all the problems or one fewer successfully, and their achievement was considered insufficient if they only solved up to two problems.

Table 3.8: Three categories of achievement according to the number of WPs successfully solved

Category of achievement	Pre-test	Teaching Intervention	Post-test
Excellent	5 or 6	5 or 6	5 or 6
Good	3 or 4	3 or 4	3 or 4
Insufficient	0, 1 or 2	0, 1 or 2	0, 1 or 2

Different “routes” for pupils can be distinguished. For example, some pupils moved from the “insufficient” category in the pre-test to “good” or even “excellent”, while others remained “insufficient”. The results section presents the identified groups of pupils and examples of pupils’ solutions for three of them.

Note: Some other statistical analysis was carried out to analyse the aggregated data from the three cycles for the pre-test and post-test. It will be presented in Section 4.4 together with the findings as it was not used for the analysis of data from the cycles separately.

3.9.2 Interviews

The interviews were transcribed and analysed qualitatively using thematic analysis to gain insights into the participants’ perspectives and experiences. In the transcribed recordings of interviews, units in which the teacher’s and pupils’ views were visible regarding the SIMPLE Approach, focusing on the categories stemming from our research questions and study of literature were identified. The unit of analysis in each episode was a meaningful unit that could be categorised into one category such as “Identifying areas for improvement in the Approach”. In some cases, the category was divided into subcategories. For example, “Evaluating SIMPLE Approach lesson success and confidence in its implementation” was divided into “How the lessons of the SIMPLE steps and BM method supports the solution of WPs” and “How confident the person was while applying it in WP solution”. “Opinions on the usability and impact of the Approach on pupil

learning” included “Perceptions of the SIMPLE Approach’s usefulness,” and “Evaluation of how the SIMPLE Approach impacted pupils’ solution of WPs”. For example, the following excerpts from the teacher interview were categorised as “Opinions on the usability and impact of the Approach on pupil learning”. Teacher 1 stated, *I observed a heightened level of engagement among pupils who were previously less active in other mathematics classes*. Similarly, Teacher 3 stated, *I would like this approach to be included in textbooks or literature on mathematics. I believe that pupils would benefit greatly from it. This approach is also attractive as drawing bar models increases pupils’ curiosity. Even pupils not interested in mathematics began to show interest and engage in these lessons*.

Similarly, the pupils’ interviews we identified units such as “Pupils’ opinions on the usability and impact of the SIMPLE approach on WP solution” and “Identifying areas for improvement in the Approach” which includes their recommendations for making the approach more effective for them. The following quotes from the interviews with pupils were coded “Opinions on the usability and impact of the Approach”: *Applying it to word problems, especially the drawing part, was very difficult in the beginning; I like to draw a lot, but not to write too much. I would like not to include every step in the solution as I got tired while writing a lot*. The results section will present fragments from both teachers’ and pupils’ responses illustrating the themes and categories identified during the analysis.

3.9.3 Teaching Intervention Analysis

The data collected during the teaching phase consisted of field notes from the lessons and chats with the teacher before and during the intervention, transcripts of audio recordings of lessons, the teacher’s diary, and pupils’ artefacts (solutions).

In these data, units were determined in which (a) the teacher’s implementation of the three features of the SIMPLE Approach and (b) pupils’ reactions while learning and applying the characteristics of this approach in the solution of WPs were visible. Such episodes were coded in terms of (a) “Adherence to the Guide”, “Adapting the constructivist principles”, “Adaptation and flexibility” and (b) “Pupil engagement” and “Pupils’ reaction to the SIMPLE approach in the WP solution”. For example, whether the teacher distinguished the types of BMs, how they introduced each SIMPLE step, whether pupils relied on the BM method and SIMPLE steps to solve WPs, whether they organised group work, how they managed discussions, what support

they provided to pupils, and whether the intervention's fidelity was upset by the teacher. Fidelity refers to how closely the teaching intervention adheres to the guidelines outlined in the Teacher Guide. It can be assessed by identifying moments when the teacher's actions do not fit the Guide's requirements, such as skipping steps in the SIMPLE Approach or not presenting the BM method as required, not including the principles of constructivism, which would indicate that fidelity was compromised. Finally, the rest of the data were searched to triangulate information from the transcripts and field notes and to find possible causes of a mismatch between the designed and implemented interventions. For example, it was determined whether and how the teacher mentioned the coded episode in the diary or the interview (see Section 3.9.2).

4. RESULTS

The results will be structured in two directions.

First, each cycle will be described in more detail in a special section and results pertinent to the data collected in that cycle will be presented to provide preliminary answers to the research questions formulated in Section 2.6. Namely, how pupils and teachers perceived the SIMPLE Approach's usefulness and their confidence in adopting it will be elaborated (RQ1), possible conditions of and barriers to implementing the proposed intervention will be identified (RQ2), and quantitative results describing the impact of the intervention on pupils' success in solving WPs and their ability to use the BM method and SIMPLE steps will be presented (RQ3). At the end of Cycles 1 and 2, feedback for the subsequent cycle will be provided.

Second, Section 4.4 will synthesise the results from all three cycles to evaluate the overall intervention refinement and its effects across the sample. It will integrate insights from all RQs and present them for all three cycles.

Notes:

- The WPs will be referred to by their distinct codes. They can be found in Appendix 3.
- The interview and quotes that illustrate some results have been translated from Albanian. The translations are faithful to the original meaning but simplified when necessary to ease understanding. For example, it was omitted when the participants stammered or could not find the correct word for some time.

4.1 Cycle 1

Cycle 1 was organised in the primary school Dream in Gjilan, a city with around 90,000 residents in Kosovo. It was opened 51 years ago and has 1,362 pupils with 54 classes in total. It consists of three educational levels: preschool (5–6 years), primary (6–11), and lower secondary (11–15).

The intervention was organised in a grade 8 class. This school had six 8th-grade classes taught by two mathematics teachers. The teachers' choice was based on their willingness to participate in the study, and the class choice was based on a schedule more suitable for observing the whole process. The participants in Cycle 1 were 20 pupils (8 boys, 12 girls). They were all of Albanian nationality, and no pupil had special needs.

The intervention was taught by a male teacher (Teacher 1). He had 25 years of experience teaching mathematics, and it was his third year teaching the class. The teacher said that WPs were part of his teaching, and he used them especially with pupils who participated in mathematics competitions. The teacher expressed willingness to include the three aspects of the SIMPLE Approach in his teaching. In the beginning, he was sceptical about the inclusion of the BM method in the solution of the WPs as, based on him, it was very new to pupils, and he was not sure if they would understand it. After a more detailed explanation, he consented to begin with WPs that were simpler so pupils could grasp their application more easily. Regarding the constructivist approach, as transpired in the initial session, Teacher 1 claimed that he did not always use pupil-centred methods, as “pupils will misuse the situation and make unnecessary noise in the classroom”.

The teacher’s participation in the study was motivated by my emphasis on the importance of this study and how its findings would contribute to Kosovo’s pupils’ learning of WPs. For the three teachers, a small financial reward to acknowledge their contribution was planned. However, Teacher 1 did not accept the reward since, according to him, this study was for the pupils’ good, and we should all contribute to it.

Regarding the pupils’ prior experience with WPs, the teacher’s monthly plan showed that the class had covered the chapter on solving WPs via linear equations, and pupils had been encouraged to solve WPs in such a way so far.

4.1.1 Teaching Intervention

This section briefly describes the course of the intervention in Cycle 1. Based on my observations of pupils’ problem-solving behaviour, I will point out important aspects of the lessons that could influence their understanding of the BM method and the SIMPLE steps. I will also mention moments when the intervention’s fidelity was upset.

The Bar Model Method

The teacher began the first lesson of the intervention by introducing the pupils to the BM method. He explained how it aided the solution of WPs and presented the three types of BMs. However, contrary to the Teacher Guide, he introduced BM types (part-whole, comparative and change models) orally without visually illustrating them on the blackboard, which apparently confused the pupils regarding the identification of the types of BMs. When practising BMs, the teacher drew the correct BMs for the pupils but did not explain which type of BM was used. For example, for WP1_I (“There were 120 visitors to Clara’s art exhibit on

the first day, which is 20 more than on the second day. How many visitors came to the exhibition on the two days?") he drew a BM for the total visitors to exhibit on the first day (120), then for the second day sold (100) and then for a total of two days (220). Still, even though the drawing was there, the teacher did not say which bar model type was associated with his drawing. The Guide advised him that the pupils should be informed that the comparative model was applied to the solution.

Optimistic that they understood how to apply BMs in a WP solution, Teacher 1 presented WP2_I for the pupils to solve independently. After some time, he noticed that many pupils struggled to draw a BM and decided to intervene. However, we identified situations in which, after being asked if the solution was correct, the teacher said that the solution was not accurate, but he failed to associate it with the pupil's drawing and explained the mistake by pointing to the solution via an algebraic equation. Sometimes, the teacher felt unsure how to proceed with the lessons and asked me, as the observer, to help explain certain aspects of the approach to the pupils. At the end of the lesson, he invited one of the pupils, P1_1, to come up to the blackboard, and together, they began to work through the problem. The teacher helped the pupil through the solution, asking questions to prompt the pupil's thinking and helping in adapting the drawn BM to the solution. The teacher frequently paused to clarify the solution for the entire class, ensuring everyone followed. Overall, he took a very active role in the whole lesson.

In the next lesson, the teacher assigned the pupils four WPs to solve independently and gave them two more as homework. When the pupils started working on the WPs, the teacher moved around the classroom, monitoring and encouraging them to use BMs in their solutions. According to his words, he was satisfied with the pupils' engagement and comprehension. Some pupils started working quietly, while others still seemed hesitant. Seeing this, the teacher approached them and provided guidance where needed. After some time, the teacher asked pupils to work in pairs and encouraged them to ask questions. The pupils did not raise questions with the teacher, as they started discussing the WPs given to them with their partners instead. As the class progressed, I observed that Teacher 1 gradually decreased his help by stepping back and allowing pupils to solve the problems more independently, intervening only when they were significantly stuck. Contrary to the Teacher Guide's requirements, the teacher did not encourage discussion among pupils enough, leaving them to work independently.

It was observed that some of the pupils had problems creating the BMs for the provided WPs and went back to their method of arithmetically solving them or converting them into an equation (see Fig. 4.1 for an example). WP4_I did not require it as it was solvable using a

straightforward calculation. However, it was our purpose to include such WPs at the beginning of the intervention for which BMs were not difficult to make.

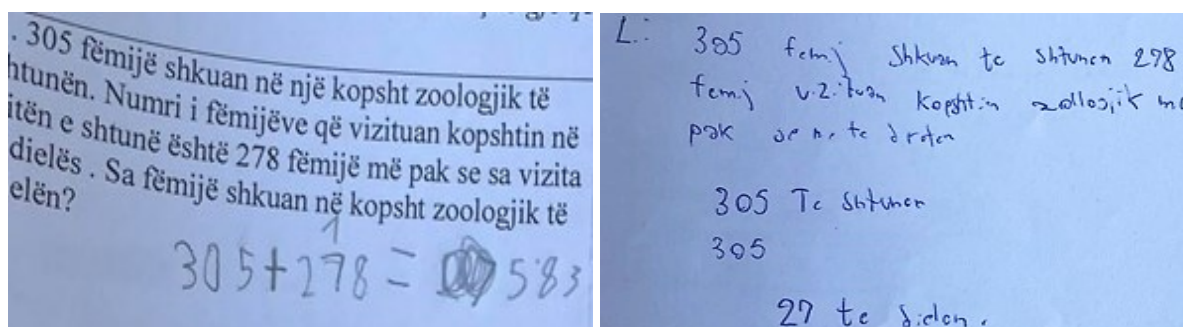


Fig. 4.1: The pupils' solutions with no BM for WP4_I (Cycle 1)

While looking at the pupils' notes, it was noticed that they tried to draw the bars in most of the given WPs, even if they were incorrectly drawn. While working on the WPs, several pupils raised their hands, eager to draw on the blackboard and explain their solutions. However, due to limited time, the teacher only selected one pupil, P2_1, to present their drawing for WP4_I in front of the class. Fig. 4.2 presents two of the pupils' correct drawings of the BM for WP4_I with the correct solution.

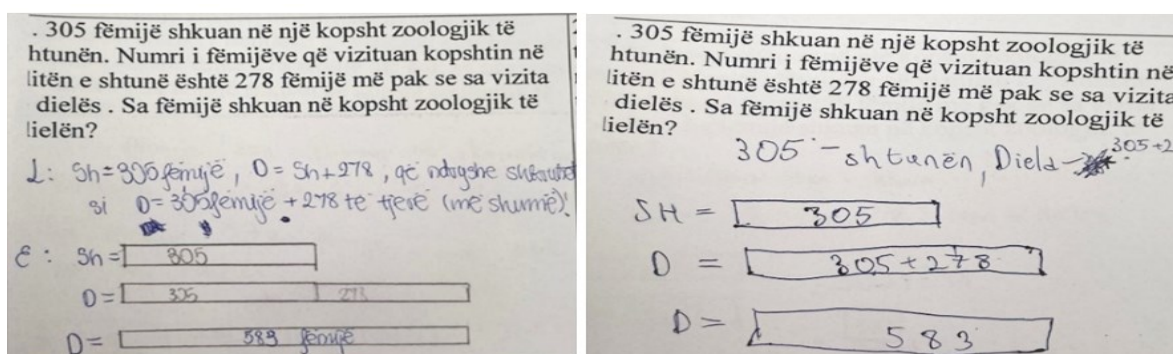


Fig. 4.2: The pupils' solutions with the correct drawing of the BM for WP4_I (Cycle 1)

The SIMPLE Steps

The following two lessons were devoted to introducing and applying the SIMPLE steps. The teacher started the lesson by introducing them to the pupils. He explained the steps alongside their indicators and highlighted their importance in solving WPs. To illustrate, he solved WP7_I by showing the pupils where each step in the solution belonged. Pupils followed the teacher and wrote down everything in their notebooks. They were encouraged to ask questions if there was something that they did not understand. The pupils mostly expressed that they understood what the teacher explained but did not understand how to implement the “monitoring the process” and “evaluating the answer” steps. The teacher explained again how

he applied these two steps in the solution of the previous WP. Next, he provided them with the SIMPLE sheet (Fig. 3.8) and instructed them to use it while solving WPs. To reinforce understanding, the teacher gave the pupils WP8_I and asked them to solve it with each step by working in pairs, just as they were sitting.

Most of the pupils successfully solved WP8_I (refer to the example in Fig. 4.3), and one of them, P3_1, shared the solution on the blackboard. The other pupils followed her solution and helped her apply the steps. The teacher provided further assistance. For example, some pupils asked why addition was used when the problem contained the word ‘less’. The teacher answered that specific words do not always tell the exact mathematical operation. He tried to re-explain the solution by referring to a simple example. He took all the pieces of chalk they had and gave seven to a pupil. Then he asked: "I have five pieces of chalk, two less than what you have. How many do you have?" This example apparently helped pupils to understand the solution to WP8_I.

The figure consists of two side-by-side photographs of handwritten mathematical solutions on a grid background. Both solutions are for a word problem in Albanian. The left photo shows a student's work with boxes for numbers and steps labeled L, H, T, E. The right photo shows another student's work with similar steps and calculations.

Left Photo Solution:

L: Emy ka mbledhë 137 pulja që janë 27 më pak se motra e saj. Sa kanë mbledhë të gjitha ato.

E: $\boxed{137}$

H: I: X- numri i pulleve që ka Emy
II: $x+27$ - numri i pulleve të motrës së Emyt
I+II

T: $x = 137+27$
 164 - motra e Emyt
 $137+164=301$

E: $301 = x+x+27$
 $301 = 137+137+27$
 $301 = 137+164$
 $301=301$

Right Photo Solution:

L: Emy ka mbledhë 137 pulja që janë 27 më pak se motra e saj. Sa kanë mbledhë të gjitha ato.

E: $\boxed{137}$

H: $137+27=x$ $x=137+27$
 $x=164$

Motra: $\boxed{137+27=x}$

T: $137+27$
 $=164$

E: $164=137+27$
 $301=137+137+27$
 $301=x+x+27$

Fig. 4.3: The pupils' solution with the SIMPLE steps for WP8_I (Cycle 1)

During the interview, the teacher admitted that he had not encountered many inconsistent WPs before the intervention. They were not used in his mathematics lessons, and for that reason, pupils mostly follow the keyword strategy, that is, determine the mathematical operation according to the words used in the text.

At the end of the lesson, the pupils were given two WPs as homework.

In the following lesson, the teacher asked pupils to share their homework solutions and to say whether they used the BM and whether the SIMPLE steps were challenging. The teacher went to each of them and checked their solutions. Many pupils claimed that applying the steps was no longer difficult because they had more time to review the method at home. After that, the pupils worked through four more WPs and were asked to use the BM method and the SIMPLE steps. Pupils started working individually and in pairs. The teacher approached every pupil at

their table to check on their progress and address the questions. Again, unlike the Teacher Guide's instruction, the teacher did not initiate discussion or group work. At the end of the lesson, the pupils seemed overall comfortable following the steps and applying the BM method.

Adhering to the Principles of Constructivism

The teacher clearly tried to follow the Teacher Guide while conducting the lessons. Above, some moments in which the intervention's fidelity was violated were described. The principles of constructivism, as depicted in Section 2.4 and included in the Teacher Guide (Section 3.3.4), are an important part of the proposed teaching intervention.

The teacher did not fully adhere to the Guide's recommendations regarding discussion encouragement. He missed some opportunities to initiate discussions and explore pupils' misunderstandings and ideas, which are crucial to developing a deeper understanding.

Second, the teacher could have asked more questions when pupils practised the BM and SIMPLE steps. This approach limited the pupils' potential to explore, which is one of the main principles of constructivism.

Lastly, the principle of facilitating group work was not implemented as recommended. Although the teacher opted for pair work when pupils were practising the BM method and SIMPLE steps, the Guide suggested that sometimes, pupils could be organised into larger groups to enhance collaborative learning.

These observations indicate that the teacher either did not study the Guide enough or the Guide did not provide sufficient help. There were important differences between the intended constructivist approach and the actual practices seen in the classroom which might impede the desired outcomes.

4.1.2 The Teacher's View: Teacher Diary and the Interview

The teacher's interview and diary provided complementary information to what I directly observed during the lessons and, to some extent, explained some events.

Fig. 4.4 shows parts of the teacher's diary. According to it, the pupils quickly grasped the BM method, making correct drawings and achieving correct solutions to WPs. However, this was in contrast to my observations during the lessons and the interview.

Researcher (R): *How do you find this approach?*

Teacher (T): *I found this method very attractive, helpful, and easy to employ.*

R: *Did you find it challenging to present the bar model to the pupils?*

T: *As this method is new to the pupils and me, it was challenging to present it in two lessons. We had just learned to solve word problems by converting them into equations, and although quite attractive, this method seemed a little difficult for the pupils to understand at first.*

Even though the teacher expressed that the approach was easy to employ, it happened in the lessons that the teacher asked me to help explain some aspects of the SIMPLE Approach. During the informal chats, he stated that since the pupils were taught to use the linear equation to solve the WPs, he sometimes had doubts while drawing the BM.

Concerning the SIMPLE steps, the teacher's diary entry indicates that the pupils understood the steps well, but he observed that more time was needed to grasp them fully. In the interview, the teacher mentioned: *At the beginning, they [the pupils] had difficulties understanding and using it, but after some practice, I think they did it well.*

Date: 13.04.2023
Bar Model method – Lesson 1
Describe the lesson learned.
The method taught today was something new for pupils not previously learned. I described first the bar model method and its type, and we practised it in some WPs. The pupils found the bar model method interesting and understood it very quickly, where even the solutions of the WPs they solved with the bar model method were correct.
How the intervention went
The first lesson went very well. The pupils first learned the method from me and then practised it together. They tried to use all the types of bar models within the solution, and I think they were good at it.
Suggestion/Opinions/New idea
I really like this method and do not have any criticism about it. It seems attractive and interesting to use, but the pupils need more practice to enhance their understanding and apply it successfully in solving WPs.

Date: 20.04.2023
SIMPLE strategy – Lesson 3
Describe the lesson learned.
In this lesson, I explained the SIMPLE steps to the pupils and applied every step to solve one WP for them. I also explained their benefits and how to incorporate them within the solution. The strategy had five steps; they were easy to use and understand. This was a new challenge for the pupils, quite interesting, not learned before.
How the intervention went
First, I model all the steps to one WP while telling them where each strategy step belongs to the solution. After that, we practised another WP together and discussed the solution. The pupils came to the table in front of others and applied a step that they thought they were more comfortable using. Then, in accordance with the prof. Qendresa, we gave pupils two WPs to solve at home and asked them to use the SIMPLE steps in the solution.
Suggestion/Opinions/New idea
I think that this approach is very good, and I would like to incorporate it more in my future teaching lessons. In order for pupils to master it, I think that they need more time to practice and feel comfortable with it.

Fig. 4.4: Two of Teacher 1's diary entries for lessons on the BM method and SIMPLE steps
(translated from Albanian)

Notably, the teacher observed a heightened level of engagement among pupils who were previously less active in other mathematics classes. According to him, the pupils responded well to the SIMPLE Approach, which not only grabbed their curiosity but also increased their accuracy on WPs and, consequently, their problem-solving skills. The teacher suggested implementing this approach more frequently in mathematics classrooms, highlighting its beneficial effects on pupils' ability to solve WPs.

When asked during the interview if he would make any changes to the intervention, he claimed: *Hmm, maybe no. Or maybe I would ask for more time to practice further, as it [Bar Model] is new to pupils and requires more time to understand it.*

Neither in the interview nor in the diary did the teacher reflect on the situations in the lessons where he felt unsure of how to work with the new approach and how to connect the BM with the equation. He mainly appreciated that it motivated pupils, led to correct answers, and repeatedly expressed his intention to use it in the future.

4.1.3 Pupils' View: Interviews

The three pupils' reactions to the SIMPLE Approach during the group interview were also positive:

R: *Did you like the SIMPLE Approach, and how did you find it?*

Pupil A: *At the beginning, it was very difficult to apply it in word problems, especially the drawing part. But after some time, we enjoyed it. Making some bars and linking them with mathematical operations is very good. But, hmm, sometimes it is tricky.*

Pupil B: *We never did it like this when we solved word problems before. We only use equations in the solution. For me, it was good but difficult to understand. But if you understand, it helps you a lot.*

Pupil C: *It is not that I did not like it. But, maybe doing some more word problems with the teacher using this method would help more.*

These responses imply that pupils find the approach beneficial but expect further clarification from the teacher to understand it better.

4.1.4 Pre- and Post-Test Results

All 25 class pupils were present at the pre-test; however, the data from 20 were only analysed since five pupils missed a lesson during the intervention phase. The pre-test analysis gave a

baseline understanding of the pupils' problem-solving abilities before the intervention. The average success rate for all the WPs was 55.8%. The distribution of scores among the problems in the pre-test is detailed in Table 4.1. Regarding consistency, the data show small differences in the success rates between consistent and inconsistent WPs. In inconsistent WPs, the solutions were less accurate compared to the consistent ones, with a difference of around 12 percentage points.

Table 4.1: The results from the pre-test (Cycle 1), inconsistent problems in bold

Pupils' solution success	WP1_ E	WP2_ E	WP3_ E	WP4_ E	WP5_ E	WP6_ E	N	%	Cons. %	Inco. %
Incorrect solutions	10	6	11	3	14	9	53	44.2	38.3	50.0
Correct solution	10	14	9	17	6	11	67	55.8	61.7	50.0
Total	20	20	20	20	20	20	120	100	100	100

Seven of the 20 pupils who participated were included in the achievement category “excellent”, meaning they had 5 or 6 correct answers out of a total of six WPs of the pre-test. Nine of them were included in the achievement category “good”, meaning they had 3 or 4 correct answers, and four were included in the achievement category “insufficient” with 0, 1 or 2 correct answers. None of the pupils used visualisation to solve the WPs. They used mainly arithmetic calculations or linear equations (Table 4.2), which can be explained by the fact that the class covered a chapter, “Solving Word Problems Using Linear Equations”, just before the intervention.

Table 4.2: Pupils' strategies used in solving WPs of the pre-test (Cycle 1)

Strategies used	Arithmetic calculation	Algebraic solutions	Direct ² or no solution	Total
Number of solutions to WPs	82	32	6	120

To illustrate pupils' results from the pre-test, we will use WP1_E (the text is in Table 3.2), an inconsistent WP. Two examples of a correct solution are in Fig. 4.5.

² By a direct solution I will mean an impulsive, superficial direct translation strategy, in which the solver only focuses on selecting the presented numbers, putting them somehow together and giving the final answer (Boonen et al., 2016).

1. Nga 450 libra të një autori, 124 i janë shitur një shtëpie botuese në ditën e parë, që i bie 24 libra më shumë të shitur se ditën e dytë. Sa nga librat e autorit nuk janë shitur ende?

450
I 124
II 124 - 24 = 100
 $124 + 100 + x = 450$
 $224 + x = 450$
 $x = 450 - 224$
 $x = 226$

1. Nga 450 libra të një autori, 124 i janë shitur një shtëpie botuese në ditën e parë, që i bie 24 libra më shumë të shitur se ditën e dytë. Sa nga librat e autorit nuk janë shitur ende?

450 libra
I = 124
II = I + 24 = 148
 $124 + 148 + x = 450$
 $272 + x = 450$
 $x = 450 - 272$
 $x = 178$

Fig. 4.5: Two pupils' correct solution to WP1_E (Table 3.2) using linear equation (Cycle 1)

Eight pupils out of 20 provided incorrect answers for WP1_E, and none of the unsuccessful solutions were left blank. The most frequent error came from misinterpreting the word “more” in the statement “which is 24 more books sold than on the second day”, causing pupils to add values when they should have subtracted them. Fig. 4.6 presents two incorrect solutions with this mistake.

Gjithsej 450 libra
D₁ = 124
D₂ = 124 + 24
 $124 + 124 + 24 = 450$
 $272 = 450$
 $450 - 272 = 178$

Zgjidhje: 450 1 autor, 124 SH.B
 $450 = 124 + 124 + 24 + x$
 $x = 450 - 272$
 $x = 178$ libra të pa shitur

Fig. 4.6: Two pupils' incorrect solutions to WP1_E using arithmetic calculations (left) and linear equations (right) (Cycle 1)

Regarding the post-test, the results reveal progress in pupils' ability to solve WPs.

Table 4.3: The results from the post-test (Cycle 1), inconsistent problems in bold

Pupils' solution success	WP1_ O	WP2_ O	WP3_ O	WP4_ O	WP5_ O	WP6_ O	N	%	Cons. %	Incon. %
Incorrect solution	2	5	5	6	7	6	31	25.8	21.7	30.0
Correct solution	18	15	15	14	13	14	89	74.2	78.3	70.0
Total	20	20	20	20	20	20	120	100	100	100

Table 4.3 shows, in total, that 74.2% of the solutions provided were correct. However, this result must be considered cautiously as the pre-test and post-test problems differed (see Section

3.4.1). There were still more incorrect solutions to the inconsistent WP, with around 8 percentage points difference from consistent ones.

Fig. 4.7 compares the pre-test and the post-test results of Cycle 1 via box plots, which provide more detail. The pupils correctly solved, on average, 3.35 WPs in the pre-test and 4.45 WPs in the post-test (out of a total of 6 WPs). Half of the pupils correctly solved 3 WPs or less in the pre-test, while in the post-test, it was only one quarter. The plot on the right shows a bigger gain for the inconsistent WPs (INC) than for the consistent ones (CON).

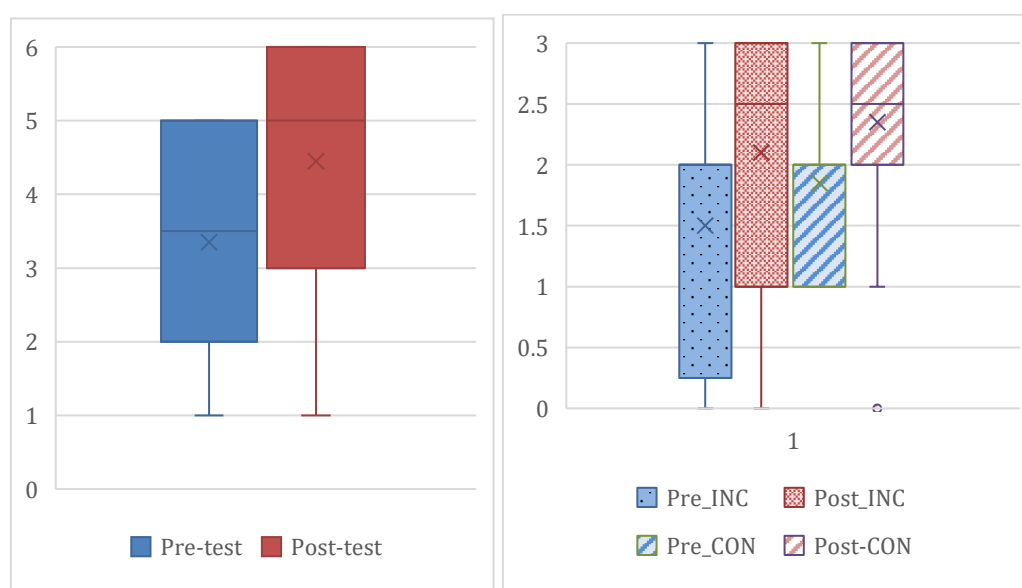


Fig. 4.7: Box plots for the number of correctly solved WPs in the pre-test and the post-test (Cycle 1, x – mean, horizontal line segment – median)

Finally, the progress of individual pupils from the pre-test to the post-test was followed. Figure 4.8 illustrates the movement of pupils between three achievement categories: “insufficient”, “good”, and “excellent” for WPs in the pre-test and post-test (based on Table 3.8, Section 3.9.1). In the post-test, 14 pupils were included in the achievement category “excellent” in their solutions (7 belong to the achievement categories “good” to “excellent”, “insufficient” to “excellent” and 7 to the achievement category “excellent” who solved the WPs successfully both in the pre-test and the post-test). Two pupils remained in the achievement category “insufficient”, and four belonged to the achievement categories of “good” to “good” or “good” to “insufficient”. The pupils whose achievement is represented by the bold arrows warrant our attention. We will analyse their solutions in more detail in Section 4.4.3, together with the solutions of pupils from the other two cycles.

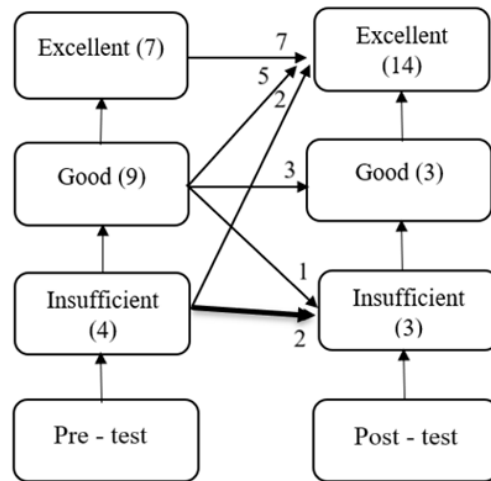


Fig. 4.8: Movements in pupils' achievement categories (insufficient, good, excellent) from pre-test to post-test (Cycle 1)

4.1.5 Presence of BMs in the Post-Test Solutions

Next, the extent to which the pupils used the BM method was followed and how it was related to the correctness of the solution. Table 4.4 shows that most of the pupils drew BMs in their WP solution, Table 4.5 shows the relationship between the BM presence and the solution success, but further analysis reveals different qualities of the BMs.

Table 4.4: The presence of the visualisation within the WP's solution of post-test (Cycle 1)

Bar model	WP1_O	WP2_O	WP3_O	WP4_O	WP5_O	WP6_O	Total	%
Not present	3	2	4	3	3	6	21	17.5
Present	17	18	16	17	17	14	99	82.5

Fig. 4.9 presents the framework that originated from our analysis of the usage of the BM and the correctness of the solution in the post-test in Cycle 1. The arrows show the relationships between three characteristics: correctness of the BM (the second column), dimensions/completeness/operation fit (the middle column), and correctness of the answer (the fourth column). For example, if the pupil drew a correct BM, the solution was always correct; however, if the pupil drew incorrect BMs whose shape did not fit the mathematical operation, that could lead to a correct or incorrect answer.

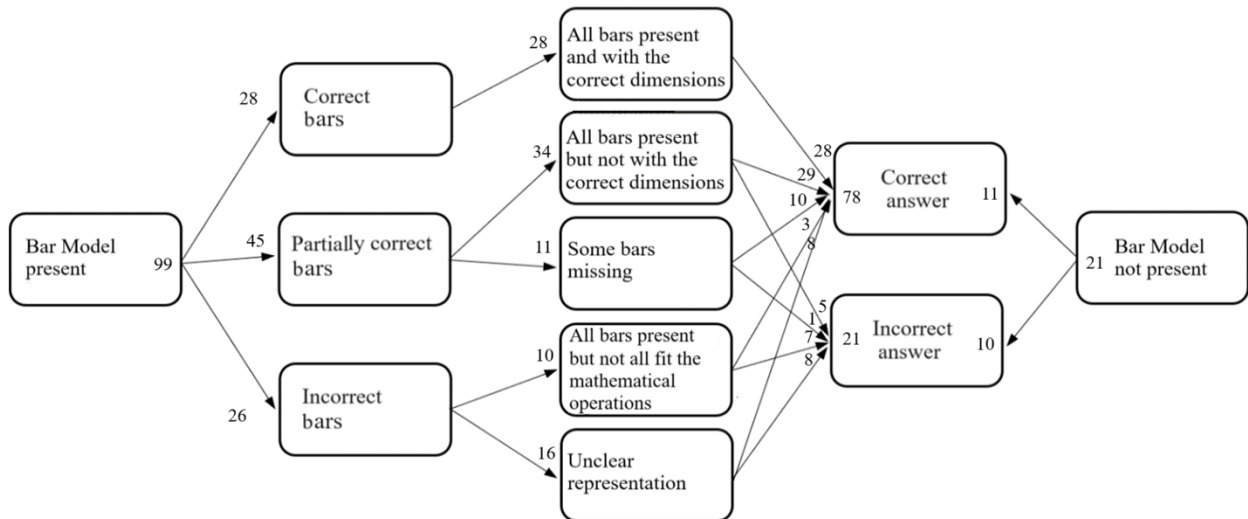


Fig. 4.9: Correctness of the solution related to the quality of BMs in the post-test (Cycle 1)

Table 4.5: The relation between the BM and the problem-solving success (Cycle 1)

BM	Incorrect solution	Correct solution	Total	%
Present	21	78	99	82.5
Not present	10	21	21	17.5

Pupils' solutions to WP2_O (see Table 3.3) will illustrate the types of BMs they used.

Fig. 4.9 shows that the solutions with a correct BM were always correct. When they were partially correct, most of the solutions were correct (39 WPs from 45) which is a good indicator of the usage of the BM method for the solution of WPs. Due to the small number of pupils, more interpretation will be provided for the whole sample in Section 4.4.3.

Fig. 4.10 includes two correct BMs, visually clear, uncluttered, and with accurate proportional sizing. They do not necessarily adhere to exact dimensions but provide a clear representation of the WP structure, quantities, and relationships between them.

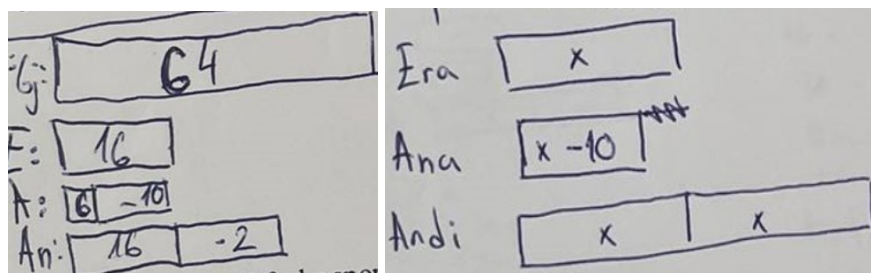


Fig. 4.10: All bars present and with correct dimension for WP2_O (Cycle 1)

For *partially correct bars*, the bars align with the mathematical operation required by the WP but (1) the BM for the second solving step is missing (e.g., multiplication by 2, see Fig. 4.11, left) or (2) all the bars are present but do not have the correct dimensions (e.g., Fig. 4.11, right – the last bar should be shorter than the bars above).

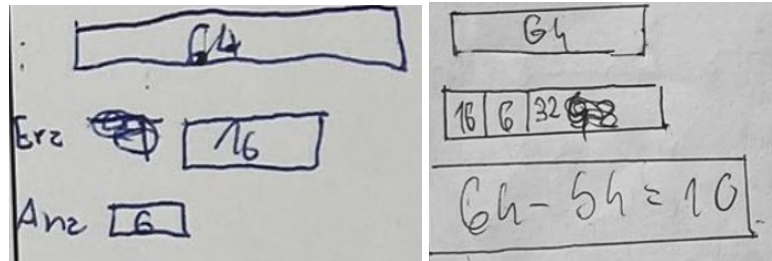


Fig. 4.11: “Some bars missing” (left) and “All bars present but not with the correct dimensions” (right) for WP2_O (Cycle 1)

In the solutions with *incorrect bars*, the BM does not match the required mathematical operation or is unclear to us. Fig. 4.12 (left) is an example of the former, as the last bar is divided into two parts rather than another bar of the same size being added, as required by WP2_O. Fig. 4.12 (right) is an example of an “unclear representation”, as there is no apparent relation of the bars to the WP2_O structure. The bars are just frames around the numbers.

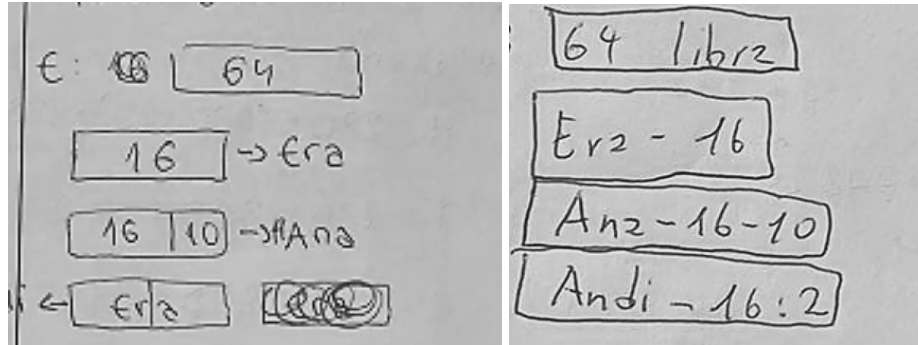


Fig. 4.12: “All bars present but not all fit the mathematical operations” (left) and “Unclear representation” (right) for WP2_O (Cycle 1)

As the number of pupils in each cycle is rather small, this data will be evaluated in more detail together in Section 4.4.

4.1.6 Presence of the SIMPLE Steps in the Post-Test Solutions

As for using the SIMPLE steps, they were present in 40.8% of solutions, while they were partially present in 35.8% of solutions. Conversely, 23.4% of solutions showed the absence of the steps (Table 4.6). Fig. 4.13 presents two pupils’ solutions of WP1_O for the category “The

Table 4.6: The presence of the SIMPLE steps in WPs of the post-test (Cycle 1)

The SIMPLE steps	WP1_O	WP2_O	WP3_O	WP4_O	WP5_O	WP6_O	N	%
Present	12	9	8	9	8	3	49	40.8
Partially present	5	7	7	7	7	10	43	35.8
Not present	3	4	5	4	5	7	28	23.4

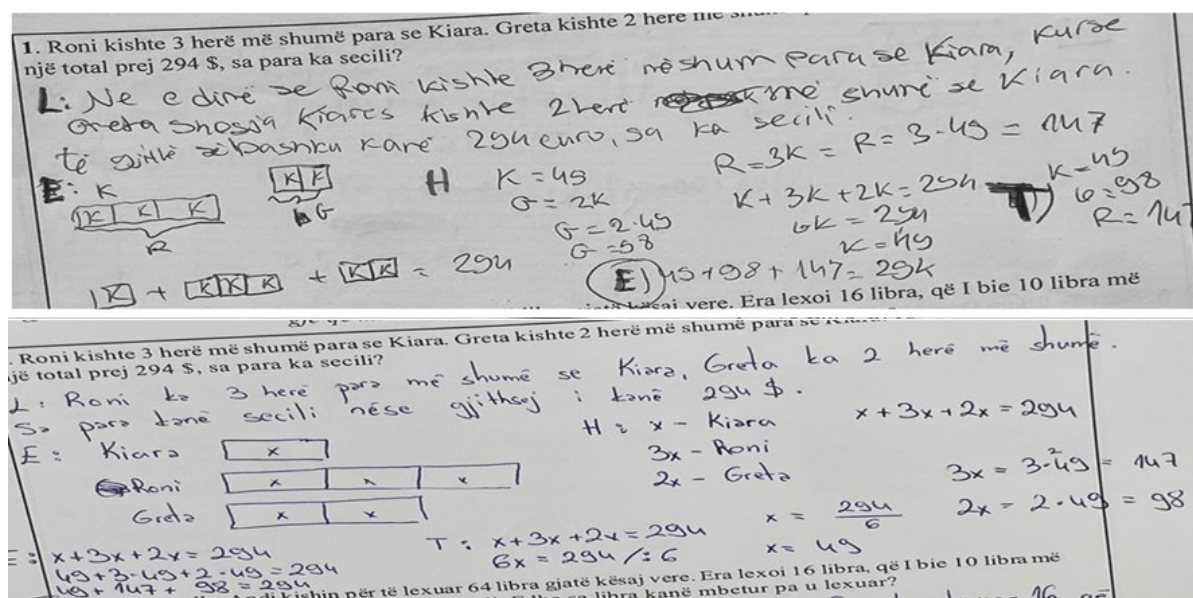


Fig. 4.13: The “presence” of the SIMPLE steps and the BM for WP1 O (Cycle 1)

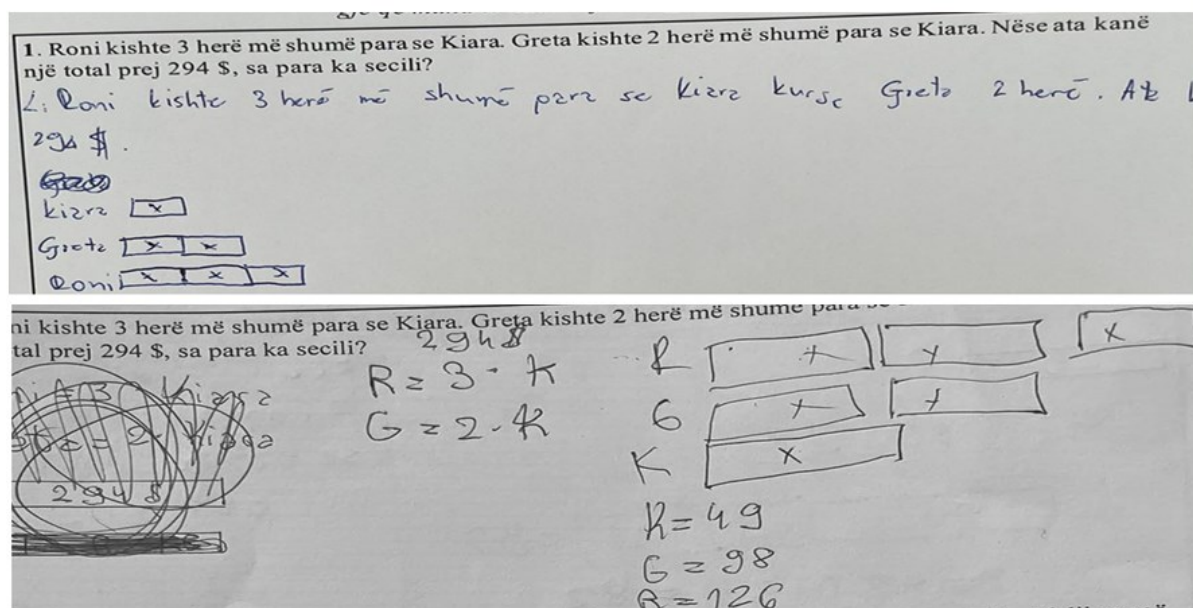


Fig. 4.14: The “partial presence” of the SIMPLE steps and the BM for WP1 O (Cycle 1)

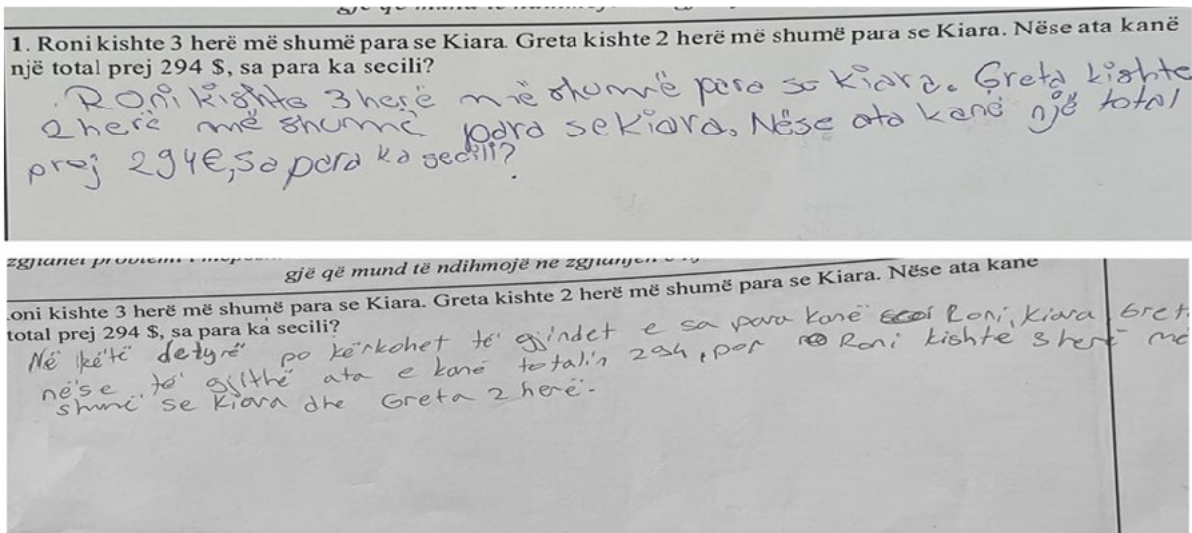


Fig. 4.15: The solutions with no indication of the SIMPLE steps and the BM for WP1_O (Cycle 1)

In Figure 4.13, the pupils evidently endeavoured to implement each step of the strategy by distinctly highlighting them. Figure 4.14 was evaluated as partial, as the pupils employed two of the SIMPLE steps, including drawing the bar model and presenting the solution, but provided no indication of utilising the remaining steps. In Fig. 4.15, the pupils offered no evidence of employing any SIMPLE step, merely describing the text of the given task.

Table 4.7 shows that when pupils presented the SIMPLE steps, they always arrived at the correct solution. The same was true for some solutions in which pupils presented steps partially. On the other hand, pupils mostly got incorrect solutions with the absence of SIMPLE steps. Fig. 4.16 presents the proportion of correct and incorrect answers based on the presence of the steps for the categories identified in the post-test.

Table 4.7: The relation between the SIMPLE steps and the problem-solving success (Cycle 1)

The SIMPLE steps	Incorrect solution	Correct solution	Total	%
Present	0	49	49	40.8
Partially present	13	30	43	35.8
Not present	18	10	28	23.4

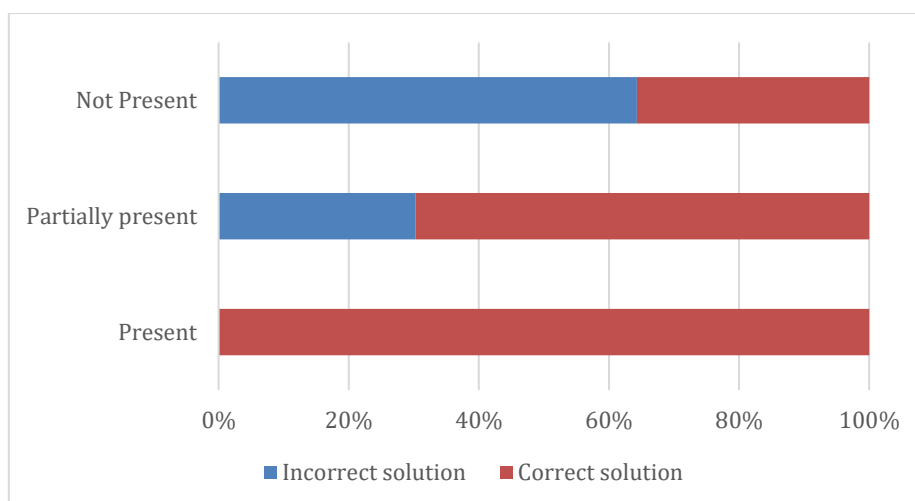


Fig. 4.16: Correctness of the answer related to the presence of SIMPLE steps (Cycle 1)

As the number of pupils in each cycle is rather small, this data will be evaluated with more detail together in Section 4.4.

4.1.7 Preliminary Answers to Research Questions Based on Cycle 1

In this section, I will summarise preliminary answers to the research questions based on the data from Cycle. To avoid repetitions, the results will be discussed against related literature in Chapter 5.

RQ1. How do pupils and teachers perceive the SIMPLE Approach's usefulness, and how confident are they with adopting it in WP solutions after the intervention?

In the interview, the teacher admitted that introducing this method was challenging for him and the pupils, especially as they had just learned to solve WPs using linear equations. Nevertheless, he claimed that pupils eventually grasped the SIMPLE steps and the BM method after some practice and effort. He emphasised the need to prolong the exposure to the new approach, suggesting that a more extended implementation period would improve understanding and effectiveness.

The teacher perceived an increase in pupil involvement and accuracy in solving WPs and in their interest and participation, especially for those less active in mathematics lessons. Recognising its benefits, the teacher stated his intention to use the approach more frequently in future mathematics lessons. In any case, he did not reflect on the situations where he felt unsure how to work with the new approach and connect the BM with the equation.

From the pupils' perspective, they admitted that they had difficulties applying the BM method to the solution initially, but after some time and practice, they liked it. They mentioned that this

method helped them solve WPs, but they would like to practise it more to comprehend the steps thoroughly.

RQ2. What are possible conditions of and barriers to implementing the proposed intervention?

Two important conditions supported the implementation of the intervention:

- The teacher's willingness to learn the new teaching approach.
- Pupils had recently learned how to solve WPs using linear equations and had immediate prior experience dealing with WPs. This prior knowledge was helpful as pupils could concentrate more on the solving method rather than on the concept of WPs as such.

On the other hand, some barriers made it difficult to implement the intervention adequately:

- The teacher was doubtful if the pupils would understand the visual part of solving the WPs.
- The teacher did not use the Guide in full to implement the intervention as designed.
- The teacher missed some opportunities to initiate discussions and ask questions to clear up any misunderstandings the pupils could have
- The teacher did not form group work during which pupils could have learned from each other.

The data showed that the teacher found it difficult to adopt the constructivist approach which was highly affected by his usual teaching style which did not fit with the principles of constructivism.

RQ3. What impact does the intervention have on pupils' success when solving WPs? To what extent can they use the BM method and the SIMPLE steps in their solutions?

The first cycle data findings show that pupils enhanced their success in solving WPs in the post-test. The post-test success rate increased to 74.2%, representing an improvement of 18.4 percentage points from the pre-test.

Regarding the BM and SIMPLE steps, the pupils always arrived at the correct solution when correctly drawing the BM and using all the SIMPLE steps. This can be considered a positive finding of the study, as the proper application of the BM approach in conjunction with the SIMPLE steps can enhance problem-solving abilities and increase the success of the WP solution.

When comparing each pupil's data individually, it was noticed that the pupils' success rates for WPs in the pre-test and post-test varied. They were mostly "excellent" in their post-test

solutions, indicating that the intervention affected their solution. Six pupils did not achieve better after the intervention. It is presumed that the limited intervention time of only four lessons was a barrier for them.

4.1.8 Implication for Cycle 2

The results of the data from Cycle 1 indicate that three aspects of the intervention need to be changed.

Addressing Teacher Involvement: In the first cycle, some moments were observed when the intervention was not appropriately implemented. To address this concern, Teacher 2 needs to be better informed about the importance of adhering to the Guide's guidelines and presenting the BM method.

Enhanced bar model practice: Since not all the pupils utilised the BM method correctly in the first cycle and did not understand it well, an additional lesson for BM practice will be included. It will enable pupils to grasp this technique better before proceeding to the SIMPLE steps. The Teacher Guide will include explicit instruction for the teacher to draw the BM types by giving an example of their application in three different WPs.

Collaborative Problem Solving: The Teacher Guide suggests including group work so that pupils can learn from each other. Teacher 1 did not follow this principle. This will be emphasised in the next cycle by giving the teacher improved instruction orally and advising the use of the Guide carefully.

4.2 Cycle 2

The second cycle was organised at the primary school, Agim Ramadani, in the same city, Gjiilan. This school has 1,192 pupils distributed in 56 classes, of which five classes belong to preschool education with 87 children, 27 classes from 1st to 5th grade, and 24 classes from 6th to 9th grade. The school was founded in 1959 and has been operating since that year.

The school has six 8th-grade classes taught by two mathematics teachers, with approximately 19 – 24 pupils per class. Teacher 2 was chosen based on her consent to participate in the study and on the class lesson plan, which includes the necessary timeframe for implementing the intervention. The class had 19 pupils (8 boys, 11 girls), all of them had Albanian nationality. All participated in this study and were present in all phases.

The intervention was conducted by a female teacher with 20 years of experience teaching mathematics and her third year teaching in that particular class. The teacher was happy to be

part of this study and was willing to interrupt her plan and implement the proposed intervention. She was informed about the research's aim and the intervention process. She agreed with all aspects of the intervention. She pointed out that since she applies the student-centred approach in her lessons, implementing the proposed approach will not be difficult for her. Although she had never used the BM method and the SIMPLE steps before in her teaching, she showed great interest in learning them and conveying them to the pupils as instructed in the Guide. The teacher received a small financial reward to acknowledge her help in carrying out the study.

Regarding WPs, in the initial session, the teacher claimed that WPs were not used much in her mathematics lessons and added that *pupils mostly solve them based on some words which are related to the mathematical operation and sometimes converting them into linear equations, but they never used visualisation to get the solution*. The teacher emphasised that these types of tasks were not present that much in mathematics textbooks. Her plan for the class showed they were supposed to learn about WPs. Later, pupils admitted that they had used them very rarely, and even when they had used them, their teacher had solved them in most cases.

4.2.1 Changes in Cycle 2

Based on the feedback from Cycle 1, at the beginning, I emphasised to the teacher her essential role in developing discussion, raising questions, and creating an environment where the pupil is put first in the learning process. The Teacher Guide was provided to her, and she was instructed in the same way as Teacher 1.

Second, Cycle 1 showed that pupils needed extra time to fully understand the BM method. Therefore, one more lesson for the BM method was added. The number and type of WPs remained the same as in the first cycle. The teacher was instructed to explain the visualisation part more slowly and carefully and to dedicate the first lesson only to introducing the BMs type to help deeper understanding.

Third, the teacher was also advised to include at least one or two WPs in the group activity throughout the intervention so that the pupils could learn from each other.

4.2.2 Teaching Interventions

As in Cycle 1, this section begins by describing the intervention course in Cycle 2. Based on the data, some key moments that may have impacted pupils' understanding of the BM method and the SIMPLE steps will be highlighted. Teacher 2's approach and the instances when the fidelity of the intervention was compromised will also be mentioned.

The Bar Model Method

The intervention started with an introduction to the BM method. In the first lesson, the teacher presented the BMs and their types and drew them on the blackboard. She provided examples of each type to help the pupils understand it. She instructed the pupils to take notes and opened a discussion with the class.

The pupils carefully followed the teacher and draw on blank paper three types of the BM presented by the teacher (Fig. 4.17). They asked questions when they encountered something they did not understand. One of their questions was, *Can you please explain again the difference between the comparison and the change BM types?*. To help the pupils understand the types of BM, the teacher once again tried to clarify all three types of BM using simple examples with money. She dedicated all 45 minutes of this lesson to explain the types of BM and discuss with pupils their usage. An increased interest was observed in pupils in learning how to use BMs to visualise the solution to WPs.

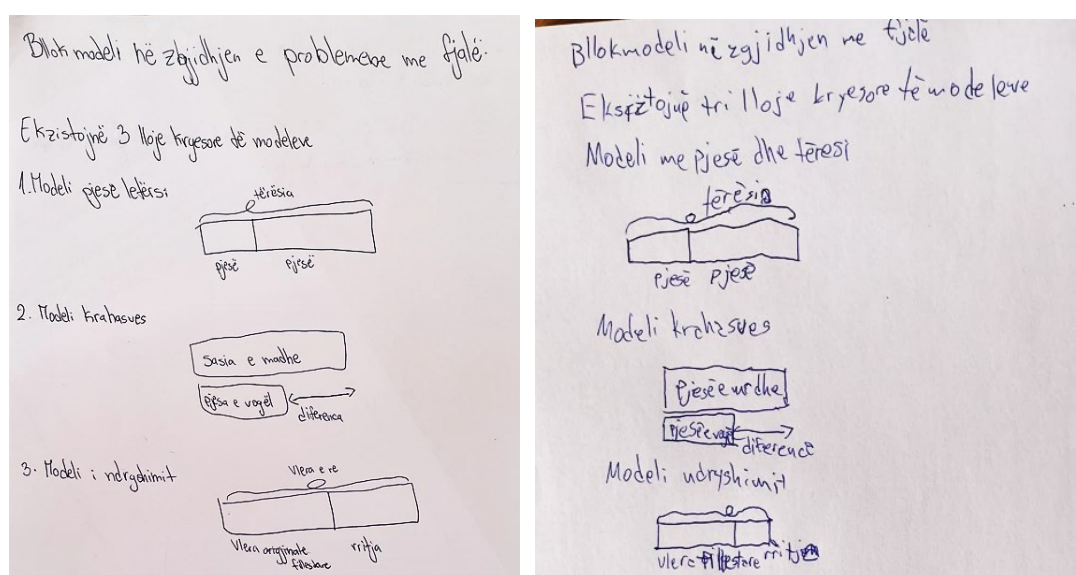


Fig. 4.17: Pupils' notes on the three BM method types (Cycle 2)

In the next lesson, the teacher demonstrated the drawing of the BM in WP1_I and asked the pupils to participate in the solution. The pupils demonstrated their understanding of the BM drawing by asking questions such as, *Why should the first bar be longer than the second?* or *Why are we subtracting when the word is more? Shouldn't we add?* With the teacher's clarification that *the word "more", in this case, is connected with the first bar, which means the first bar should be bigger and the second smaller*, they analysed these aspects and together arrived at the solution.

The pupils were given two more WPs (WP2_I and WP3_I), but this time, the teacher asked them to work on their own. They started to work on the solution, but some still faced difficulties drawing the proper BM. They felt confused, and some stopped working. When the teacher noticed it, she gave them some hints by helping them draw the BM for one part of the WP and then letting them continue further for other parts. For WP3_I, she drew pupils' attention to the solution of WP1_I explained earlier since they had similarities and guided them to work in pairs. After a few minutes of individual and pair work, she prompted a class discussion and asked two pupils to draw the BMs on the blackboard. Many pupils raised their hands, and two of them, P1_2 and P2_2, explained how they did it and why they decided mainly on that kind of BM (Fig. 4.18). At the end, the teacher assigned two WPs as homework and asked the class to use BMs and explain their solution in writing.



Fig. 4.18: Pupils solving WP3_I(left), WP2_I(right) during the intervention phase (Cycle 2)

When the teacher and I went to the class in the next lesson, pupils were already engaged in discussing their homework solutions. They immediately asked the teacher which of the answers was correct. Rather than encouraging the pupils to analyse and make sense of the mistakes themselves, the teacher checked two or three pupils' solutions with different results and clarified mistakes in front of the class. This approach reflects a more traditional method of error correction rather than a constructivist one, where pupils are guided to reason through their mistakes. In Fig. 4.19 left, even though the pupil had the correct answer, she did not draw the correct BMs. As for the second step, she just multiplied by 1.5. In Fig. 4.19 middle, the pupil added two similar BMs and did not present the solution, and in Fig. 4.19 right, the pupil added a BM, representing the value 1.5 and arrived at the incorrect answer. The teacher tried to clarify each of these mistakes orally and demonstrated how the solutions should be done.

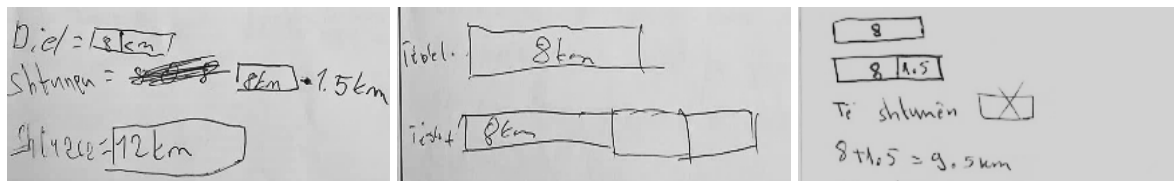


Fig. 4.19: Three pupils' solution to WP1_H (Cycle 2)

After that, the pupils were assigned three other WPs and allowed to work on the solution with their friends if they wanted. Some pupils started working individually, and others collaborated with their friends sitting next to them. The teacher let the class work alone for some time. She was walking among them, monitoring their work and assisting some in drawing the BMs and finding solutions. The pupils seemed engaged in their work. Fig. 4.20 presents two of the pupils' drawings of the BM for WP4_I with the correct solution.

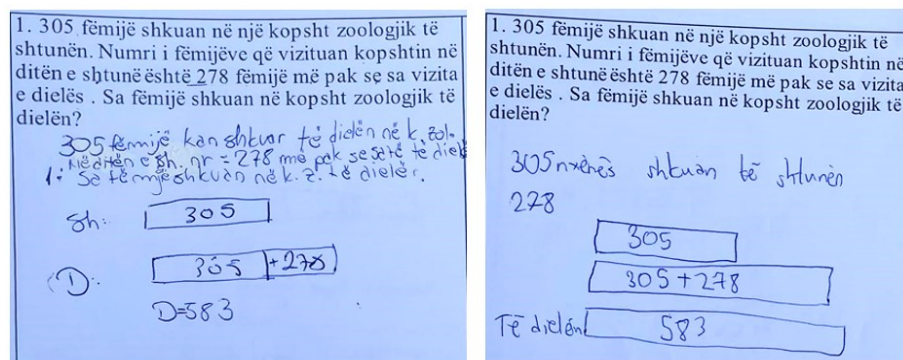


Fig. 4.20: The solution of WP4_I with the inclusion of the BM method (Cycle 2)

The SIMPLE Steps

The teacher presented the SIMPLE steps to the pupils in the next lesson of the intervention. She first explained the steps and asked the pupils to write down everything (Fig. 4.21).



Fig. 4.21: The explanation of the SIMPLE steps by the teacher (Cycle 2)

The teacher solved WP7_I by showing where each step belonged to the solution. She incorporated the pupils into the process by asking questions continuously. For example, for the

first step, “S”, the teacher asked two or three pupils to rephrase the problem in a way they understood. In this way, she involved the pupils in the process of learning. It was noticed that the teacher’s focus was not the same for all the SIMPLE steps. Steps like “evaluating your answer” or “monitoring the process” received less attention. For example, instead of encouraging pupils to explore different ways to solve the WP or assess their solutions, she often concluded the “evaluate your answer” step by simply stating that “*based on the result and the proof, the solution is correct*”. This approach limited pupils’ opportunities to think critically and consider alternative ways to arrive at the correct answer. Instead, the teacher could have guided them to reflect on their solutions, discuss different possible methods, and consider the reasoning behind their answers.

To reinforce understanding, the teacher engaged the pupils in another WP (WP8_I) and asked them to solve it by writing down all the steps that belonged to the solution. She provided the pupils with the SIMPLE instructional sheet (Fig. 3.5). The pupils could decide whether to work alone or with friends.

The pupils started the solution and often asked the teacher to confirm whether they were doing it correctly. While they initially worked alone, they frequently shared the solution with other pupils in the class. They moved from one table to another, discussing their answers. The teacher observed and intervened where needed. Most of the pupils understood the solution to that WP, so they were very interested in coming to the blackboard to share it with others. The teacher chose one of them, P3_2, to present it to everyone. Two of the pupil’s solutions for WP8_I are presented in Fig. 4.22 (the division into columns for the right figure was made by the pupil so he can arrange the space for the solution based on each of the SIMPLE steps). In Fig. 4.22 left, the pupils wrote for the letter L “Here we need to find how many steps they collected together” while in Fig. 4.22 right, the pupil wrote “We know that Emmy collected 137, 27 less than her sister. But we do not know how many they collected together”. After that, the pupils continued to work on the solution by preparing, presenting it and evaluating the answer.

Because there was no time left, the teacher gave the pupils two WPs to work on at home and instructed them to use the SIMPLE steps.

In the next lesson, the pupils showed the teacher their homework solutions. The teacher quickly checked each one by going to the pupils’ desks. To avoid losing time, she gave general feedback by commenting on the solution and the steps involved in each part of it. The teacher encouraged pupils to ask questions if they needed further explanation.

S	I	M	P	L	E	S	I	M	P	L	E
L- kh duhet me gjet sa i kane gjet ab bache						Ve e dime	137	137+27	M=137+	301-164-137	
S- 137 - Emy						Se Emy	137	M-X	27=164	= 0	
M- 137 - motra e saj						mbledhi	137 dhe	M=E+27	Total 137+		
E- 301 = 137+164						27 me	301	E+M=?	164=301		
301 = 301						pak se					
H- x-sa i kane gjet ab bache						mota e tij					
137 - Emy						Nuk e dime					
137+27 - motra e saj						Sa pilla					
T- X = 137 + 137 = 27						mbledhen					
Y = 137+164						se bashko					
X = 301											

Fig. 4.22: The solution to WP8_I by the pupils (Cycle 2)

Some pupils admitted that they had discussed their homework solution together before starting the lesson, and with this feedback from the teacher, they had already addressed their prior misunderstandings. As there were no questions, the teacher assigned the pupils four other WPs. Again, the pupils could decide whether to work alone or collaborate with their friends. They started individually but often consulted with the teacher and their friends when encountering difficulties. The teacher observed the process and intervened by giving them hints when they struggled with the solution. The hints were mainly related to the BM method drawing in which the teacher guided the pupils to return to previously solved WPs, or sometimes she even started the drawing, leaving the continuation to the pupil.

The pupils seemed to appreciate the SIMPLE steps, as they quickly adapted and implemented them. However, some pupils found it challenging to implement all the steps of the SIMPLE strategy in all four given tasks because, based on their complaints to the teacher that were taken as notes in the last lesson intervention, there was not enough time.

Unlike the Guide's instruction, the teacher did not initiate the group work, and because pupils were free to choose their way of working, they often chose to work in pairs.

Despite some challenges, the class as a whole seemed comfortable using the SIMPLE steps and the BM method.

Adhering to the Principles of Constructivism

The teacher showed strong motivation and dedication in implementing the intervention as designed to help pupils solve WPs. She mostly followed the Teacher Guide and presented the intervention's lessons by putting the pupils in the centre. She included them by asking questions, fostering class discussions, and creating a pleasant environment in which the pupils

felt free to participate. They could also move around the classroom to discuss with other peers. The teacher helped the pupils individually and gave them some hints if needed.

Furthermore, it was observed that the teacher appreciated the BM method and integrated it into her teaching as instructed in the Guide. Regarding the SIMPLE steps, she initially introduced the pupils to each step by applying them in one WP and then letting them practise individually. Still, some moments in which the fidelity of the intervention was violated were uncovered.

First, as noted above, the teacher did not place equal emphasis on all SIMPLE steps. Second, like in the first cycle, the absence of group work was noticed, an important component for pupils to learn from each other and foster collaborative learning. Contrary to the Guide's guidelines supporting group work, the teacher allowed pupils to choose how they wanted to work. Most pupils either worked alone or with a friend sitting next to them which is beneficial, too, but the intended approach was to include group work. Furthermore, the way how mistakes of WPs were addressed reflected a more traditional method of error correction rather than a constructivist one. Handling errors is an important aspect of constructivist teaching since it helps pupils' reasoning in the solution of WPs.

4.2.3 The Teacher's View: Teacher Diary and Interviews

The second teacher's interview provided further insights into her perception of the BM and the SIMPLE steps in the solution of WPs. She admitted limited use of WPs in her mathematics lessons and limited use of strategies for their solution. She appreciated the SIMPLE Approach.

Researcher (R): *What do you think about the SIMPLE Approach?*

Teacher (T): *I really liked it. After I did the intervention here, I also explained it to the 7th-grade class. We used it in some simple word problems, which helped them find a solution easily.*

R: *How difficult was it to present it to the pupils?*

T: *No, it was not challenging at all. I think the bar model made the approach attractive; it awakened pupils' curiosity and helped them to form the equation more easily.*

The teacher appreciated this approach for her own learning:

T: *I don't want to spoil you, but I think it helps. I have noticed that this approach also affects me as a teacher. Honestly, it never occurred to me to use bars to solve problems, but I liked them.*

The teacher expressed the intention to continue using the SIMPLE Approach with 8th graders as well as with 6th and 7th graders in the future. While steps like “monitoring the process” or “evaluating the answer”, in her opinion, were a bit time-consuming, the teacher favoured the visualisation step.

R: Are there any steps you think need to be changed, as they hinder pupils from obtaining the solution?

T: Maybe the “evaluating the solution” step; in my opinion, it is not always necessary to put it in writing. However, drawing the bars helps to visualise the problem and facilitates finding a solution, which I like the most.

Notably, the teacher suggested not obligating the pupils to write each step or to write them briefly due to the limited time. In her diary, she stated that making pupils write a lot may result in insufficient time to complete all the post-test WPs. She expressed overall satisfaction with the approach and emphasised that its application helps solve WPs (see Fig. 4.23).

Date: 13.06.2023	Date: 20.06.2023
Bar Model method – Lesson 1	SIMPLE strategy – Lesson 4
Describe the lesson learned.	Describe the lesson learned.
In the beginning, the pupils were informed about WPs and their application to real life. Then, they were presented with the bar model method. I started by presenting the types of bar model methods and giving an example for each. The bar model method is very helpful because it converts visualization into mathematical expression.	In this lesson, pupils were informed about the SIMPLE steps. Throughout the lessons, pupils expressed interest in learning this approach. The steps that they liked the most, in my opinion, were the „Involve the bar model“ method, „Present the solution“, and "Look over the solution again“.
How the intervention went	How the intervention went
The lesson went well because pupils understood the lesson very well and the way bar models were incorporated into the solution. They felt satisfied, and in my opinion, this method aroused their curiosity, as they immediately started to draw bar for each of the WPs they gave to them. In general, I think that the bar model method could be a good tool for helping pupils solve problems.	The intervention went well. First, the pupils observed while I explained all the steps. Then, I solved a WP and included pupils to the solution by taking their opinions. After that, they worked on another WP to practice the SIMPLE steps. I think that they liked it, but they struggled to write every step of it in detail.
Suggestion/Opinions/New idea	Suggestion/Opinions/New idea
I have no specific suggestion; maybe we can draw lines or another figure sometimes beside the bar.	My only suggestion is that we do not need to ask pupils to write everything on paper if they are sure how to do it. Maybe they need to write every step, as they are helpful but very short so as not to take a lot of time while doing it. I suggest implementing this strategy often because it is good, easy, and practical. I will continue to use it in mathematics classes.

Fig. 4.23: Two of Teacher 2’s diary entries for lessons on the BM method and SIMPLE steps
(translated from Albanian)

4.2.4 Pupils' View: Interviews

Like the teacher, the pupils had positive opinions about using the new approach.

Researcher (R): *Did you like the SIMPLE Approach, and how did you find it?*

Pupil A: *I liked this approach because we had not learned anything like it before, and it was interesting to use while solving word problems.*

Pupil B: *It was helpful for me as I don't like these kinds of problems very much. But it made me want to solve another word problem when I started using it.*

Pupil C: *I liked it because you could go step by step and solve the problem very easily, but I got tired in the end because there were many problems to solve.*

The last points to the fact that the lessons might be rather repetitive for some.

The pupils agreed that they would use it in the future.

Pupil A: *I started to use it at home, too. I explained a word problem to my little brother, who is in the fourth grade. I told him how to construct the bar models,*

Pupil B: *Yes, I will [use it], as I liked it, and it helped me a lot.*

4.2.5 Pre- and Post-Test Results

The pre-test and post-test solutions were analysed in the same way as in Cycle 1.

In the pre-test, 52.6% of the solutions were correct (Table 4.8). As in the first cycle, there is a slight difference in correctness (around 10.4 percentage points) between consistent and inconsistent WPs. The solutions in inconsistent WPs were less accurate than the consistent ones.

Table 4.8: The results from the pre-test (Cycle 2), inconsistent problems in bold

Pupils' solution success	WP1_ E	WP2_ E	WP3_ E	WP4_ E	WP5_ E	WP6_ E	N	%	Cons. %	Incon. %
Incorrect solutions	11	4	10	6	14	9	54	47.4	42.2	52.6
Correct solution	8	15	9	13	5	10	60	52.6	57.8	47.4
Total	19	19	19	19	19	19	114	100	100	100

Four of the 19 pupils who participated were included in the category of achievement “excellent”, meaning they had 5 or 6 correct answers out of a total of 6 WPs of the pre-test.

Twelve were included in the category of achievement “good”, meaning they had 3 or 4 correct answers, and three were included in the category of achievement “insufficient” with 0, 1, or 2 correct answers. The pupils' work confirmed the teacher's expectations, as none used visualisation or diagrams in the pre-test. They mostly tried to solve problems by direct calculation (Table 4.9).

Table 4.9: Pupils' strategies used in solving WPs of the pre-test (Cycle 2)

Strategies used	Arithmetic calculation	Algebraic steps	Direct or no solution	Total
Number of solutions to WPs	89	13	12	114

Similarly to Cycle 1, problem WP1_E will be used to illustrate pupils' work (the text is in Table 3.2), an inconsistent WP. Two examples of a correct solution are in Fig. 4.24.

Fig. 4.24: Two pupils' solutions to WP1_E of pre-test (Cycle 2)

For the WP1_E, 11 out of 19 pupils provided incorrect answers, and none of the solutions were blank. Most of the solutions were achieved using direct arithmetic calculations, and very few of them contained algebraic solutions. Similarly to Cycle 1, most of the wrong answers were from misinterpreting the inconsistent word “more” (see Fig. 4.25 for examples).

Fig. 4.25: Two pupils' incorrect solutions to WP1_E (Cycle 2)

The post-test results were better than the pre-test ones as 81.6% of the solutions provided were correct, while 18.4% were incorrect (Table 4.10). Notably, there was hardly any difference between the success rates for the consistent and inconsistent WPs. When comparing the incorrect solutions of WPs in the pre-test and post-test, it can be observed that the improvement of pupils' success was greater for the inconsistent WPs. Fig. 4.26 depicts the success rate for WPs of the pre-test and post-test.

Table 4.10: The results from the post-test (Cycle 2), inconsistent problems in bold

Criteria/ Task	WP1_ O	WP2_ O	WP3_ O	WP4_ O	WP5_ O	WP6_ O	N	%	Cons. %	Incon. %
Incorrect solution	3	5	3	3	3	4	21	18.4	17.6	19.2
Correct solution	16	14	16	16	16	15	93	81.6	82.4	80.8
Total	19	19	19	19	19	19	114	100	100	100

Fig. 4.26 compares the results of the pre-test and the post-test of Cycle 2 via box plots, which provide more detail. The pupils correctly solved, on average, 3.16 WPs in the pre-test and 4.95 WPs in the post-test (out of a total of 6 WPs). Half of the pupils solved 4 or fewer WPs in the pre-test while 11 out of 19 pupils solved all 6 WPs correctly in the post-test. The plot on the right shows a slightly bigger gain for the inconsistent WPs (INC) than for the consistent ones (CON).

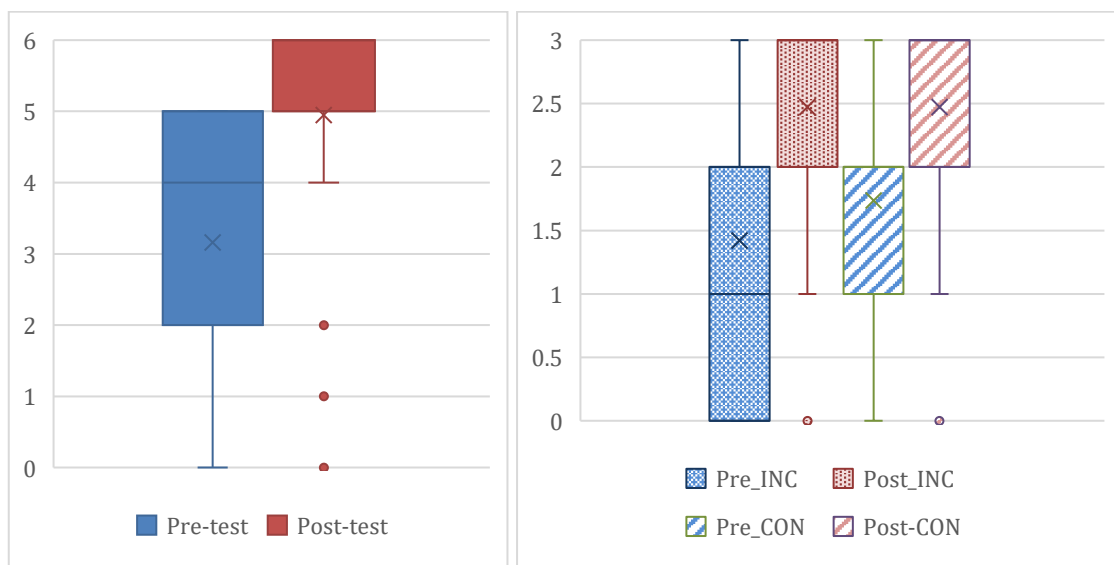


Fig. 4.26: Box plots for the number of correctly solved WPs in the pre-test and the post-test (Cycle 2, x – mean, horizontal line segment – median)

Figure 4.27 illustrates the movement of pupils between three achievement categories: “insufficient”, “good”, and “excellent” for WPs in the pre-test and post-test. In the post-test, eleven pupils improved after the intervention. Ten pupils went from the achievement categories “good” to “excellent”, and one from “insufficient” to “excellent”. Four of the pupils were included in the achievement category “excellent” in the pre-test and remained in the achievement category “excellent” in the post-test. A deeper analysis was needed for three pupils’ work (see the bold arrows in 4.27). Their solutions will be further explored in Section 4.4.3. The pupils whose achievement is represented by the bold arrows warrant our attention. We will analyse their solutions in more detail in Section 4.4.3, together with the solutions of pupils from the other two cycles.

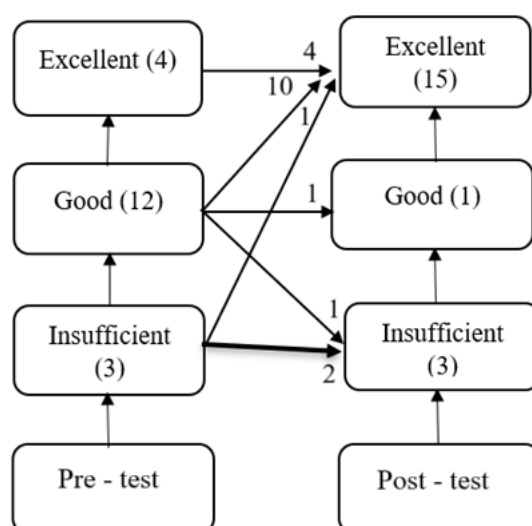


Fig. 4.27: Movements in pupils’ achievement categories (insufficient, good, excellent) from pre-test to post-test (Cycle 2)

4.2.6 Presence of BMs in the Post-Test Solutions

As indicated in Table 4.11, 94.7% of the post-test solutions included a BM.

Table 4.11: The presence of the visualisation within the WP’s solution of post-test (Cycle 2)

Bar Model	WP1_O	WP2_O	WP3_O	WP4_O	WP5_O	WP6_O	Total	%
Not present	1	0	1	0	2	3	6	5.3
Present	18	19	18	19	17	16	108	94.7

Based on the framework for categorising pupils’ solutions in terms of BMs and correctness of the answers (Fig. 3.7, Section 3.9.1), Fig. 4.28 presents the results for the post-test of Cycle 2. Table 4.12 presents the relationship between the presence of the BM and the solution’s success.

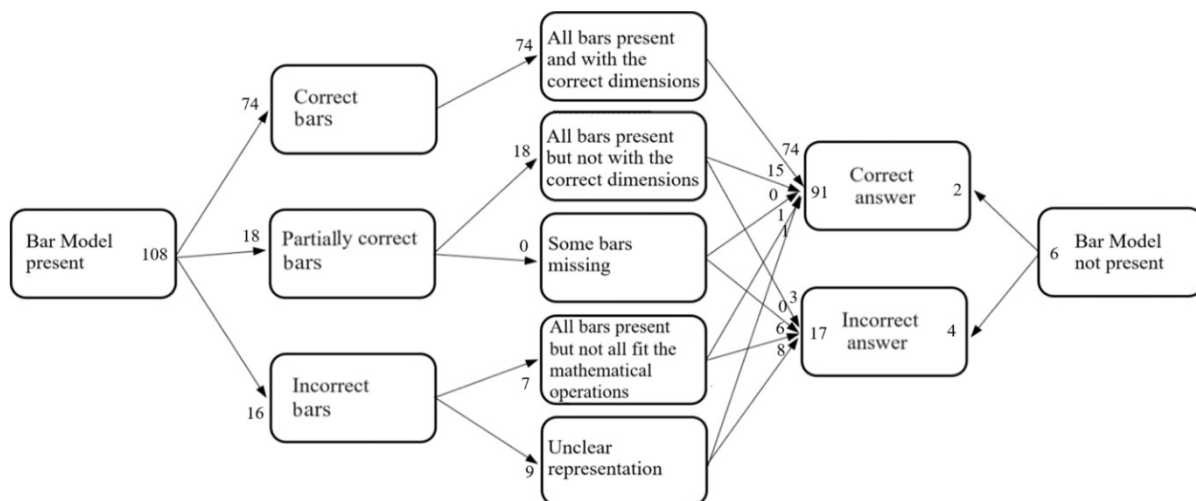


Fig. 4.28: Correctness of the solution related to the quality of BMs in the post-test (Cycle 2)

Table 4.12: The relation between the BM and the problem-solving success (Cycle 2)

BM	Incorrect solution	Correct solution	Total	%
Present	17	91	108	94.7
Not present	4	2	6	5.3

Fig. 4.28 shows that the solutions with a correct BM were always correct. When the BMs were partially correct, most of the solutions were correct (15 WPs from 18), which is a good indicator of the usage of the BM method for the solution of WPs. Due to the small number of pupils, more interpretation will be provided for the whole sample in Section 4.4.3.

4.2.7 Presence of the SIMPLE Steps in the Post-Test Solutions

The post-test WP solutions demonstrated the presence of the steps in 71.1% of cases, partial presence in 21% and absence in 7.9% (Table 4.13). For an example of the application of the SIMPLE steps, refer to Fig. 4.13, 4.14 and 4.15, respectively.

Table 4.13: The presence of the SIMPLE steps in WPs of the post-test (Cycle 2)

The SIMPLE steps	WP1_O	WP2_O	WP3_O	WP4_O	WP5_O	WP6_O	N=114	%
Present	15	14	11	15	13	13	81	71.1
Partially present	3	4	6	4	5	2	24	21.0
Not present	1	1	2	0	1	4	9	7.9

Table 4.14 shows the correctness of the solution in relation to the presence of indications of the SIMPLE steps' usage. The pupils consistently got the right answer if their solution included the presence of the SIMPLE steps (see Fig. 4.29). If there was no indication of the steps'

application, the majority of pupils got incorrect answers. Due to the small number of pupils, more interpretation will be provided for the whole sample in Section 4.4.3.

Table 4.14: The relation between the SIMPLE steps and the problem-solving success (Cycle 2)

The SIMPLE steps	Incorrect solution	Correct solution	Total	%
Present	0	81	81	71.7
Partially present	13	11	24	21.0
Not present	8	1	9	7.9

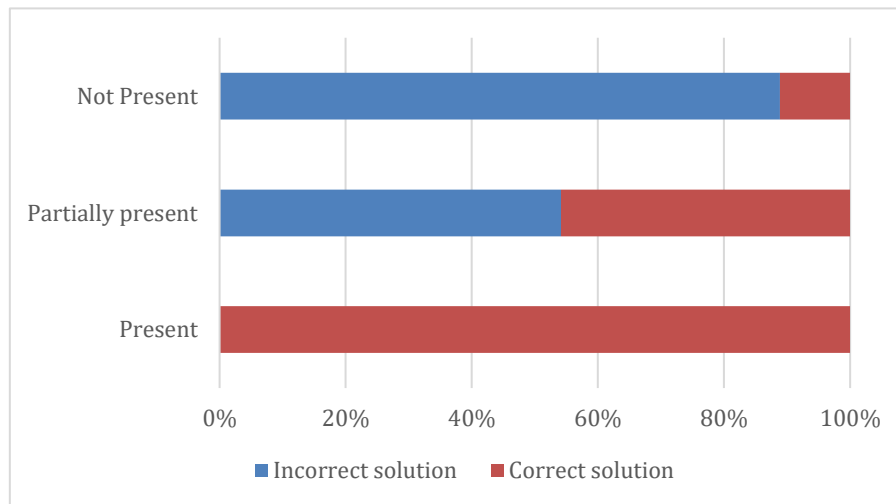


Fig. 4.29: Correctness of the answer related to the presence of SIMPLE steps (Cycle 2)

4.2.8 Preliminary Answers to Research Questions Based on Cycle 2

In this section, I will provide preliminary answers to the research questions based on the data from Cycle 2.

RQ1. How do pupils and teachers perceive the SIMPLE Approach's usefulness, and how confident are they with adopting it in WP solutions after the intervention?

The teacher perceived the new approach positively, reflecting a strong appreciation of its usefulness in solving WPs. She found it valuable not only for the pupils but also for her teaching practice. According to her, the BM method and the SIMPLE steps offer a clear and structured way of dealing with WPs, making it easier for her to guide pupils through the problem-solving process. The teacher's high motivation and willingness to learn and apply the approach were evident both during classroom observations and in her written diary. She was confident in its long-term usage. While doing the intervention with the 8th graders, she stated that she had also started incorporating the approach with the 7th graders, and they appreciated it, too.

The pupils also had a positive attitude toward the Approach, finding it helpful in solving WPs. They particularly enjoyed drawing BMs, which helped them visualise the WP and link it with mathematical operations. The interviewed pupils expressed a strong desire to keep using it in the future, recognising its importance in making problem-solving easier. However, they also noted a practical challenge: they got tired in the post-test by having to explain the steps in writing for all six WPs. Although they accepted that these steps were helpful for a deeper understanding, they asked to simplify their usage by allowing them to let some things do in their minds without writing everything on paper.

RQ2. What are possible conditions of and barriers to implementing the proposed intervention?

Two conditions supported the implementation of the intervention:

- The teacher's willingness to pause her regular curriculum and carry out the intervention, her strong motivation and enthusiasm to learn the new approach and to present it to her pupils, and her creation of a friendly environment where pupils felt free to participate in every aspect.
- The pupil's high engagement and interest in solving WPs through the new approach learned.

On the other hand, some barriers hindered the implementation of the intervention as planned.

- The pupils' lack of familiarity with WPs and strategies for their solution. Introducing them directly to the BM method and the SIMPLE steps was quite demanding.
- The teacher did not fully adhere to the Guide's instructions.
- Inconsistency with the principles of constructivism regarding reasoning through their mistakes.
- The absence of group work. In contrast to the Guide, the teacher allowed the pupils to choose whether they wanted to cooperate, and most pupils worked alone or with a friend sitting next to them.

RQ3. What impact does the intervention have on pupils' success when solving WPs? To what extent can they use the BM method and the SIMPLE steps in their solutions?

The comparison of the pre-test and post-test results reveals a notable positive impact of the intervention on pupils' success in solving WPs as there was an increase in the average success rate in the post-test. None of the pupils made visualisations in the pre-test, typically solving problems with direct arithmetic calculations.

The results of our study regarding the drawing of the BM method and the presence of SIMPLE steps are promising. Similar to Cycle 1, when pupils followed each SIMPLE step and drew correctly BMs, they consistently arrived at the correct answer. In cases where the BM or SIMPLE steps were not applied, a high percentage of solutions was incorrect.

Further analysis of each pupil's individual solutions revealed that most pupils achieved excellent scores in the post-test, demonstrating improvement after the intervention. There were a few cases in which pupils did not perform well in either test or had decreased performance after the intervention. For such pupils, a longer intervention period might be necessary.

4.2.9 Implication for Cycle 3

Cycle 2 confirmed the usefulness of some aspects of the intervention and uncovered some areas for improvement.

The continuation with a three-hour lesson on the BM method: It was found that adding one lesson to the BM lessons in the second cycle seemed to help pupils better grasp the basis of the BM method. Therefore, it is suggested to continue in the same way in Cycle 3.

Application of the SIMPLE steps by the teacher: Even though Teacher 2 was determined to follow the Guide's instructions, she deviated from the plan by not paying equal attention to all the steps. Teacher 3 will be asked to ensure balanced attention to all the steps when presenting and applying them in the WP solution and working with pupils.

Following the constructivist principles: Based on the observation of the teacher not encouraging pupils to explore and correct their errors, I realised that the Guide did not explicitly emphasise that. Therefore, for the next cycle, the Guide should be rewritten to add this aspect and Teacher 3 should be asked to involve pupils more actively in identifying and discussing the mistake.

Time constraints: Based on our observation and the pupils' reactions to the post-test WPs, it was noticed that they got tired by writing up all the steps for all the WPs. For Cycle 3, it is suggested that the intervention continues in the same way as in Cycle 2, the teacher asking pupils to apply all the steps in each WP. However, in the post-test, it is proposed that pupils include each step briefly but not write all the details. For instance, in the initial step, rather than rephrasing the WP in detail, they are asked to highlight key aspects of the WP. This practice can be applied throughout all the SIMPLE steps.

Collaborative Problem Solving: Similarly to Cycle 1, group work was not included in any intervention lesson and the teacher let the pupils choose their way of working, not encouraging collaborative learning. It is suggested that this deviation be carefully considered in the third cycle.

Intervention timing: For the next cycle, it is suggested to choose the class that has not covered the chapter on solving WPs with linear equations. We want to see if introducing the intervention before the pupils learn to solve the WPs with linear equations would help them understand the meaning of the BM method and SIMPLE steps more easily, potentially improving its effectiveness.

4.3 Cycle 3

The third intervention cycle was conducted at the primary school Pavaresia in the same city, Gjilan. The school was founded in 2012 and has 650 pupils. There are three levels of education: preschool education (pupils of age 4 to 6), elementary level (pupils of age 7 to 11) and lower secondary level (pupils of age 12 to 15). The school had three 8th-grade classes taught by two mathematics teachers. The teachers' choice was based on the school principal's suggestion. He specifically recommended this teacher because, according to him, she is more suitable to help conduct the intervention. The teacher teaches only one 8th-grade class. Therefore, the class choice became mandatory.

The class had 25 pupils (10 boys, 15 girls). Two pupils had special needs: one could not write and read, and the other had a problem with concentration and often left the lessons in the middle. It was observed that both needed particular time and attention, which could not be given because of the limited intervention time. As the goal of the study is to design teaching for usual heterogeneous classes and the number of pupils with special needs is very small, it was decided not to analyse the data for these pupils, even though they were present in all the lessons. So, in total, 23 pupils participated in our study (9 boys, 14 girls). They were all of Albanian nationality.

The class was taught by a female teacher with 12 years of experience teaching mathematics and informatics and this was her first year teaching in that class. She was ready to carry out the intervention. She believed that WPs are important in mathematics, so she felt happy to be part of this study and gain new insights. After being introduced to the SIMPLE Approach, she felt satisfied with all its aspects. According to her, as the visual part of the solution is not presented in any of the Kosovo textbooks, using this approach will arouse pupils' curiosity and help them

stay focused. She mentioned that she used various teaching methods in her regular classes, such as working in groups, putting pupils in the centre, and letting them explore, some of which align with the constructivist principles outlined in the Guide.

The teacher had no complaints about the lessons prepared and expressed a willingness to follow the Guide's instructions and implement them fully. She said she would try her best to contribute to this study and accepted the financial reward as an acknowledgement of her involvement.

Since the school year only started two months earlier, till then, Teacher 3 had not employed WPs in her mathematics lessons and pupils had not been introduced to their solution by using linear equations.

4.3.1 Changes in Cycle 3

The feedback from Cycle 2 led to the continuation of the three lessons for the BM method practice. Teacher 3 was instructed to give equal attention to each SIMPLE step when practising it in the WP solution to ensure a balanced understanding of the steps. She was explicitly encouraged to use group work when practising the WPs and involve pupils more actively in identifying and discussing the mistake. Finally, after ensuring that the pupils understood each of the SIMPLE steps during the intervention, in the post-test phase, they were allowed to write down only the essential elements rather than going into every detail.

4.3.2 Teaching Intervention

The Bar Model Method

The first intervention lesson was dedicated to familiarising pupils with the BM method and its types. To help with comprehension, the teacher introduced pupils to the types of BM by using three simple WPs with some elements in the classroom and demonstrated how each type of BM could be applied to the solution. For the part-whole type, she took an example: "I have three pens, two more than Ema has; how many pens are there?" The teacher drew the BMs on the blackboard and the pupils were asked to write everything that the teacher explained on paper. After that, she asked questions regarding their understanding and assigned the pupils WP1_I to solve, asking them to draw a BM while solving it. The pupils expressed their understanding of the lesson by discussing the drawing of the BM for these three examples and answering the teacher's questions (for example, "Did you understand when we should apply part-whole BM, comparative, and change BM?").

The pupils started to work on WP1_I. They started to make some drawings, but only three pupils drew BMs correctly and found the solution. The teacher let the pupils think for a bit, and when she saw them struggling, she explained on the blackboard how the BM for WP1_I can be constructed and how the answer can be found in it.

The teacher started the next lesson by recalling what they had learned in the previous lesson. Before starting to practise the BM method again, she asked pupils questions regarding the type of BM, like “Which type of BM did we learn last lesson?” or “In which type of WPs do we use the part-whole type of BM?”, etc. They answered quickly by mentioning the same example that the teacher took in the first lesson. One of the pupils raised her hand and told the teacher that she had practised BMs at home with her mother in some WPs and enjoyed doing it.

Afterwards, the teacher gave pupils three other WPs and asked them to work individually or in pairs with the friend next to them. The teacher went to each desk and monitored the pupils. One of the pupils’ solutions to WP3_I is presented in Fig. 4.30 right, in which the pupil wrote “Xhamie picked 17 flowers, while Linda 12. They gave 20 of the flowers”, drew the BMs and presented the arithmetic solution. After 10 minutes of working that way, the teacher rearranged pupils into a group of four. She then asked them to discuss solutions with each other, identify any mistakes and aid those who struggle to draw the proper BM. She went to each group’s desk to observe their work and further instructed those struggling to find the correct answer. It was noticed that the teacher did not provide the necessary feedback to all the pupils. In an informal chat during this lesson, the teacher said that the pupils were very active and that she could not give more extended feedback due to the limited time. She claimed that for this reason, she put them in groups so they could consult with each other occasionally.

The pupils were constantly engaged and appeared enthusiastic about applying the BM method. They repeatedly asked the teacher to go to their group and check whether the drawing and the solution were correct. When struggling with the drawing, they did not hesitate to ask questions, such as, *based on the sentence, should the bar be bigger or smaller? Or why should we make a smaller bar when in WP, this word says more books?* Although the teacher tried to address every question, she occasionally omitted some due to time constraints and the many questions that were raised. At the end of the lesson, she chose one of the pupils (P1_3) to explain the WP2_I solution on the blackboard in front of the class (Fig 4.30, left) and gave the pupils two WPs as homework.

The third lesson started with a discussion about homework solutions. The pupils seemed satisfied, as they claimed that the problems were simple, and they believed the solutions were correct. The pupils raised their hands to explain how they made the BMs. In this case, the teacher confirmed whether they were correct in their explanation and went to three or four pupils' desks to ensure that the solutions were done as described. She asked pupils to analyse each other solutions shortly, identify any mistakes presented and discuss them. The pupils admitted that they discussed the homework before the teacher came to class and improved any mistakes they had. Because of this, the teacher went straight ahead and assigned two additional WPs to practise the BM method.

Left side (WP2_I solution):

x	$\frac{1}{2}x + 4 = 12$
$\frac{1}{2}x$	$\frac{1}{2}x = 12 - 4$
$\frac{1}{2}x$ 4	$\frac{1}{2}x = 8$
	$x = 16$

Right side (WP3_I transcription):

1. Xhemi zgjodhi 17 lule, qe eshte 5 lule me shume nga ajo qe zgjodhi Linda. Ata dhuruan 20 nga lulet. Sa lule kishin mbetur?

L: Xhemi zgjodhi 17 lule nders Linda
 12. Ata dhuruan 20 nga lulet.

I 17 lule
 II 12 lule
 III 29 lule

I = 17
 II = 12
 III = 29 - 8 = 20 lule kane mbetur.

Fig. 4.30: Pupils' solution to WP2_I (left; the transcription of the text on the blackboard is provided) and WP3_I (right) (Cycle 3)

The teacher asked the pupils to use the same way of work, that is, starting alone or with a friend and then forming a group to discuss the solution. Then the teacher chose pupils P2_3 and P3_3 to write the solution on the board, with the rest of the class observing and helping if needed.

The SIMPLE Steps

The remaining two intervention lessons were dedicated to presenting the SIMPLE steps. The teacher explained all the steps by applying them in WP7_I. She involved pupils in the solution by including them in implementing each step. For example, for the first step, she asked the three to four pupils to rephrase the problem in a way that they understood, or for the second step, as they were already familiar with the BM method, she invited one pupil to draw it for

this WP. After the solution was achieved with every step, the teacher led a discussion in which pupils were asked to elaborate on anything they would like to add to the solution and whether they understood all the steps. The pupils responded by posing questions about specific steps they found challenging, such as “Can you repeat again what we should do in the last two steps?”. Some pupils expressed their understanding by summarising parts of the solution process in their own words, while others sought to give them some examples to reinforce their comprehension. Meanwhile, the pupils wrote everything on paper.

Afterwards, the teacher engaged the pupils in solving WP8_I. This time, she asked the pupils to work individually and provided them with the SIMPLE instructional sheet (Fig 3.9). Pupils started to work and seemed enthusiastic about using the steps. According to the teacher, even the pupils who usually are not active in mathematics lessons started to show interest in solving the WP. During this time, she monitored their solutions and helped those struggling by going to their desk and showing how some steps of the solution should be done.

Fig. 4.31 presents two of the pupils’ solutions for WP8_I. In Fig. 4.31 (left), the pupil initially rephrased the requirements of the WPs, identifying known and unknown quantities (Fig. 4.31 left: Emmy “has collected 137 stamps, that means 27 stamps less than her sister. How many stamps they collected together”; Fig. 4.31 right: “Emmy has collected 137 stamps, that means 27 stamps less than her sister. How many stamps they collected and how many are left”). Afterwards, the pupil constructed the BM; in the third phase, the pupil wrote the number of stamps Emmy collected, found the number of stamps for her sisters after adding Emmy’s one with 37 and denoted by x the number of stamps collected together. Following this, the pupil solved the WP and verified the solution, demonstrating its correctness, yet did not evaluate the answer. When the teacher asked about the absence of the final step, the pupil responded, *I believed the solution was correct and that no alternative approach was necessary to achieve the solution*. A similar solution was presented in Fig. 4.31 (right).

At the end, the teacher gave two WPs as homework asking them to use all the steps.

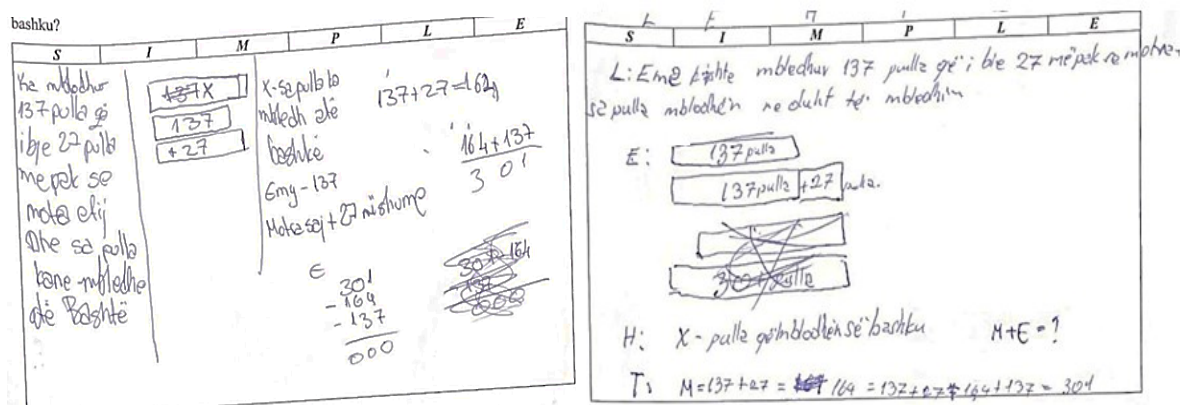


Fig. 4.31: The solution of WP8_I of the intervention using the SIMPLE steps (Cycle 3)

The first 10 minutes of the last intervention lesson were dedicated to discussing the homework solution. The teacher asked the pupils how they presented each step and how the answer was achieved. The pupils apparently understood the solution as they showed a great interest in answering these questions. Next, the teacher gave them the other four WPs to practise and asked them to work in pairs. After approximately 15 minutes, she formed five groups and asked pupils to work together and help each other. Pupils seemed satisfied with this arrangement, and some even mentioned that time was going faster when they were engaged in such problems.

Finally, the teacher's interest in introducing the new approach to other pupils outside the class was noticed. She invited three 9th-grade pupils to participate in the lesson with the SIMPLE steps. She believed that this approach would help them prepare for mathematics competitions. The data of these three pupils were not analysed as they were not part of all the lessons.

Adhering to the Principles of Constructivism

Teacher 3 showed great interest in conducting the intervention. She was very careful in following the Guide instructions, ensuring that each lesson was delivered as planned.

For each lesson, the teacher began by explaining and giving clear instructions to the pupils and practising some WPs with them. Whenever she was sure that the pupils understood the concepts, she stepped aside and let them practise individually and in groups, fostering both independent and group learning. She created a positive environment in the classroom by motivating her pupils to ask questions and discuss the WP solutions. Her teaching approach, which balanced individual and group work, and her ability to foster an open discussion environment greatly benefited the pupils' learning. As Teacher 3 encouraged pupils to work in groups, they were also encouraged to discuss their solutions and learn from each other's

mistakes. It was observed during the lessons that the teachers encouraged them specifically to see each other's works and identify any possible errors they made and discuss them.

Rare moments were observed when the teacher could not give the necessary feedback due to the lack of time. The pupils were quite active, asking many questions, which she sometimes could not answer. Despite these challenges, it can be concluded that the Guide was largely adopted by the teacher. Even though the teacher had not previously used such approaches for solving WP, she quickly adapted and correctly conveyed them to the pupils.

The successful implementation of the constructivist teaching approach was apparently eased by the fact that the teacher had already adopted pupil-centred teaching in her work.

4.3.3 The Teacher's View: Teacher Diary and Interviews

The teacher's perceptions of the BM method and the SIMPLE steps became clearer after the interview at the end of the study. The teacher acknowledged that she did not frequently use WPs and was not very familiar with strategies for solving them. Despite this (or maybe because of that), she showed a great interest in the new approach, stating that she would like to apply it in her regular teaching practice.

Researcher (R): *How tough was presenting the SIMPLE Approach to the pupils?*

Teacher (T): *It wasn't difficult because I explained it step by step. We started by practising the BM method and then continued to the SIMPLE steps. I think that even the pupils grasped them quickly.*

The teacher emphasised her belief that this approach was beneficial for pupils as she noticed great engagement even in the pupils who were not active in her regular lessons:

T: *I would like this approach to be included in the textbooks or literature on mathematics. I believe that pupils would benefit a lot from it. This approach is also attractive as it increases pupils' curiosity by drawing bar models.*

Her enthusiasm for the steps' effectiveness was apparent when she invited three 9th-grade pupils to learn the steps, believing it would aid them in the competition.

In response to the question of whether there was anything she would suggest changing about the steps, she responded that every step plays an important role in the solution and would not change any part of it. She observed that combining the BM with the other steps improved learning. This statement was also reflected in her diary, noting her intention to use the approach frequently and advocating its inclusion in textbooks (Fig. 4.32).

Date: 13.11.2023	Date: 20.11.2023
Bar Model method – Lesson 1	SIMPLE strategy – Lesson 4
Describe the lesson learned	Describe the lesson learned
I started the first lesson by presenting the pupils with the bar model method. I described its type and took an example for each of the types.	In the fourth lesson, I presented the pupils with the SIMPLE steps and its benefits in the WP solution.
How the intervention went	How the intervention went
The intervention went well. After explaining the bar model method, we started discussing its application in the WP solution with the pupils. They raised some questions, and we tried to elaborate on the answers together, as the method was also new to me. After that, the pupils had to solve one WP and were asked to use the bar model method in its solution. They were allowed to ask questions and collaborate together with other pupils. In the end, one of the pupils came to the board and explained the drawings and the solution.	The lesson went wonderfully, as the pupils understood very quickly the steps. First, we modelled each step in one WP by explaining where each of them can be applied within the solution. After that, I assigned the pupils a WP and asked them to use the SIMPLE steps in the solution. Pupils worked individually but were free to ask questions if they needed to. I noticed that even the pupils who were not very interested in mathematics began to show interest and engage in this lesson. I think that the approach grew their curiosity to learn.
Suggestion/Opinions/New idea	Suggestion/Opinions/New idea
I have no suggestion; I appreciate this method and think it is very beneficial for pupils.	I hope this approach will be incorporated into the mathematics literature and textbooks because it is logical and understandable for pupils. I will not change anything, but I want to add that such an approach has not been practised before, and I will continue to practice it in the future.

Fig. 4.32: Two of Teacher 3's diary entries for lessons on the BM method and SIMPLE steps (translated from Albanian)

4.3.4 Pupils' View: Interviews

A positive attitude toward the new approach was again observed in the pupils' interview. The pupils claimed that they usually did not use WPs in their teaching. However, they liked solving such WPs and found solutions more easily through the BM and the SIMPLE steps.

Pupil A: At first, I thought using all the steps was challenging, but after we practised two word problems, it helped me understand the solution better and solve the problem logically.

Pupil B: I would not have known how to solve the 3rd and 4th word problem if I had not used the bar model method. I understood how much I should add and subtract when I drew them.

Pupils C: *The SIMPLE steps simplified the solution even though there were many to use. However, as we had not learned any strategy for the solution before, I really liked this one and will use it again.*

R: *Would you continue to use it in the future?*

Pupil B: *I plan to participate in the mathematics competition Kangaroo next year. I will practice it until then, and I will definitely use it.*

Pupil A: *Yes, especially the drawing of bars I will use a lot.*

4.3.5 Pre- and Post-Test Results

The results of the total 138 solutions were analysed based on the same criteria as in the two previous cycles. The average success rate for all the WPs was 37%. The distribution of scores among the problems in the pre-test is detailed in Table 4.15. Regarding consistency, the data show that in inconsistent WPs, the solutions were less accurate compared to the consistent ones, with a difference of around 13 percentage points.

Table 4.15: The results from the pre-test (Cycle 3), inconsistent problems in bold

Pupils' solution success	WP1_E	WP2_E	WP3_E	WP4_E	WP5_E	WP6_E	N	%	Cons. %	Incon. %
Incorrect solutions	18	13	11	8	18	19	87	63.0	56.4	69.6
Correct solution	5	10	12	15	5	4	51	37.0	43.6	30.4
Total	23	23	23	23	23	23	138	100	100	100

Three of the 23 pupils who participated were included in the category of achievement “excellent”, nine of them were included in the category of achievement “good”, and 11 were in the category of achievement “insufficient”. None of the pupils presented any drawings for the solution. They mainly solved the WPs directly, but in a few cases, they approached them algebraically by assigning variables to unknown quantities. Although they had not learned how to use linear equations in solving WPs, they had been exposed to basic algebraic reasoning in simpler tasks and thus some used an algebraic strategy (Table 4.16).

Table 4.16: Pupils' strategies used in solving WPs of the pre-test (Cycle 3)

Strategies used	Arithmetic calculation	Algebraic steps	Direct or no solution	Total
Number of solutions to WPs	110	6	22	138

Problem WP6_E, an inconsistent WP, will be used to illustrate pupils' work (the text is in Table 3.2). Two examples of a correct solution are in Fig. 4.33. For WP6_E, 19 out of 23 solutions were incorrect, which indicates that pupils did not understand the WP properly. Five pupils left their solution blank, while others made errors mainly because they misinterpreted the inconsistent expression "5 more toys" (Fig 4.34).

6. Ema bleu 22 lodra për fëmijë, që është 5 lodra më shum nga ajo që bleu Andrra. Ato dhuruan 22 nga lodrat e tyre. lodra kanë mbetur pa u dhuruar? Ema → 22 lodra Andrra → $22 - 5 = 17$ lodra $22 + 17 = 39$ 22 i dhuruan 17 u mbetur	6. Ema bleu 22 lodra për fëmijë, që është 5 lodra më shum nga ajo që bleu Andrra. Ato dhuruan 22 nga lodrat e tyre. lodra kanë mbetur pa u dhuruar? Ema: 22 lodra Andrra: $22 - 5$ lodra = 17 lodra $\begin{array}{r} 22 \\ + 17 \\ \hline 39 \end{array}$ 39 lodra bashkë $39 - 22 = 17$ lodra të mbetura
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Fig. 4.33: Two pupils' correct solution for the pre-test WP6_E (Cycle 3)

6. Ema bleu 22 lodra për fëmijë, që është 5 lodra më shum nga ajo që bleu Andrra. Ato dhuruan 22 nga lodrat e tyre. lodra kanë mbetur pa u dhuruar? $\begin{array}{r} 22 \\ - 5 \\ \hline 17 \end{array}$ Mbetur 5 pa u dhuruar Pasi që $17 + 5 = 22$ X	6. Ema bleu 22 lodra për fëmijë, që është 5 lodra më shum nga ajo që bleu Andrra. Ato dhuruan 22 nga lodrat e tyre. lodra kanë mbetur pa u dhuruar? Kërkë mbetur pa u dhuruar 5 lodra.
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Fig. 4.34: Two pupils' incorrect solutions for the pre-test WP6_E (Cycle 3)

The post-test results show progress compared to the pre-test results, with a 34 percentage points increase in the average success rate from 37% in the pre-test to 71.7% in the post-test (see Table 4.17).

Table 4.17: The results from the post-test (Cycle 3), inconsistent problems in bold

Pupils' solution success	WP1_O	WP2_O	WP3_O	WP4_O	WP5_O	WP6_O	N	%	Cons. %	Incon. %
Incorrect solutions	6	7	3	9	5	9	39	28.3	26.2	30.4
Correct solution	17	16	20	14	18	14	99	71.7	73.8	69.6
Total	23	23	23	23	23	23	138	100	100	100

The number of incorrect solutions decreased in both consistent and inconsistent WPs. From approximately 69.6% of the inaccuracy of inconsistent WP solutions in the pre-test, this percentage was reduced to 30.4% in the post-test. Therefore, it can be assumed that the pupils can benefit from the BM method and SIMPLE steps even when WPs are inconsistent.

Fig. 4.35 compares the results of the pre-test and the post-test of Cycle 3 via box plots, which provide more detail. The pupils correctly solved, on average, 2.22 WPs in the pre-test and 4.26 WPs in the post-test (out of a total of 6 WPs). Half of the pupils correctly solved 2 WPs or less in the pre-test, while in the post-test, 8 pupils (out of 23) solved 5 WPs correctly and 5 pupils solved all 6 WPs correctly. The plot on the right shows a slightly bigger gain for the inconsistent WPs (INC) than for the consistent ones (CON).

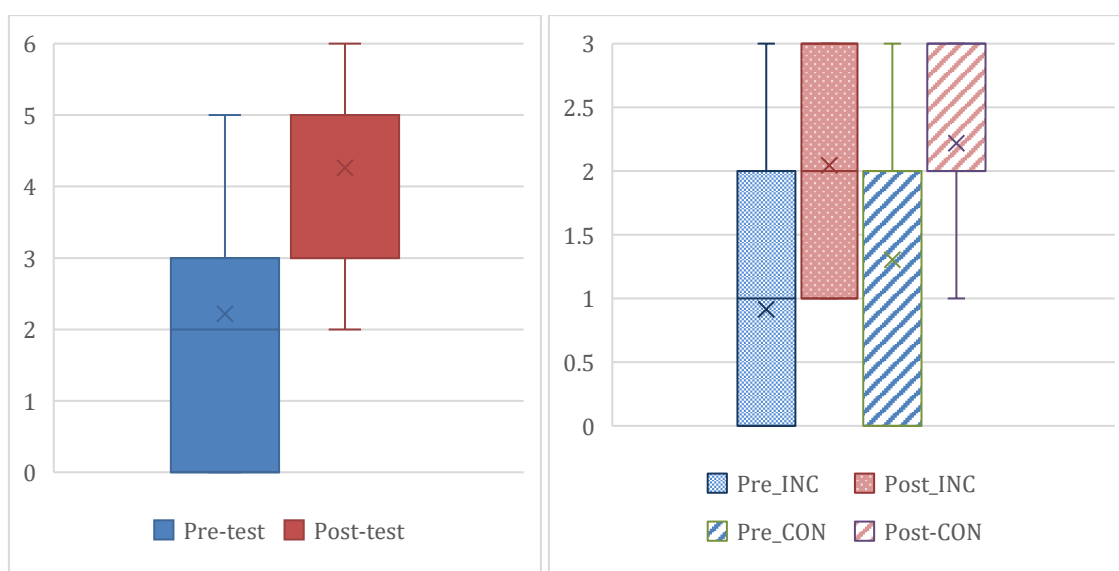


Fig. 4.35: Box plots for the number of correctly solved WPs in the pre-test and the post-test (Cycle 3, x – mean, horizontal line segment – median)

Figure 4.36 illustrates the movement of pupils between three achievement categories: “insufficient”, “good”, and “excellent” for WPs in the pre-test and post-test (based on Table 3.8, Section 3.9.1). In the post-test, 12 pupils improved after the intervention. Four pupils went from the achievement categories “good” to “excellent”, three from the achievement categories “insufficient” to “good”, and five went from the achievement categories “insufficient” to “excellent”. Three of the pupils were included in the achievement category “excellent” in the pre-test and remained in the achievement category “excellent” in the post-test. A deeper analysis was needed for three whose achievement was categorised as insufficient even at the end of the intervention, see the bold arrows in Fig. 4.36. Their work will be analysed in Section 4.4.3. The pupils whose achievement is represented by the bold arrow warrant our attention.

We will analyse their solutions in more detail in Section 4.4.3, together with the solutions of pupils from the other two cycles.

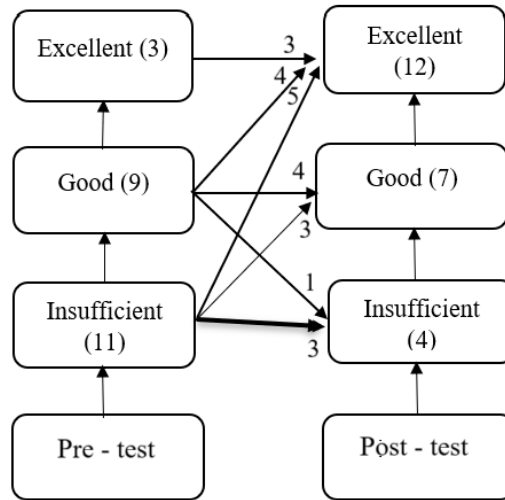


Fig. 4.36: Movements in pupils' achievement categories (insufficient, good, excellent) from pre-test to post-test (Cycle 3)

4.3.6 Presence of BMs in the Post-Test Solutions

The results in Table 4.18 and Table 4.19 indicate that pupils used BMs in 91.3% of solutions.

Table 4.18: The presence of the visualisation within the WP's solution of post-test (Cycle 3)

Bar Model	WP1_O	WP2_O	WP3_O	WP4_O	WP5_O	WP6_O	Total	%
Not Present	2	3	1	3	1	2	12	8.7
Present	21	20	22	20	22	21	126	91.3

Table 4.19: The relation between the BM and the problem-solving success (Cycle 1)

BM	Incorrect solution	Correct solution	Total	%
Present	99	27	126	91.3
Not present	12	0	12	8.7

Based on the framework for categorising pupils' solutions in terms of BMs and correctness of the answers (Fig. 3.7, Section 3.9.1), Fig. 4.37 presents the results for the post-test Cycle 3. It shows that the solutions with a correct BM were always correct. When the BMs were partially correct, most of the solutions were correct (20 solutions out of 35) which is a good indicator of the usage of the BM method for the solution of WPs. Due to the small number of pupils, more interpretation will be provided for the whole sample in Section 4.4.3.

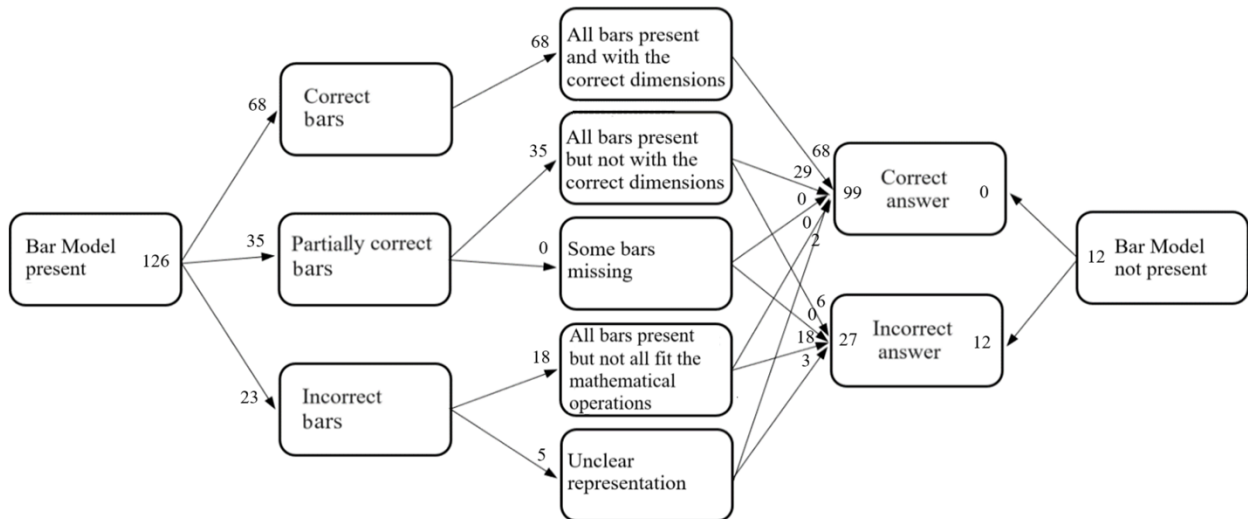


Fig. 4. 37: Correctness of the solution related to the quality of BMs in the post-test (Cycle 3)

4.3.7 Presence of the SIMPLE Steps in the Post-Test Solutions

Regarding the SIMPLE steps, the findings show that 46.4% of pupils presented the steps in the solution, 42.7% partially presented them, and 10.9% did not indicate their presence (Table 4.20). For an example of the application of the SIMPLE steps, refer to Fig. 4.13, 4.14 and 4.15, respectively.

Table 4.20: The presence of the SIMPLE steps in WPs of the post-test (Cycle 3)

The SIMPLE steps	WP1_O	WP2_O	WP3_O	WP4_O	WP5_O	WP6_O	N=138	%
Present	14	13	7	10	13	7	64	46.4
Partially present	8	7	15	9	9	11	59	42.7
Not present	1	3	1	4	1	5	15	10.9

Table 4.21 shows the correctness of the solution in relation to the presence of the SIMPLE steps.

Table 4.21: The relation between the SIMPLE steps and the problem-solving success (Cycle 3)

The SIMPLE steps	Incorrect solution	Correct solution	Total	%
Present	0	64	64	46.4
Partially present	27	32	59	42.7
Not present	12	3	15	10.9

The results indicate that when pupils were presented with the SIMPLE steps, they constantly got the right answer, representing 46.4% of the total. In most cases where there was no

indication of their usage, they gave incorrect answers. Due to the small number of pupils, more interpretation will be provided for the whole sample in Section 4.4.3.

Fig. 4.38 shows the proportion of correct and incorrect answers based on the presence of the steps for the categories identified in the post-test.

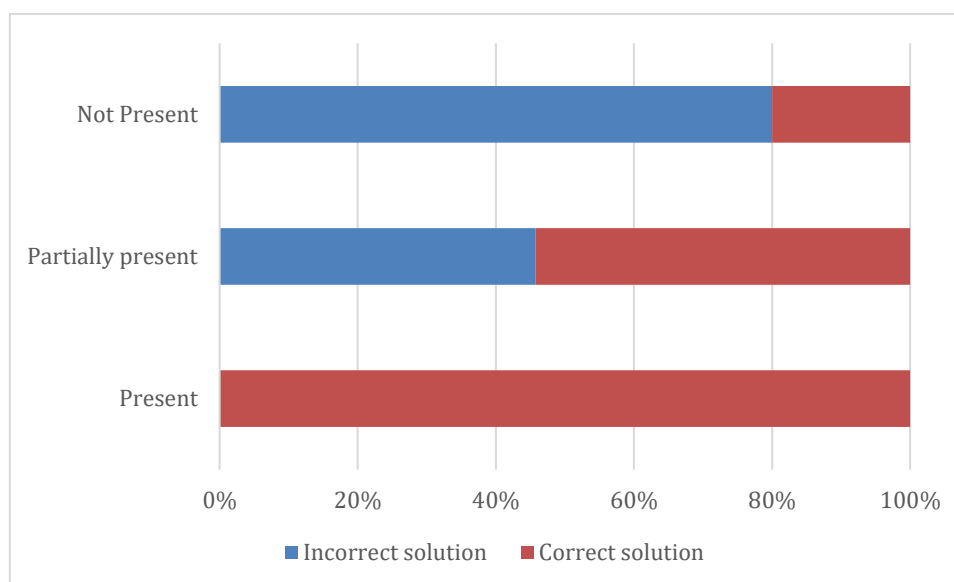


Fig. 4.38: Correctness of the answer related to the presence of SIMPLE steps (Cycle 3)

4.3.8 Preliminary Answers to Research Questions Based on Cycle 3

RQ1. How do pupils and teachers perceive the SIMPLE Approach’s usefulness, and how confident are they with adopting it in WP solutions after the intervention?

The new teaching approach was well-received by both the teacher and pupils. After being instructed by me and reading the Guide, the teacher was able to deliver the intervention. She highly valued BMs and the SIMPLE steps and believed they helped pupils solve WPs. According to her, even though the pupils were unfamiliar with BMs, they did not find it difficult to learn the new methods. She believed this approach made the lessons attractive, especially when they had to draw BMs.

Her appreciation of this approach was even more evident when she invited three other pupils from the 9th grade who were chosen to participate in the school mathematics competitions to follow the lessons. According to her, this approach can help these pupils as it develops their logical thinking. She further stated that she wished this approach could be included in elementary school mathematics textbooks from the early grades. Even if this does not happen, she appreciated its usefulness and emphasised that she would continue using it.

The pupils also appreciated this approach. They stated that even if it was something new, they were very interested in the approach and would continue using it in the future in solving WPs. According to them, this approach becomes even easier after constantly practising it. My observations of the lessons confirm that pupils valued the strategy's usefulness and were eager to use it in every WP.

RQ2. What are possible conditions of and barriers to implementing the proposed intervention?

Similarly, as in the previous two cycles, two main conditions helped the implementation of the proposed intervention.

- The teacher's motivation to learn and convey the new approach to her pupils and her willingness to pause the current curriculum to prioritise the proposed intervention.
- The teacher's usual way of teaching, similar to the constructivist principles noted in the Guide.
- The pupil's high engagement and interest in learning the new approach to the solution of WPs

Teacher 3 followed the Guide to a great extent with minor omissions, but some barriers were identified.

- The teacher could not respond to all of the pupils due to the pupils' high engagement during the intervention lessons and the many questions they asked. The teacher's feedback could have been more extensive for all the pupils.

This class had two pupils with special needs. The teacher attempted to teach the lessons individually to them but was unsuccessful. Since there was no assistance for these pupils in that class, teaching this approach to these pupils was almost impossible. Therefore, it cannot be said that our intervention was inclusive to all the pupils present.

RQ3. What impact does the intervention have on pupils' success when solving WPs? To what extent can they use the BM method and the SIMPLE steps in their solutions?

In the third cycle, the intervention positively impacted pupils' success in solving WPs. The average success rates advanced the most among the cycles. This progress might indicate that the continuous improvement and refinement of the intervention help to gain better results. However, among others, it must also be considered that the three classes differed in their baseline achievement at the beginning of the intervention and the teacher used the student-centred approach in her usual teaching, too.

Applying the BM method and the SIMPLE steps also produced positive results. Most of the pupils were able to describe them correctly and always reached an accurate solution. From the individual analysis of pupils' success, most pupils rose from the achievement category "insufficient" to "good" or "excellent" or from the achievement category "good" to "excellent", meaning that more than half of them improved after the intervention.

4.4 Synthesis of Findings across Cycles

Each cycle offered novel insights and contributed to a deeper understanding of the impact of the intervention on developing pupils' problem-solving skills. In this section, the findings across the cycles will be synthesised to answer RQs.

4.4.1 How Do Pupils and Teachers Perceive the SIMPLE Approach's Usefulness, and How Confident Are They with Adopting It in WP Solutions after the Intervention?

The three teachers had positive attitudes towards the SIMPLE Approach from the beginning and appreciated its benefits at the end of the intervention. All reported that the approach was new to the pupils and that conveying it to them in such a short timeframe was not easy. All could see an increase in pupil involvement and accuracy in solving WPs (which is confirmed by pupils' results in the post-test; see Section 4.4.3).

The first teacher seemed to be less confident in his use of the SIMPLE Approach. For example, he often preferred to return to his way of solving WPs through linear equations rather than helping pupils see how BMs could facilitate their solution. The second and third teachers were more confident, which is evidenced by the fact that they started incorporating the new approach with pupils outside the intervention classes. They were determined that they would use the new approach in the future and said they would like to include this approach in elementary school mathematics books.

As the lesson observations showed, the pupils, in general, seemed engaged during the lessons. The ones who were interviewed admitted that they had difficulties applying the BM method initially, but after some time and practice, they liked it and were confident in using it. They expressed a desire to keep using BMs and the SIMPLE steps in the future, recognising their importance in making their problem-solving easier.

4.4.2 What Are the Possible Conditions and Barriers to Implementing the Proposed Intervention?

The study revealed common conditions across three cycles that helped implement the proposed intervention and uncovered some barriers.

From the pupils' perspective, they seemed interested in learning the new approach and tried to do their best while learning and practising. One significant barrier for the pupils, specifically in Cycle 1, and less so in Cycle 2, was their prior familiarity with solving WPs using linear equations. Since the pupils were accustomed to this way of solution, introducing the BM method with SIMPLE steps posed challenges for them. Furthermore, from the lesson observations, it was noticed that pupil-centred learning, especially in the first cycle, was another challenge for the pupils since they lacked experience with it. This was evident in how pupils relied on the teacher for guidance and struggled to adapt to activities requiring independent thinking. Over time, with encouragement from the teacher, the pupils gradually adjusted their way of learning. However, this adaptation took longer for the first teacher's class, suggesting that the shift to pupil-centred learning requires sustained effort and practice.

From the teacher's perspective, an important condition was their appreciation of the SIMPLE Approach and their willingness to pause their regular curriculum and carry out the intervention. However, there were moments when the fidelity of the intervention was compromised in terms of the content (introduction of BMs and SIMPLE steps) and pupil-centred teaching. The following differences between the intervention-as-implemented and the intervention-as-designed (Nelson et al., 2012) were identified:

- Introducing BM types without visual illustrations (Teacher 1)
- Missing opportunities to initiate discussions and explore pupils' misunderstandings (Teacher 1)
- Failing to associate BM types with the inner structure of the WP (Teacher 1)
- Failing to use visual representations to explain the solution; resorting to algebra (Teacher 1)
- Little group work (Teacher 1, Teacher 2)
- Failing to give the same emphasis to all SIMPLE steps (Teacher 2)
- Introducing pupils to the types of bar models by using three simple WP with some objects in the classroom (Teacher 3)

- Failing to respond to all of the pupils' questions and give more extensive feedback (Teacher 3)

As for the content, the fidelity was high for the third teacher, whereas the first teacher did not adequately present the types of BMs, and the second teacher neglected some SIMPLE steps. Furthermore, the first teacher often preferred to return to the solution of WPs through linear equations rather than helping pupils see how BMs could facilitate their solution. The same can be said regarding implementing the constructivist ways of teaching – a gradual improvement can be seen among the cycles. At least two causes can account for that.

First, there were differences in the teachers' initial training. In the first cycle, as there was still no feedback from the intervention implementation, the training primarily relied on the Teacher Guide, which may have underestimated the preparation needed for successful implementation. The first teacher admitted that introducing the SIMPLE Approach and adapting the way of solving WPs was challenging for him and the pupils, especially as they had just learned to solve WPs using linear equations. Therefore, the initial training plays an important role. It is perhaps obvious that a teacher's knowledge of the tools they present to their pupils is paramount for the success of the teaching. Thus, it is recommended that future initial training be enhanced by incorporating practical demonstrations enabling teachers to engage directly with the bar model method and the SIMPLE steps. Furthermore, the inclusion of more detailed examples in the Guide that illustrate the transition from traditional methods, such as solving with equations, to the SIMPLE Approach could offer more precise guidance. Regular monitoring and follow-up training sessions throughout the implementation phase would ensure that teachers are adequately supported and confident in adopting the new method.

Second, the teachers' usual teaching style played an important role. Even though we can infer only indirectly what their teaching styles were (from the interviews and observations), we can say that the first teacher's teaching style was quite different from the required constructivist underpinning of the intervention. The second teacher's approach aligned better with pupil-centred teaching. However, she did not initiate group work, which the Teacher Guide advocated. One can argue that group work is not an indispensable way of working within the intervention; pupils can take an active part in their learning when working individually or in pairs. The third teacher's approach fitted very well with the constructivist approach. Thus, while the second and the third teachers had to master the BM method and the SIMPLE steps only, the first teacher had to change his way of teaching substantially.

These conditions and barriers were also influenced by several factors that were out of my control, such as class composition, pupils' characteristics, prior knowledge of WPs, etc. Therefore, these factors also influence the intervention's implementation and the cycles' results.

4.4.3 What Impact Does the Intervention Have on Pupils' Success When Solving WPs? To What Extent Can They Use the BM Method and the SIMPLE Steps in Their Solutions?

In the sections devoted to the three cycles, pupils' solutions to the pre-test, post-test and intervention WPs were analysed, and results pertinent to a single class were presented. In this section, the same will be done for the whole sample.

First, the results pertinent to the internal consistency of the tests will be presented. The correlation coefficients between individual WPs and the total score were between 0.41 and 0.65 for the pre-test and between 0.42 and 0.75 for the post-test, indicating moderate to strong positive relationships. Cronbach's alpha, which measures internal consistency, was 0.60 for the pre-test and 0.68 for the post-test, indicating acceptable reliability given the small sample size of six problems.

Pre-Test Results for the Whole Sample

The data included 372 solutions, in total, from 62 pupils to the pre-test WPS. The average success rate for all the WPs was 47.8%. Table 4.22 details the number of correct and incorrect solutions for the pre-test across three cycles. Table 4.22 shows that pupils of Cycle 3 had weaker results compared to the previous two cycles. A possible cause could be the fact that they did not cover the chapter on WPs with the linear equation before the intervention contrary to the two previous classes.

Table 4.22: The distribution of pre-test results across three cycles

	Cycle 1		Cycle 2		Cycle 3		Total	
Pupils' solution success	N	%	N	%	N	%	N	%
Incorrect solutions	53	44.2	54	47.4	87	63.0	194	52.2
Correct solution	67	55.8	60	52.6	51	37.0	178	47.8
Total	120	100	114	100	138	100	372	100

Table 4.23 shows that inconsistent WPs were more difficult for the pupils than consistent WPs in all cycles, with nearly 54% of correct solutions for the consistent ones and less than 42% for

the inconsistent ones. WP5_E was the most difficult for pupils across cycles. The reason might be that it requires understanding comparative statements involving multiple relationships which is cognitively more demanding for pupils at this level. On the other hand, WP4_E was the easiest for pupils. Contrary to WP5_E, WP4_E is an arithmetic-based problem that follows a straightforward computational process.

Table 4.23: Pre-test results across three cycles for each WP, inconsistent problems in bold

Pupils' solution success	WP1_E	WP2_E	WP3_E	WP4_E	WP5_E	WP6_E	N	%	Cons. %	Incon. %
Incorrect solutions	39	23	32	17	46	37	194	52.2	46.2	58.2
Correct solution	23	39	30	45	16	25	178	47.8	53.8	41.8
Total	62	62	62	62	62	62	372	100	100	100

Of the 62 pupils who participated in the study, 14 were included in the category of achievement “excellent” in the pre-test, 30 belonged to the category of achievement “good”, and 18 were in the category of achievement “insufficient”. It must be emphasised that none of the pupils used visualisation or presented any drawing to solve WPs in the pre-test. Their preferred method was using arithmetic calculation or algebraic steps (Table 4.24).

Table 4.24: Pupils' strategies used in solving WPs of the pre-test

Strategies used	Arithmetic calculation	Algebraic steps	Direct or no solution	Total
Number of solutions to WPs	281	51	40	372

Post-Test Results for the Whole Sample and Their Comparison to the Pre-Test

In total, the data included 372 solutions to the post-test WPS from 62 pupils. The post-test results show progress in the solutions' correctness compared to the pre-test. The average success rate for all the WPs was 75.5% (Table 4.25).

Table 4.25: The distribution of post-test WP results across cycles

	Cycle 1		Cycle 2		Cycle 3		Total	
Criteria/Task	N	%	N	%	N	%	N	%
Incorrect solution	31	25.8	21	18.4	39	28.3	91	24.5
Correct solution	89	74.2	93	81.6	99	71.7	281	75.5
Total	120	100	114	100	138	100	372	100

The difficulty of all the WPs in the post-test was quite similar (Table 4.26). The inconsistent WPs were still more difficult for the pupils, but the difference from the consistent ones was only 5 percentage points (as opposed to 12 percentage points in the pre-test).

Table 4.26: The results from the post-test across three cycles, inconsistent problems in bold

Pupils' solution success	WP1_ O	WP2_ O	WP3_ O	WP4_ O	WP5_ O	WP6_ O	N	%	Cons. %	Incon. %
Incorrect solutions	11	17	11	18	15	19	91	24.5	22.0	27.0
Correct solution	51	45	51	44	47	43	281	75.5	78.0	73.0
Total	62	62	62	62	62	62	372	100	100	100

Fig. 4.39 compares the results of the pre-test and the post-test for the whole sample via box plots, which provide more detail. The pupils correctly solved, on average, 2.9 WPs in the pre-test and 4.5 WPs in the post-test (out of a total of 6 WPs). The median is 3 WPs in the pre-test and 5 WPs in the post-test. Half of the pupils correctly solved 3 WPs or less in the pre-test, while in the post-test, it was only one quarter. The plot on the right shows a bigger gain for the inconsistent WPs (INC) than for the consistent ones (CON).

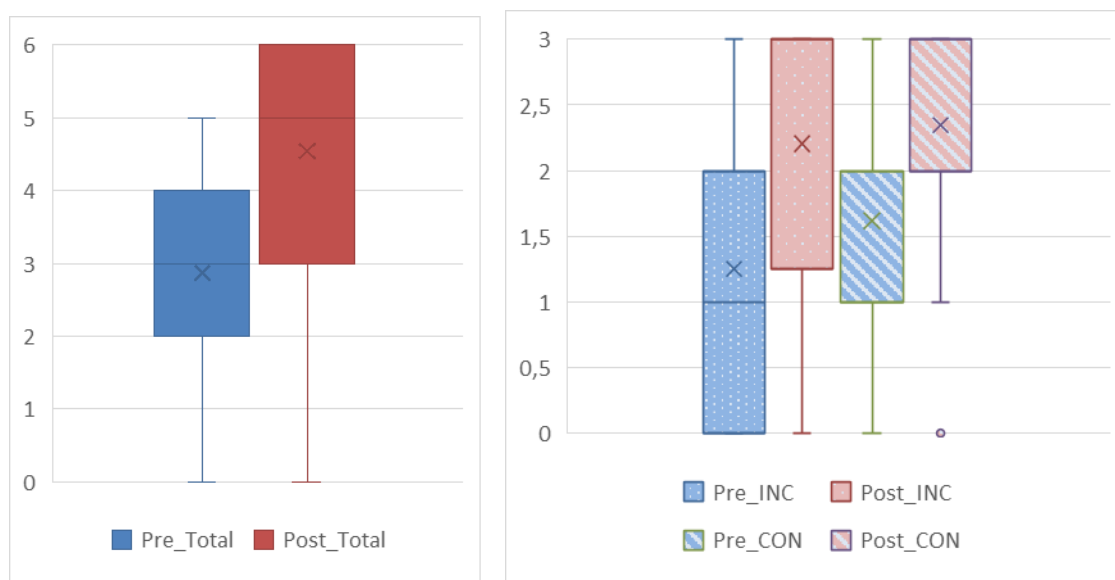


Fig. 4.39: Box plots for the number of correctly solved WPs in the pre-test and the post-test for the whole sample (x – mean, horizontal line segment – median)

Fig. 4.40 presents the same data for each cycle. There is an increasing trend in the solution's success from Cycle 1 to Cycle 3. The pupils in Cycle 1 exhibited the lowest increase in their success scores from pre-test to post-test and the pupils in Cycle 3 the biggest.

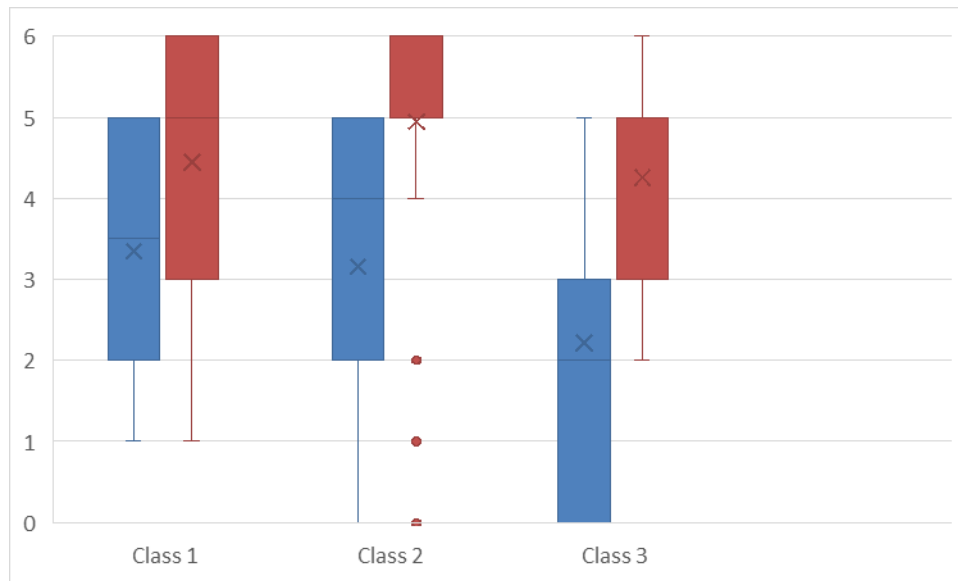


Fig. 4.40: Box plots for the number of correctly solved WPs in the pre-test and the post-test for each cycle

Achievement Categories in the Pre-Test and the Post-Test for the Whole Sample

Next, the progress of individual pupils from the pre-test to the post-test for all cycles will be summarised.

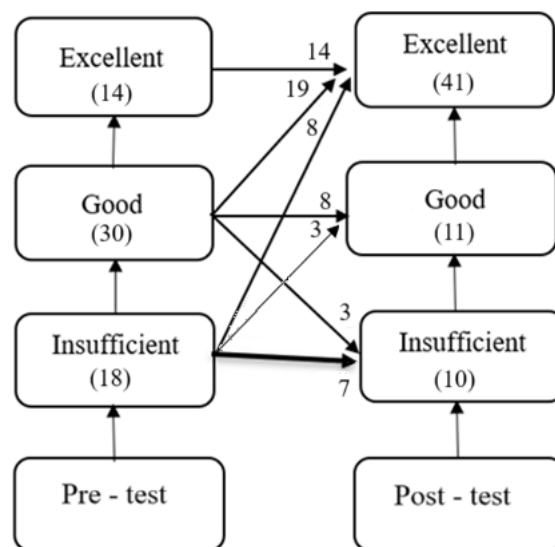


Fig. 4.41: Movements in pupils' achievement categories (insufficient, good, excellent) from pre-test to post-test across three cycles

Fig. 4.41 illustrates the movement of pupils between three categories of achievement: “insufficient”, “good”, and “excellent” for WPs in the pre-test and post-test. In the post-test, 41 pupils were included in the category of achievement “excellent” in their solutions. 14 were identified in the category of achievement “excellent” from the beginning to the end of the

interventions, while 27 improved after the intervention (8 who moved from the category of achievement “insufficient” to the “excellent”, and 19 from the category of achievement “good” to the “excellent”). Three pupils moved from the category of achievement “insufficient” to “good”, which is considered an improvement again. Seven pupils remained in the category of achievement “insufficient”, eight in the category of achievement “good” and three moved from the category of achievement “good” to “insufficient”. Fig. 4.44, 4.45 and 4.46 present examples of solutions for pupils in different category groups.

Pupils who were categorised in the achievement category “excellent” in both the pre-test and post-test demonstrated a ceiling effect, meaning they had no space to show improvement in the post-test.

A notable group included pupils who remained in the insufficient achievement category. Even after receiving lessons on the BM and SIMPLE steps, the analysis of their solutions shows that they continued to rely on arithmetic solutions. Their use of BMs was often superficial—making frames around numbers rather than using BMs to structure their solution. The most common mistakes involved incorrect application of mathematical operations, especially in WPs with inconsistent structures (Fig. 4.42). This raises questions about potential barriers to their improvement, such as their insufficient understanding of the bar model method, the SIMPLE steps, or limited intervention time. Identifying these barriers is essential for refining the intervention to support struggling pupils more effectively.

The image shows two examples of handwritten mathematical work. The left example is for a word problem (WP4_O) and shows a bar model with three boxes containing the numbers 25, 3, and 12. Below the bar model, the equation $25 + 3 - 12 =$ is written, followed by the answer 16 . The right example is for a word problem (WP5_O) and shows a bar model with three boxes containing the numbers 280, 89, and 62. Below the bar model, the equation $280 - 89 - 62 =$ is written, followed by the answer 129 .

Fig. 4.42: The incorrect solution to WP4_O and WP5_O of post-test

Another small group included the three pupils whose performance worsened in the post-test. A closer look at their solutions suggests potential causes for this decline. For instance, inability to draw the BM method, non-adaptation of the SIMPLE steps, and errors in setting up equations. Additionally, time constraints, misunderstanding certain concepts, or difficulties in moving from their usual way of solving WPs. However, as there were no follow-up interviews with these three pupils, a deeper understanding of their difficulties remains limited, which can be considered a study’s limitation.

Presence of BMs in the Post-Test Solutions

Most pupils drew BMs when solving WPs in the post-test. The number of correct BMs was slightly bigger than the number of incorrect ones (Fig. 4.43).

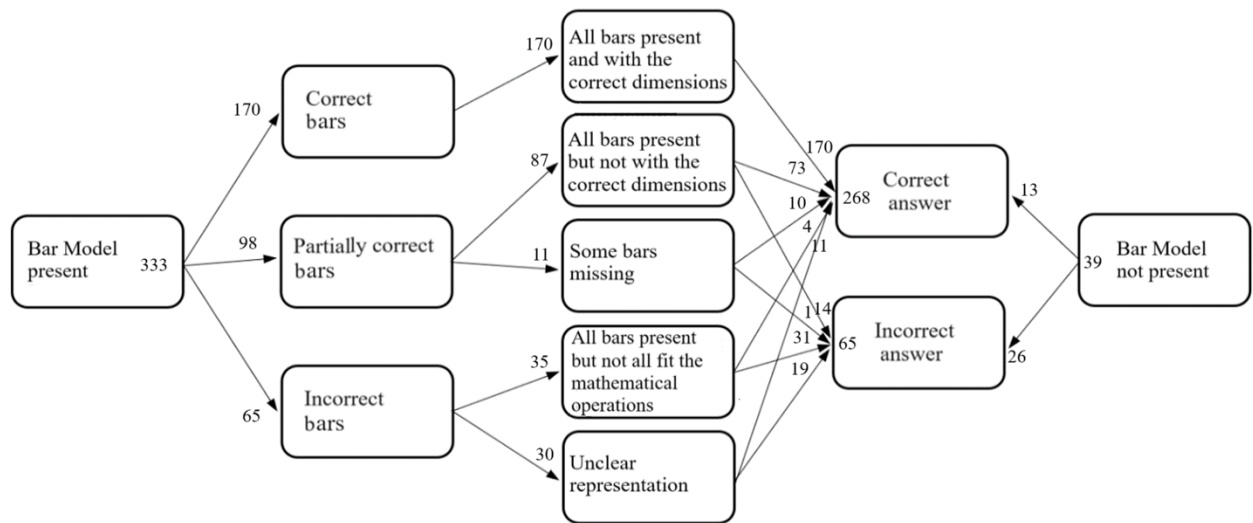


Fig. 4.43: Correctness of the solution related to the quality of BMs in the post-test

Table 4.27: The relation between the BMs and the problem-solving success for the whole sample

BM	Incorrect solution	Correct solution	Total	%
Present	65	268	333	89.5
Not present	26	13	39	10.5

The calculation of the correlation between the presence and quality of the applied BMs and the correctness of the solution enabled the assessment of the relationship between the accuracy of BM use and problem-solving outcomes. Their correlation was 0.66, indicating a strong positive correlation. Fig. 4.43 shows that the solutions with a correct BM were always correct, with 84% of the solutions with partially correct BMs being correct (see also Table 4.28; an example of a pupil's solution is in Fig. 4.11). Approximately 25% of the solutions with incorrectly drawn BMs were correct, while 33% of those who did not draw any BM were correct.

As no follow-up interviews with pupils were conducted, no firm conclusions can be made. For the pupils who were in the excellent achievement category already at the beginning of the intervention (see above), it is probable that they would have solved the WPs without BMs, too. For those who moved to the “excellent” category of achievement, it is likely that the BMs facilitated their solutions (as an example, see Fig. 4.44). However, we cannot rule out the

possibility that the pupils made the BM only to illustrate their strategy, which they found differently.

Table 4.28: Average success rates for solutions per category of BMs for the post-test (%)

Category	No BM	Incorrect BM	Partially correct BM	Correct BM
Post-test success	33	25	84	100

Table 4.29 reports the results for consistent and inconsistent WPs. No notable difference was observed in the correlation coefficients, which were 0.64 for consistent WPs and 0.67 for inconsistent WPs. However, closer inspection of the bolded data in Table 4.29 reveals that pupils tended to solve consistent WPs more successfully without employing BMs than inconsistent WPs. Interestingly, there were more solutions to the inconsistent WPs, which were correct in the post-test. We may speculate that even if the pupil did not manage to draw a correct BM, this effort brought insight into the structure of the WP, which helped them to make a correct mathematical model and reach the correct answer. Nevertheless, we cannot be sure about that without interviewing the pupils.

Table 4.29: Average success rates for solutions per category of BMs for consistent and inconsistent WPs (%)

Category	No BM	Incorrect BM	Partially correct BM	Correct BM
Consistent WPs	42	19	83	100
Inconsistent WPs	25	29	85	100

Presence of the SIMPLE Steps in the Post-Test Solutions

Regarding the presence of the SIMPLE steps in the post-test, clear usage was shown in 52.1%, partial usage in 33.9%, and no indication of their use in 14.0% of the solutions (Table 4.30).

Table 4.30: The presence of the SIMPLE steps in WPs of the post-test across three Cycles

	Cycle 1		Cycle 2		Cycle 3		Total	
	N	%	N	%	N	%	N	%
SIMPLE steps presence/ Number of WP (%)								
The SIMPLE steps present	49	40.8	81	71.1	64	46.4	194	52.1
The SIMPLE steps partially present	43	35.8	24	21.0	59	42.7	126	33.9
The SIMPLE steps absent	28	23.4	9	7.9	15	10.9	52	14.0
Total	120	*	114	*	138	*	372	*

* The sum is not 100% due to rounding.

Table 4.31 shows that for the whole sample, the solutions in which pupils described all the steps were correct. On the other hand, around 14.2% of solutions with partially presented steps

were incorrect and 19.6% were correct. The solutions with no indication of the usage of the step were mostly incorrect.

Table 4.31: The relation between the SIMPLE steps and the problem-solving success

The SIMPLE steps	Cycle 1		Cycle 2		Cycle 3		Total			
	Incorrect	Correct	Incorrect	Correct	Incorrect	Correct	Incorrect	%	Correct	%
Present	0	49	0	81	0	64	0	0.0	194	52.2
Partially present	13	30	13	11	27	32	53	14.2	73	19.6
Not present	18	10	8	1	12	3	38	10.2	14	3.8
Total	31	89	21	93	39	99	91	24.5	281	75.5

Nevertheless, care should be taken to consider the difference between writing the SIMPLE steps and using them. For example, some pupils said during the interviews that they did not have enough time to write everything on paper and took some of the steps in their minds. However, it is also plausible that because there was an increase in correct answers between the pre-test and post-test, even a partial usage of the SIMPLE steps facilitated the creation of the situation model and helped some pupils in the same way as the inclusion of BMs was hypothesised to do.

To gain deeper insight into the relationship between the success of solving a WP and the usage of the SIMPLE steps, three pupils' solutions from the achievements category "excellent" to "excellent", "insufficient" to "insufficient" and "insufficient" to "good" or "excellent" (one from each cycle, see Table 3.8 for the achievements categorisation) to the WP1_E from the pre-test and WP1_O from the post-test are presented.

Fig. 4.44 presents the solutions of three pupils (one per cycle) that were in the achievement category "excellent" in the pre-test and remained "excellent" in the post-test with the presence of the SIMPLE steps. As these pupils provided no visual representation in the pre-test and were successful in it, it is difficult to determine whether the SIMPLE Approach helped them solve the WPs in the post-test.

Fig. 4.45 shows the solutions of three pupils who were in the achievement category "insufficient" in the pre-test and remained in the achievement category "insufficient" in the post-test. The pupils show no grasp of the meaning of the BM method and the SIMPLE steps.

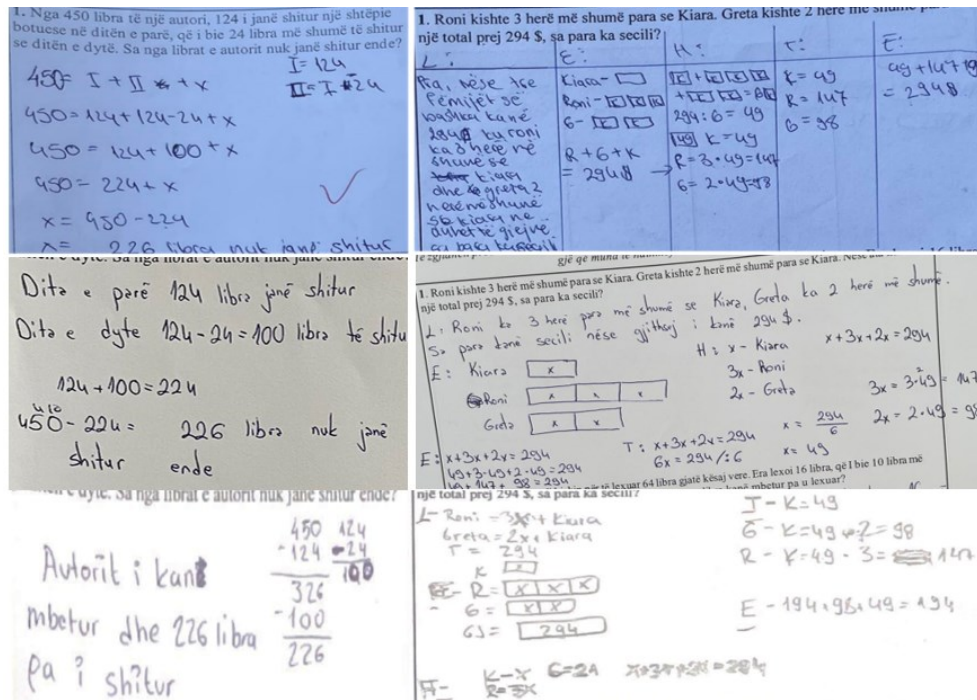


Fig. 4.44: Correct solution of WP1_E and correct solution of WP1_O with the application of the SIMPLE steps from three pupils (up-Cycle 1, middle-Cycle 2, down-Cycle 3)

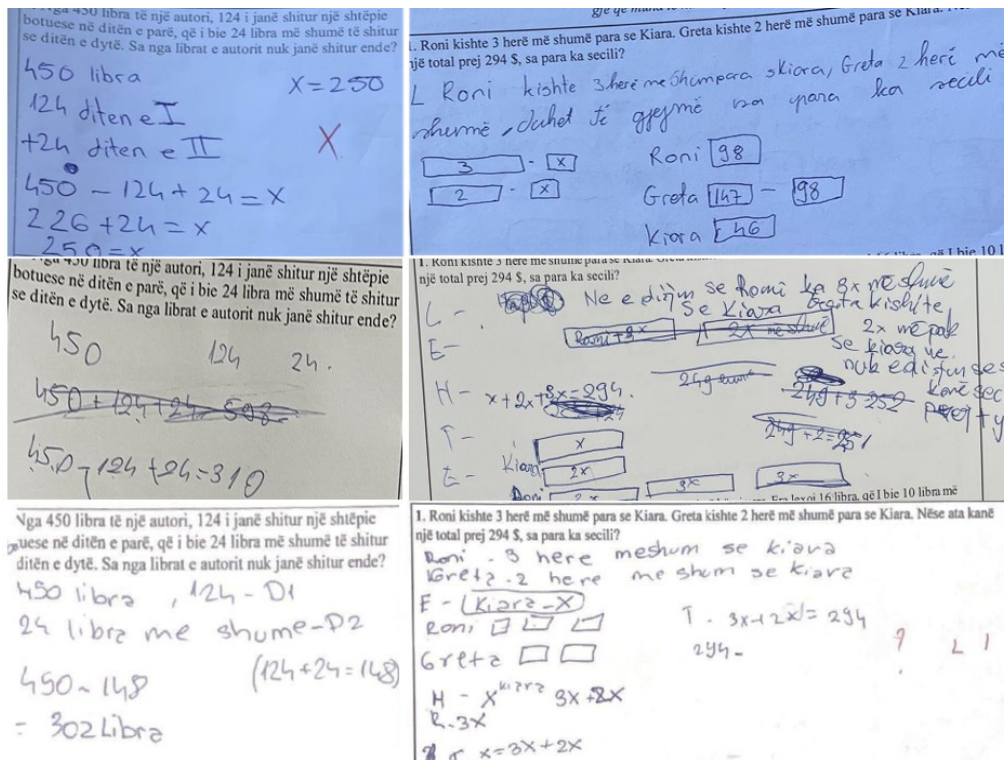


Fig. 4.45: Incorrect solutions of WP1_E and of WP1_O with the partial application of the SIMPLE steps from three pupils (up-Cycle 1, middle-Cycle 2, down-Cycle 3)

Finally, Fig. 4.46 presents the solutions of three other pupils who moved from the achievement category “insufficient” to “good” or “excellent”. The pupils’ solutions were better in the post-test. In this case, the SIMPLE Approach probably impacted the pupil’s success in the post-test solutions.

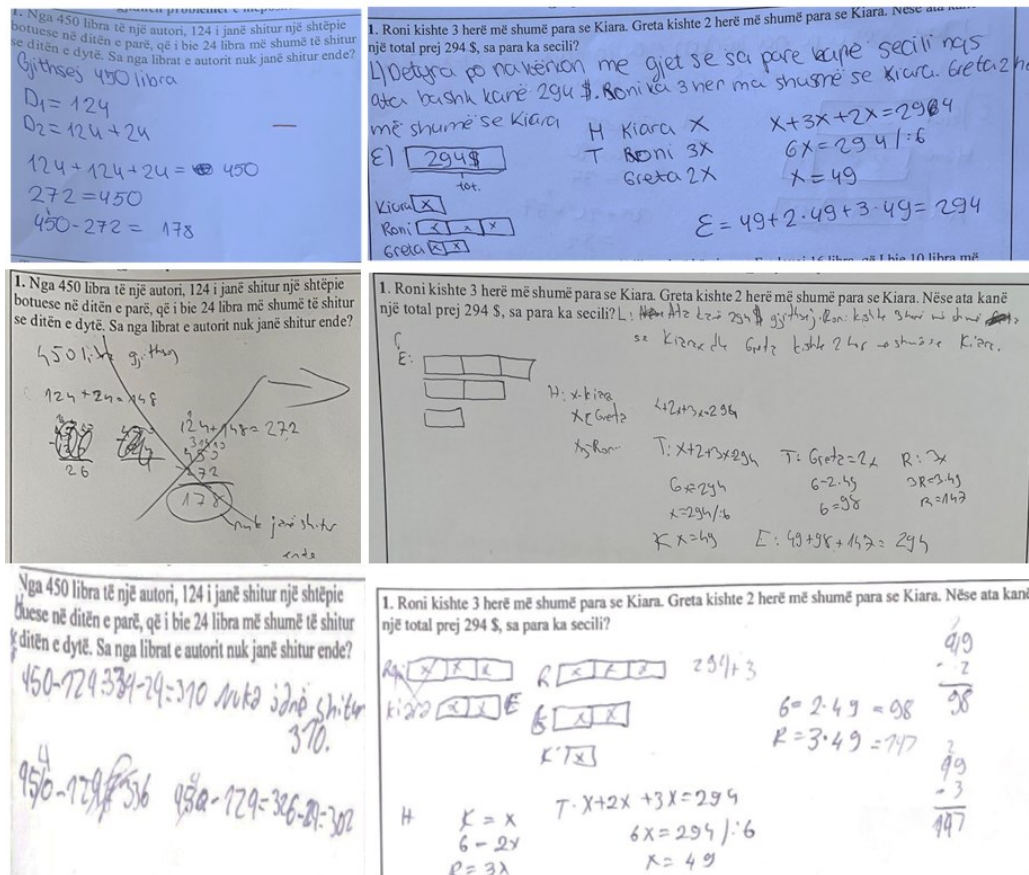


Fig. 4.46: Incorrect solutions of WP1_E and correct solutions of WP1_O with the SIMPLE steps from three pupils (up-Cycle 1, middle-Cycle 2, down-Cycle 3)

5. DISCUSSION

This chapter will revisit the research questions that guided the study and integrate findings with the literature.

The primary aim of this study was to examine the pupils' learning and teachers' teaching of the SIMPLE Approach, its usage in the WPs solutions and its connection to problem-solving success. Furthermore, the study aimed to address the research gap in that intervention studies in DBR recognise the critical role of teachers but provide little insight into their actual work and the fidelity of their implementation. Using a DBR methodology, three iterative cycles of intervention were implemented, with each cycle building upon the findings of the previous one. This iterative process optimised the intervention by continuously refining the teaching approach based on pupils' work, teachers' way of teaching, and real-time feedback from classroom implementations.

In the sections below, the findings of our study are compared with previous research to determine how they support, contradict, or add to the body of knowledge, thus contributing new insights to the field. Additionally, the implications of our study, considering how our findings may influence future research and teaching, are presented. These implications are not just theoretical, but they have the potential to inspire and shape the future of problem-solving teaching. Additionally, the practical implications for teachers will be considered, the study's limitations acknowledged, and potential areas for future research suggested.

5.1 Pupils' and Teachers' Perceptions of the SIMPLE Approach, Conditions and Barriers to its Implementation

The three teachers had positive attitudes towards the SIMPLE Approach from the beginning and appreciated its benefits at the end of the intervention. They all expressed their perception of pupils' more active engagement in the lessons and belief that the new ways of solving WPs helped the pupils to be more successful.

The Approach was new to the pupils and Teacher 1 warned that conveying it to them in such a short timeframe was not easy. This probably mirrored his insecurities in using the new strategies (especially the BM method) mentioned in Chapter 4 and resulted in lower confidence in utilising the SIMPLE Approach. In contrast, the second and third teachers exhibited greater confidence, as evidenced by their initiation of the new approach with pupils even outside the intervention classes. They expressed conviction in the continued application of the new approach and indicated a desire to see it incorporated into elementary school mathematics

textbooks. On the other hand, interviewed pupils admitted that initially, they had difficulties applying the BM method, but after some time and practice, they liked it and were confident in using it. They expressed a desire to keep using bar models and the SIMPLE steps in the future, recognising their importance in making problem-solving easier but also pointing out the time demands of the latter.

Regardless of the good perception, three main factors impacted the implementation of the intervention.

First, it is the pupils' familiarity with WPs and the strategies for their solutions. WPs are not used much in mathematics lessons in Kosovo, and pupils are unfamiliar with the visual aspect of WP solutions (Berisha et al., 2013; Morina, 2022). Indeed, none of the pupils used any visualisation to solve WPs in the pre-test. The prior knowledge of solving WPs through equations or even directly based on keywords created a challenge, as pupils initially found it difficult to shift their problem-solving approach to incorporate the bar model method and reasoning through SIMPLE steps. This result aligns with literature showing problems pupils have when using the keyword strategy (Hegarty et al., 1992; Powell & Fuchs, 2018).

Second, another key factor is the teacher, namely, how they master the new approach, how they interpret it to the pupils and how they organise the pupils' ways of working (Mcnamara et al., 2002; Ventistas et al., 2024).

Teachers' mastering of strategies to be taught is paramount. In the present study, this was evident especially for Teacher 1 as there were some situations in which he restored to using his usual way of solving WPs through algebra rather than helping pupils modify their drawing to grasp the situation model. It can be inferred that this preference stems from his long-standing reliance on linear equations in teaching WPs, which made a shift to the SIMPLE Approach particularly challenging. There was a difference between intervention-as-designed and intervention-as-implemented (Nelson et al., 2012). Teacher 1 would have benefited from a longer initial training, even though it must be stressed that he did not seem hesitant about the content before the intervention. It only became apparent during his teaching. Another option would have been to include Teacher 1 in all stages of the intervention as it would foster a deeper understanding of instructional strategies and increase the likelihood of sustainable implementation (Brown et al., 2020; Sterner, 2019). No problems with mastering the BM method were observed for the other two teachers. Teacher 2 did not seem to value the

importance of some steps of a metacognitive nature in the scaffolding provided (SIMPLE steps) and would require some attention to this feature in the initial training.

The three teachers also differed in their usual teaching style and how “constructivist” it was. According to the principles of constructivism, the teacher’s role moves from being the primary source of knowledge to that of a learning facilitator who does not simply transmit information but guides, supports, and challenges pupils to construct their understanding (Khalid & Azeem, 2012; Mcnamara et al., 2002; Simon, 1995). In the present study, the first teacher did not want to use pair or group work at all (as he stated), the second and especially the third teacher welcomed it as they used it in their usual teaching. In cycles when the group work was present, it was observed that it supported the solution of WPs through the SIMPLE Approach as pupils often relied on each other and collaborated when encountering difficulties during the WPs solutions. These findings complement statements made by Ventistas et al. (2024), who demonstrated that problem-solving in groups is a rich learning experience that supports WP-solving skills. By supporting group work, the teacher’s role moved from being the primary source of knowledge to that of a learning facilitator who supports and challenges pupils to construct their understanding (Mcnamara et al., 2002; Simon, 1995).

Additionally, the way errors were addressed varied across teachers. While Teacher 1 primarily corrected errors himself, Teacher 3 took a different approach, involving pupils more actively in identifying and discussing them. This contrast in handling errors is an important aspect of constructivist teaching and highlights different levels of alignment with its principles. Thus, Teacher 1 and his class had “the longest way to travel” to implement the intervention faithfully to the principles of pupil active learning. For him and his class, the intervention required a total re-negotiating of the didactic contract (Brousseau, 1997) which would not be possible in such a short timeframe.

Thus, our study confirmed that adopting constructivist teaching approaches presents initial obstacles for teachers. With time and experience, teachers and pupils adjust to and value the new learning and teaching approach (Windschitl, 2002). The question remains to what extent it is reasonable to expect such a development in the case of the teachers in the present study, as this was a one-time (and rather short) intervention.

The design-based character of our study may be considered an important factor. There is a gradual improvement among the cycles both in terms of presenting the content (bar models, SIMPLE steps) and adhering to constructivist principles. One cause of this lies in the repetitive

nature of our study with the changes made after the first and second cycles. These results support Ford et al.'s (2017) claim that interventions often result in significant improvements after identifying and addressing the first-cycle challenges. Nevertheless, it is essential to acknowledge that while the intervention was carefully planned, several factors previously not envisaged and/or beyond our control may have influenced its implementation. These factors can be, for example, class composition, individual characteristics of pupils, pupils' familiarity with strategies for WPs solution, teacher and classroom dynamics, pupils' behaviour and cooperation, pupils' unique learning abilities, motivation levels, and prior knowledge of solving WPs.

5.2 The Impact of the SIMPLE Approach on Pupils' Achievement in WPs

Our analysis of pupils' solutions to the WPs in the pre-test and the post-test confirmed results pertinent to the difficulty of WPs for pupils. For example, pupils in our study had consistently lower results when WPs were inconsistent than consistent, which aligns with previous research (e.g., Verschaffel et al., 1992; Vondrová, 2020). The pupils who were unsuccessful in inconsistent problems often applied a keyword strategy and associated, for example, the term "more" with addition even though subtraction was needed, a phenomenon amply documented in the literature (see, e.g., Powell & Fuchs, 2018).

Section 4.4.3 presents some favourable findings concerning the impact of the intervention on the pupils' success when solving WPs. As already mentioned, (and also based on the teachers' experience of the intervention they reported), we can only tentatively conclude that BMs and the SIMPLE steps helped pupils to solve the problems. The same results might have been reached if the pupils devoted the same amount of time to solving WPs using other means. However, there is a body of literature showing the benefits of visualisation (Boonen et al., 2014; de Koning et al., 2022; Ho & Lowrie, 2014) and development of pupils' awareness of solving steps for WPs (Freeman-Green et al., 2015; Teong, 2003; Veenman et al., 2004), which adds support for claiming that the intervention was successful in achieving its aim at least for some pupils. Our study combined the two aspects (visualisation and metacognition) and added a new perspective to the body of results. According to our findings, the combination of these two tools likely increased post-test success rates and induced a positive attitude in teachers and pupils towards WPs, a desire and a motivation to deal more with such tasks, and a willingness to use the tools in the future.

The data analysis from the pre-test, intervention and post-test uncovered that pupils evolved differently regarding their ability to apply the new teaching approach. Around 14 pupils (23%)

demonstrated proficiency in solving WPs without utilising the new approach in the pre-test, and they maintained this proficiency in the other two phases when incorporating it. There was no room for improvement for them. Eight pupils (13%) remained in the category of achievement “good” in all the phases, and 30 pupils (48%) improved their ability to solve WPs in the post-test. On the other hand, 10 pupils (16%) did not improve their ability to solve WPs (Fig. 4.41). Furthermore, the classroom observations and pupils’ interviews made it clear that some pupils would require more time and individual attention to the core aspects of the approach to be able to use it effectively.

As per the BM method and the SIMPLE steps usage, it can be inferred that most pupils understood and adopted these tools. The data from the three cycles shows a lower application of this approach in the first cycle. Following a revision of the intervention based on the first cycle’s recommendations, a greater use of the approach in Cycles 2 and 3 was noticed. The post-test success rates improved after the teaching intervention in three Cycles (Table 4.26). It was noticed that the pupils in Cycle 1 exhibited the lowest increase in their success scores from pre-test to post-test and the pupils in Cycle 3 the biggest. These results show an increasing trend in the solution’s success from Cycle 1 to Cycle 3. There may be a combination of reasons for this success. First, the enhancement of the intervention contributed favourably to the solution’s successful growth. Second, the approach is more beneficial in classes which did not learn the WP solution with linear equations yet. They are not hindered by an algebraic method and can use BMs to help them make a situation model (while in Class 1, on the contrary, pupils may have tried to make a BM to fit their equation, which they created without it). This would support the idea of introducing the SIMPLE Approach in earlier grades (de Koning et al., 2022; Kaur, 2018; Murata, 2008; Ng, 2003).

A notable finding of our study is that if pupils were able to describe all the steps correctly, they always arrived at the correct answer (Table 4.31). This result aligns with the conclusions of related studies (de Koning et al., 2022; Ng & Lee, 2009), showing the benefits of introducing metacognitive scaffoldings for improving problem-solving skills.

To sum up, research has demonstrated that the presence of metacognitive steps (Freeman-Green et al., 2015; Teong, 2003; Veenman et al., 2004, etc.) and the presence of BM (Boonen et al., 2014; Ho & Lowrie, 2014; de Koning et al., 2022, etc.) can both be helpful when solving WPs. This study showed that these tools can be successfully combined. This makes the study unique and adds knowledge to the existing literature.

5.3 Implications

This study developed a local instruction theory for teaching WPs to 8th-grade pupils in Kosovo. The local instruction theory consists of lessons using the SIMPLE Approach to address the challenges pupils in Kosovo face. This sequence includes detailed lesson plans that introduce and practise the bar model method, structured steps of the SIMPLE Approach to guide pupils through solving WPs, and instruction to foster pupil engagement and encourage collaborative learning, aligned with constructivist principles. The study's significance lies in showing that the BM method and the SIMPLE steps may function well in a cultural context where visual representations, metacognition and problem-solving skills are not promoted in the classroom. Therefore, the study has three types of implications: the theoretical, the practical, and the methodological.

Regarding the theoretical implications, the findings of this study provide important contributions to the existing educational theories, particularly in two areas: the study reinforces the effectiveness of constructivist methods in enhancing problem-solving skills, showing that pupils benefit from active engagement and the opportunity to build their understanding, and the introduction and drawing of the bar model method offer insights into how visual representations can significantly aid in solving WPs, particularly in contexts where pupils may have limited experience with such methods. Therefore, this study enriches the body of the literature in the field since, in addition to promoting a new approach to teaching and learning WP, it also strengthens the teacher's constructivist approach to teaching, which supports the pupils' participation in the learning process, the teamwork, discussion and exploration.

The study has practical implications. The SIMPLE Approach can successfully be used in Kosovo, a cultural context in which pupils have little experience with WPs and the development of metacognition and no experience with visualisations in WP solutions. An obvious implication would be to include it in Kosovo's mathematics textbooks or provide it to teachers as a handbook so they can use it whenever dealing with WPs.

However, introducing the BMs coupled with the metacognitive skills to solve WPs in grade 8 might have come too late for some pupils. Pupils might have grasped the tools better if they had been introduced at least in the 5th grade when the curriculum includes less complex WPs and linear equations are not known to them.

Finally, as this approach helped pupils more profoundly in inconsistent WPs, types of WPs shown to be more difficult than the consistent ones, it is reasonable to expect that this approach

would benefit solving WPs with another complicating variable, such as an irrelevant number or mixed order of data (Englert et al., 1987; Pongsakdi et al., 2020; Vondrová, 2020). These types of WPs were not included in our intervention. In future work, it is planned to investigate how this approach can impact the development of problem-solving skills, even among pupils with special needs.

Regarding the methodological implications, the DBR was crucial for developing the new teaching approach to WP solving. Its iterative nature allowed ongoing refinement of the intervention across three cycles. Each cycle offered us new information and knowledge for improvements, which impacted the quality of teaching through the SIMPLE approach. Some breaches of implementation fidelity and problems the teachers experienced when implementing the new tools could be (Anderson & Shattuck, 2012; Brown et al., 2020; Sterner, 2019).

The findings suggest that similar DBR approaches could be applied in various educational contexts, considering social and cultural differences. Future studies could focus on implementing this approach to other grade pupils, a larger sample size, and pupils with special needs.

5.4 Study's Limitations

Our findings must be considered within the limitations of the study.

Even though the new teaching approach benefited most pupils, these findings cannot be generalised. The sample is small, limited to one school grade and three classes from one city. The teachers' individual implementation of the intervention and the specific characteristics of each class influenced the results of the study. However, the fact that the BM method and the metacognitive skills have been successfully implemented by others (e.g., Baysal & Sevinc, 2022; Gani et al., 2019; Ho & Lowrie, 2014; Kaur, 2018; Özsoy & Ataman, 2009; Teong, 2003; Veenman, 2004; Xin, 2019) lends credibility to our results.

Second, the mere practice of WPs during the teaching intervention influenced pupils' success in the WP solutions in the post-test. Thus, the study's success cannot be solely attributed to the SIMPLE Approach.

Third, even though the intervention proved successful for many pupils, seven pupils did not improve in the WP solution. During the interview, they claimed that a more extensive intervention time could help them grasp the approach better. Another limitation is that the intervention was not delivered to all pupils, as in one of the classes, two pupils with special

needs could not participate. The teacher could not convey the new approach to them as they would need because the intervention time was limited, and there were no assistants present in the lessons. A special study should be aimed at pupils with special needs.

Furthermore, a potential limitation of the findings pertinent to pupil perception lies in the fact that only volunteers participated. This may have introduced bias, as those who were less receptive to the SIMPLE approach may have chosen not to talk.

Another limitation of our study is that the teachers' regular teaching before the intervention was not observed, and reliance was placed only on their descriptions. Without this baseline observation, it is difficult to determine if the teachers' adherence or non-adherence to constructivist principles during the intervention directly resulted from their typical teaching style or was impacted by the intervention itself.

Furthermore, unlike in some design-based research frameworks where teachers are co-designers, the novelty of the bar model method would have necessitated a more guided approach at least for one of the teachers.

6. CONCLUSION

The primary aim of this study was to examine the pupils' use of the SIMPLE Approach after a brief instructional intervention and determine whether it was linked to their problem-solving success. Furthermore, the study aims to address the research gap in that intervention studies in design-based research recognise the critical role of teachers but provide little insight into their actual work and the fidelity of their implementation.

Initially, in the pre-test, in all three cycles, no pupils used visualisation, or at least there was no evidence of the use of metacognition in their solution, highlighting their unfamiliarity with these aspects of the SIMPLE Approach. Following the intervention, however, pupils not only used BMs and metacognitive steps more frequently but also demonstrated marked improvements in their post-test results. This suggests that even brief instructional interventions can significantly enhance the success of WP solutions. On the other hand, while some pupils quickly grasped the SIMPLE Approach, others struggled to follow the steps and interpret BMs as representations of quantities and operations. This highlights the need for more sustained or individualised instruction for those who face difficulties adopting the new approach.

A key finding of the study is the strong correlation between the usage of the SIMPLE Approach and problem-solving success. Pupils who presented the SIMPLE steps and drew correct BMs consistently arrived at the correct solutions. Furthermore, even partial presentation of the SIMPLE steps led to correct solutions in most cases, indicating that this approach helps pupils better understand the problem's situation and the necessary mathematical model.

As for implementation fidelity, data from classroom observations, teacher diaries, and interviews revealed varying fidelity levels, improving over time. Teacher 1 showed the lowest fidelity, struggling with the bar model method and limiting group work, while Teacher 3 demonstrated the highest fidelity, effectively incorporating all aspects of the SIMPLE Approach and promoting active pupil engagement.

The DBR methodology was crucial for developing the new teaching approach to WP solving. Its iterative nature made it possible to refine the intervention continuously across three cycles. Each cycle offered novel insights and contributed to a deeper understanding of the impact of the SIMPLE Approach on developing pupils' problem-solving skills.

This study contributes to the existing body of research in important ways. First, it confirms the efficacy of the SIMPLE Approach in improving problem-solving accuracy, particularly for more complex WPs. Second, our findings have important implications for education in contexts

where visual representation and metacognitive strategies are not widely emphasised, as they suggest that the SIMPLE Approach can be successfully adapted to such environments. Third, by framing this research as a local instruction theory development, our study provides a valuable framework for future educational interventions to improve WP-solving skills in diverse classroom settings. In short, if the intervention is prepared by researchers and implemented by teachers not involved in its elaboration, it acts as a hindrance to its successful implementation as intended due to limited teacher ownership of the approach, which may lead to lower adherence, for the tendency to go back to their usual teaching methods, mainly when challenges arise or due to short initial training that can sometimes leave gaps in understanding and confidence in using the new approach. Thus, for an intervention to be effectively implemented, it is recommended that teachers should be more actively involved in its design and be supported through continuous monitoring and adaptation.

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APPENDICES

Appendix 1: Teacher Guide

TEACHER GUIDE THE SIMPLE APPROACH

WORD PROBLEMS

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Qëndresa Morina

1. Introduction

Word problems have become a vital part of learning mathematics. They help pupils develop critical thinking and deal with various real-world scenarios. Verschaffel et al. (2000) describe a word problem as a mathematical task that asks pupils to apply their mathematical knowledge to ‘real-life’ situations. They are distinct from other mathematical tasks since they aim to describe a situation through words and seek the solution using mathematical operations (Pongsakdi et al., 2019).

Numerous strategies and methods have been developed to help pupils solve word problems more easily. Many findings have demonstrated that using these strategies has helped pupils develop word problem-solving skills. Therefore, we have developed this Guide to help mathematics teachers implement a specific teaching approach called the SIMPLE Approach.

1.1 Who Is This Guide For?

This Guide is designed for primary school mathematics teachers, specifically those teaching 6th to 9th grade. It supports the teachers in implementing an intervention for solving word problems based on a specific approach called the “SIMPLE”. This approach consists of three factors: visual representation, specifically the application of the bar model method, the development of metacognitive skills and teaching based on constructivism principles. Studies have shown that these factors are important components in developing critical thinking, and we hope this guide will help teachers and pupils regarding the solution of word problems.

1.2 Why Has This Guide Been Written?

This Guide fosters mathematical reasoning and equips pupils with the skills to navigate life’s problems successfully. Worldwide, teachers have tried many approaches to teaching mathematics word problems to foster higher-order thinking abilities. We hope that teachers, with the instruction of this Guide, will help pupils increase their mathematical knowledge and problem-solving abilities.

This manual will help mathematics teachers:

- Solve word problems,
- Guide them through constructivist principles
- Develop problem-solving skills
- Make mathematics more attractive to their pupils.



1.3 When to Use This Handbook?

Use this handbook:

- For every word problem that requires mathematical operations in its solution.
- When using mathematical activities related to everyday life.
- To foster pupil’s critical thinking,
- To develop mathematics reasoning and, most important,
- To encourage a positive attitude in pupils towards word problems.



2. What Are Word Problems?

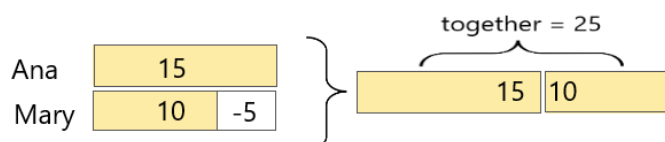
Numerous situations in everyday life demand the use of mathematical knowledge for their solution. Applying word problem-solving skills is one of the best ways to deal with these situations. As stated by Wyndhamn and Säljö (1997), developing problem-solving skills is a crucial mathematics ability that aids students in solving word problems. A word problem, on the other hand, is described as a mathematical task that presents real-world scenarios and seeks solutions through mathematical operations. To solve word problems, we must translate language into mathematical expressions. Such a transformation requires careful attention to ensure the correct connection with mathematical operations. For example, mathematics teachers utilise the modelling approach to link the necessary operations with the known and unknown quantities in a given problem. For instance, *Ana has 15 apples, which is five more apples than Mary. How many apples do they have together?*

The solution to this problem can be modelled with the bar model method.

$$\text{Mary} = \text{Ana} - 5 = 15 - 5 = 10 \text{ and, together} = \text{Mary} + \text{Ana} = 15 + 10 = 25$$

There are different types of mathematical word problems, such as Change, Combine and Compare problems with a one or multi-step solution.

In the change problems, an initial quantity is changed by a new value, which can increase or decrease the original quantity.

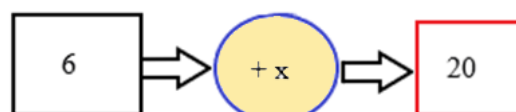


Example 1: Andy has 5.5£ (initial value). His father gives him an extra £2.50 (change – increase). How many pounds does Andy have now (new value – unknown)?



Combine problems to connect two or more parts to form a whole. They may also be known as part-whole problems. In these exercises, pupils must determine the total quantity given in a problem or the missing parts.

Example 2: 6 pupils (part 1) are taking a mathematics exam in class A, while some other pupils (part 2- missing part) are in class B. There are 20 pupils in both classes (total). Find the number of pupils in class B.



The compare problems find the difference between two quantities in a given problem. In these exercises, pupils must determine if one quantity is greater or smaller than the other.

Example 3: The little girl has three candies (a smaller set). Her friend has seven candies (a greater set). How many more candies does her friend have (difference—unknown)?



2.1 Consistent vs Inconsistent Word Problems

Consistent word problems use straightforward language and do not contain contradictions or misleading cues (de Koning et al., 2022). They are easier to solve because their solutions require direct application of mathematical operations. See Example 1, 2 and 3. On the other hand, inconsistent WPs use words, phrases, or cues that refer to an operation inverse to the one required. Pupils are usually less successful with inconsistent WPs. A reason for this might be the teaching in which teachers teach pupils to solve WPs by connecting terms like “more,”

“less,” “together,” and “divide” to specific operations. In the example above, *Ana has 15 apples, which is five more apples than Mary. How many apples do they have together?* It is an inconsistent WP as it has the word “more apples” but requests subtraction instead.

$$\text{Mary} = \text{Ana} - 5 = 15 - 5 = 10 \quad \text{Together} = \text{Mary} + \text{Ana} = 15 + 10 = 25$$

2.2 Their importance in the process of learning

Word problems are essential in mathematics lessons since they enable pupils to develop their cognitive skills, logical reasoning, and creative thinking. Hence, they must be adequately treated and promoted to the pupils from the beginning of their mathematics learning to foster a positive attitude towards them. They enable pupils to develop a more logical view of life and better handle it. They teach pupils to perform a particular activity by combining logic and creative thinking. Most importantly, it encourages pupils to take responsibility for their actions and face the reality of life. Given the significance of word problem-solving, this Guide explains how to solve word problems with the SIMPLE Approach.

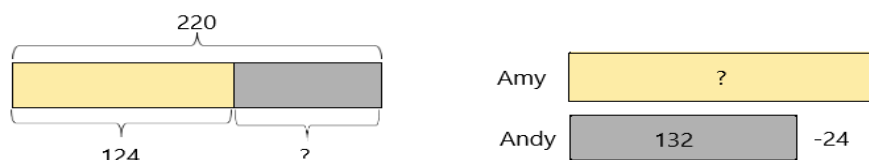
3. Teaching intervention

The teaching intervention consists of three factors: the bar model method, metacognitive steps, and teaching based on constructivist principles. This section will describe each of them in detail and explain how to convey them to the pupils.

3.1 Bar model method

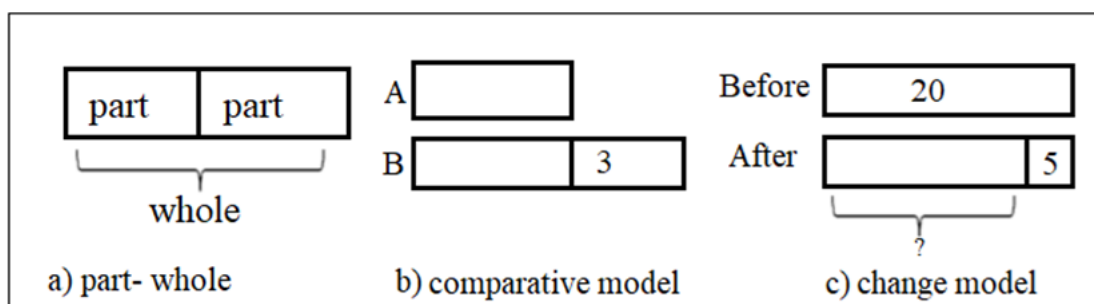
Lesson 1 – The introduction to the bar model method

The bar model method originates from Singapore and is one way of introducing pupils to visual representations of the structure of word problems. Pupils depict unknown quantities and their interactions in a word by drawing a pictorial model using strips, bars, or rectangular regions. For example, the bars in the figure below show the relationship between two objects. The figure on the left shows how to find a part by subtracting the given part from the whole. The picture on the right shows how to find the whole by adding the given part and the difference.



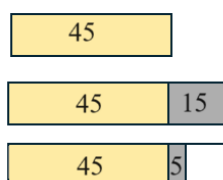
The lessons on the bar model method begin with a description and its importance in solving word problems. For further clarification, the teacher is asked to explain the three types of bar models, illustrate them for the pupils, and use real-world objects in the classroom to demonstrate how they can be used.

There are three main types of bar models. The part-whole bar model describes the relationship between the whole and its constituent parts, and it is based on the fact that the whole is made up of two or more parts. The length of each rectangle indicates the (known) quantity it represents and reflects the relationship between these quantities. In problem situations involving comparative bar models, a quantity is compared to another, showing how much one part is greater/smaller than the other. In this case, the length of each rectangle indicates the (unknown) quantity that the rectangle represents. Finally, problems that involve change situations can be solved using the before-after model. A change bar model depicts the relationship between a quantity’s new value and its original value after an increase or decrease.



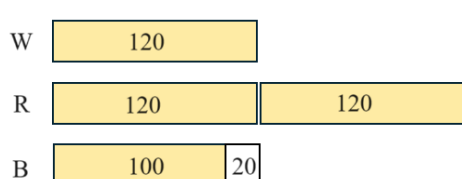
Take three word problems and show the pupils in which of them we can apply these three types of bar models. You can utilise the word problems listed below with your pupils.

Example 4 (Part-whole bar model): The mathematics exam consists of 3 parts. In the first part, Andi reached 45 points; in the second, 15 points more than in the first one; and in the third part, 10 less than in the second half. If the exam had 200 points, how many points did Andy fail to get?



$$\begin{aligned}
 I: x &= 45 \\
 II: x + 15 &= 45 + 15 = 60 \\
 III: x + 15 - 10 &= 60 - 10 = 50 \\
 T = I + II + III &= 45 + 60 + 50 = 155 \\
 \text{Left: } 200 - 155 &= 45
 \end{aligned}$$

Example 5 (Comparative bar model): There are 120 crayons in the white box, which is two times less than in the red box and 20 more than in the blue box. Which box has more crayons?



$$\begin{aligned}
 W &= 120 \\
 R &= W * 2 = 120 * 2 = 240 \\
 B &= W - 20 = 120 - 20 = 100 \\
 \text{The red box has more crayons.}
 \end{aligned}$$

Example 6 (Change bar model): Sara has 17 pencils, five more than what Emy has. They are giving 10 of them to their friend Andy. How many pencils are left?



$$\begin{aligned}
 E &= x \\
 S &= x + 5 = 17 \rightarrow x = 12 \\
 E + S &= x + x + 5 = 12 + 12 + 5 = 29 \\
 \text{Left: } 29 - 10 &= 19
 \end{aligned}$$

After taking an example for each type of bar model, you can give pupils some word problems to practice and ask them to use the bar model in their solution. You can start by giving pupils an easy word problem to solve.

WP1_I³. There were 120 visitors to Clara’s art exhibit on the first day, which is 20 more than on the second day. How many visitors came to the exhibition on the two days?

I	100	20	$I = 120$
II			$II - x = 120 - 20 = 100$
			$T = I + II = 120 + 100 = 220$

Ask pupils to work individually, and after some minutes, make them work in pairs and discuss the solution between them. When the pupils finish their solution, raise questions like “Can somebody explain what type of bar is suitable for the solution (WPs used) and why?” or “How did you draw the bar models?” “Why is one shorter than the other?” etc. You can also ask a pupil to draw it on the blackboard and explain how the solution is obtained from that drawing.

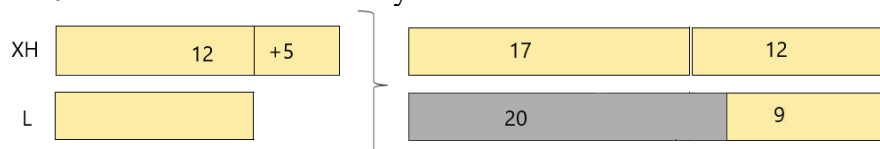
Lesson 2 – Practice of the bar model method

The word problems below can be used for individual practice and homework. For their solution, ask pupils to work in groups of four and discuss the solution together. Help them if they struggle to continue, raise questions to prompt their thinking, and present some solutions on the blackboard. Give pupils at least two word problems to practice at home.

WP2_I. Klara gives half of her paintings to her sister, Suzanne, and then buys herself four more. If she now has 12 paintings, how many did Klara have at the beginning?

I		$\frac{1}{2}x + 4 = 12$
II		$\frac{1}{2}x = 8$
III		$x = 16$

WP3_I. Xhamie picked 17 flowers, which is five more flowers than Linda picked. They donated 20 of the flowers. How many flowers were left?



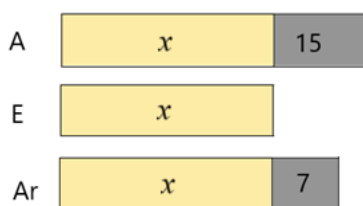
$L = x$	$L + Xh = x + x + 5 = 12 + 17 = 29$
$Xh = x + 5 = 17 \rightarrow x = 12$	$Left: 29 - 20 = 9$

WP4_I. 305 children went to a zoo on Saturday. The number of children who visited the Zoo on Saturday was 278 less than on the Sunday visit. How many children went to the zoo on Sunday?

St		$Sn = St + 278$	
Sn			$Sn = 305 + 278 = 583$
T			

³ The acronyms (such as WP1_I referred to the word problem 1 of intervention, WP2_I referred to the word problem 2 of intervention, or WP1_H referred to the word problem 1 of homework, etc.) were added later for the sake of the thesis.

WP5_I: Andi, Era, and Ari scored 106 points in a volleyball match. Andy scored 15 points more than Era. Era scored seven less than Ari. How many points did Ari score?



$$x + 15 + x + x + 7 = 106$$

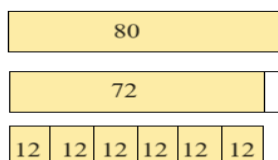
$$3x + 22 = 106$$

$$3x = 106 - 22$$

$$3x = 84$$

$$x = 28 \quad Ar = 28 + 7 = 35$$

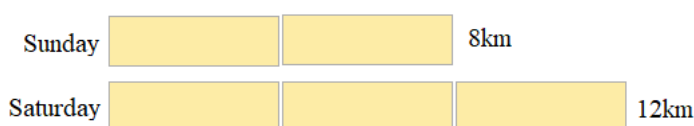
WP6_I: Brenda is an art teacher. To prepare for tomorrow's project, she opens a box of 80 crayons. She removes eight broken crayons and divides the rest equally into six boxes. Find the number of crayons in each box.



$$I: 80 - 8 = 72$$

$$II: 72 : 6 = 12$$

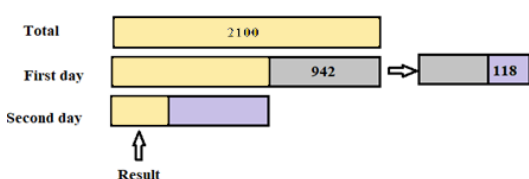
WP1_H: Some friends decided to go for a run on the weekend. They run 8 kilometres on Sunday. Sunday's run was 1.5 times shorter than Saturday's. How many kilometres did they run on Saturday?



$$\frac{x}{1.5} = 8$$

$$x = 12$$

WP2_H: From 2100 kilograms of potatoes in the warehouse, 942 kilograms were sold on the first day, while on the second day, 118 kilograms less than on the first day. How many kilograms of potatoes are left in the warehouse?



$$\text{First day: } 2100 - 942 = 1158$$

$$\text{Second day: } 1158 - (942 - 118) = 1158 - 824 = 334$$

$$\text{Total left: } = 334$$

3.2 Introducing the Pupils to the SIMPLE Steps

Lesson 1: The presentation of the SIMPLE steps

The SIMPLE steps have been introduced as a part of the SIMPLE Approach to support pupils in solving mathematical word problems. The name is an acronym from the initial letters of the following stages: Study the problem, Involve the model method, Monitor the process, Prepare/Present the solution, Look over the solution again, and Evaluate your answer so that pupils may recall it easily while solving word problems (Table 1).

Table 1: The SIMPLE steps and their indicators

Study the problem	Read the problem and rephrase it. “Have I understood what I am supposed to find?”
Involve the Bar Model method	Draw a diagram (create a visual representation of the word problem). How are the pieces of information in the word problem related? How can I represent these relations by rectangles?
Monitor the process	Go back to the beginning, including the problem formulation. Ask yourself: “Is the bar model method helping me to understand how to reach the solution?” / “Am I getting any closer to my objective?”/ “Am I still implementing the solving strategy I picked?”
Prepare / Present the solution	Make the expression/calculation/equation, write the solution, and present it.
Look over the solution again	Check the whole solution, solving steps and the calculations.
Evaluate your answer	Evaluate whether your solution makes sense and if there is a better way to solve the problem. Write up the answer.

The SIMPLE steps combine metacognition and visualisation. They include six steps that aim to develop problem-solving skills. Metacognition helps pupils comprehend when, why, where, and how to apply their knowledge to solve problems. On the other hand, visualisation can assist in the early stages of a word problem’s solution by providing much information about the task.

Studying the problem is the first step, which is the ability to understand what the problem is about. This is better achieved when you read the problem carefully and attempt to rephrase it in your own words. It is suggested that pupils ask themselves and others questions at this stage and not go further until they are confident that they know the problem’s objectives.

The second step involves visualisation skills, which involves understanding and applying the bar model method to connect the known and unknown quantities in a given problem. It is important to explain how to relate the model to an appropriate word problem and how to use proper operations.

Monitoring means observing all the processes before, during, and after the solution of the problem. It helps pupils understand how to reach the solution. In this step, instruct pupils to consider whether they are following the plan and strategy they picked and whether that approach is helping them find the correct answer.

The fourth step is when pupils should implement the plan, create a mathematical expression, prepare the solution, and present it.

Looking over the solution again entails going through the entire solution process. Instruct the pupils to conduct the proof and determine whether the result is correct.

The final phase requires pupils to evaluate the answer, determine whether it makes sense, and consider whether there is a better way to approach the problem.

Present to pupils all the SIMPLE strategy steps

S – Study the problem

Knowing how to read a problem and understand its meaning is critical. Pupils must read the problem more than once and not skim any word within it until they are sure that they fully comprehend it. The teacher can check it by asking pupils to repeat the statement in their own words and identify the main part of the problem, the unknown, the data, and the condition. If that does not happen, the teacher can ask questions like: *What are the unknown, the data? What is the true aim of the problem? Which part is more important, which can be left out, etc.?*

Let us illustrate some of the points mentioned with an example.

Example 5: Of the 450 books of an author, 124 were sold to one publishing house on the first day, which is 24 more books sold than on the second day. How many of the author's books have not been sold yet?

A possible dialogue between the teacher and a pupil can be:

Teacher (T): Can you tell us what the problem is asking for? Paraphrase in your own words.

Pupil (P): The problem is asking to find the number of unsold books after two days.

T: What are the unknown quantities? “Which letters would you choose to represent the unknown quantities?”

P: Number of books sold on the second day (we can denote it with the letter x) and those left unsold (with the letter y)

T: “What are the data?”

P: The number of books at the beginning and those sold on the first day.

T: What is the condition?

P: Finding the x value through the number of sold books on the first day, and y through the x .

T: Are you ready to continue to the next step?

P: As now I know what I must find and how I will find it, I think I'm ready to move to the next step.

I – Involve the bar model method

In this phase, pupils will be informed about the bar model method, its benefits, and its types. After a brief introduction, they will practice utilising it in certain problems with guidance from the teacher until they are comfortable with it. For examples, refer to section 3.1.

M – Monitor the process

After understanding the problem and constructing the visual part, namely the bar model, we have a plan or know at least which calculations or computations to perform to obtain the unknown quantities. This is when the “monitoring the process” starts and ends at the last step of the strategy. The role of monitoring involves a series of steps that eventually lead to the main solution of the entire problem. Every step requires a decision over how to proceed to the next step. So, it is crucial to monitor all the processes to evaluate whether you successfully implement those steps or if you need to review the problem again. In our case, advise pupils to go back to the beginning, including the problem formulation. Ask yourself: “Is the bar model method helping me to understand how

to reach the solution?” / “Am I getting any closer to my objective?”/ “Am I still implementing the solving strategy I picked?”

P – Prepare/ Present the solution

Conceiving the idea of a solution is not easy. A detailed plan is needed, which means following the data gathered from the previous steps, monitoring it, and being patient. Preparing a solution entails making a mathematical expression, calculating, or converting words into equations to reach the solution. It is often required to recollect pertinent facts, such as previously solved problems or counterexamples, to have an idea for the solution. Thus, it is often appropriate to start this step with a question: *Do you know a related problem? Consider the unknown and try to think of a familiar problem with the same aim as the presented problem.*

If that does not work, we must look for an alternative solution and explore different aspects of the problem. We must use the information from the bar model step, the data, and the conditions. Again, a teacher can ask questions like: *Did you use all the data from the previous step? Can you relate the known and unknown quantities using proper operation?*

L – Look over the solution again.

Looking back to the solution is an important and instructive phase of the solution. By looking back at the completed solution and reconsidering the process that led to it, pupils could improve their knowledge and develop their ability to solve other problems. This way, we can always improve our understanding of the solution. Mistakes can still exist even if we are sure the solution is correct. Hence, proofs are desirable. At this point, it is possible to ask: *Can you check the result? Consider whether the results are correct or not.* One way to verify this is by working backwards.

E – Evaluate your answer

This is the phase when pupils must reflect on their own solutions, identify any errors, verify that the solution satisfies all the given conditions, and interpret the answer.

It is recommended that the answer be evaluated using self-monitoring questions, including whether it made sense, was reasonable, was accurate, and had another way to solve the problem. If, in any case, the answer is not correct, the teacher should encourage pupils to explore by looking at previously solved problems, counterexamples, or anything that might provide information about the solution to the task.

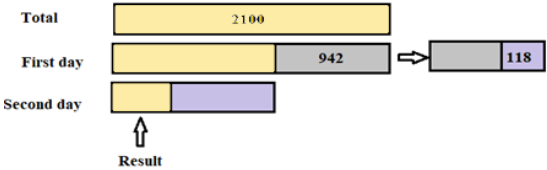
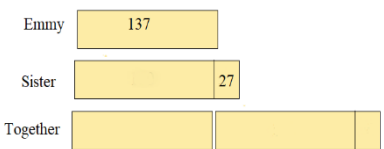
To gain a deeper understanding, use a word problem as an example and show where each step belongs to the solution. Ask pupils to write down the steps and provide them with the instructional sheets (Figure 1). While solving the problem, ask questions and develop discussion so pupils can understand the steps more deeply. *“Is there a step that needs further explanation?” “Can somebody explain why we did this drawing and how we can get the solution?” etc.*

S	I	M	P	L	E
Study the problem Read the problem and rephrase it. "Have I understood what I am supposed to find?"	Involve the bar model method Draw a diagram (create a visual representation of the word problem). How are the pieces of information in the word problem related? How can I represent these relations by rectangles?	Monitor the process Go back to the beginning, including the problem formulation. Ask yourself: Is the bar model method helping me to understand how to reach the solution?/ "Am I getting any closer to my objective? Am I still implementing the solving strategy I picked?"	Prepare / Present the solution Make the expression/ calculation/ equation, write the solution, and present it.	Look over the solution again Check the whole solution, solving steps and the calculations.	Evaluate your answer Evaluate whether your solution makes sense and if there is a better way to solve the problem. Write up the answer.

Figure 1: The front and back of the SIMPLE steps instructional sheet

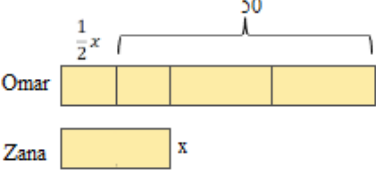
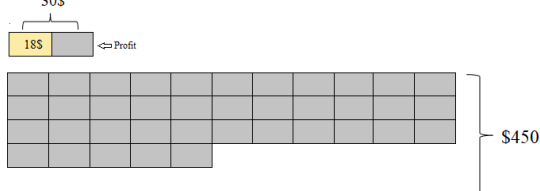
You can use the WP7_I below as illustrations for the application of SIMPLE steps. While solving it, ask pupils to follow every SIMPLE step and encourage them to ask questions if they struggle to understand the solution. Ask pupils for their opinions on each step and provide discussion when needed. After that, provide them with another WP8_I and ask them to solve it individually. You should monitor all the processes of pupils' work and, when needed, help them. Help can be at every step that they struggle with. If they struggle in the second step, you can help them by drawing the bar model for one part of the problem and leaving the other parts for pupils. Or if they struggle to monitor or evaluate their answer, you can ask them questions as described above that can orientate them to continue the solution.

	WP7_I: Of the 2100 kilograms of potatoes in the warehouse, 942 kilograms were sold on the first day, while on the second day, 118 kilograms less than on the first day. How many kilograms of potatoes are left in the warehouse?	WP8_I: Emmy collected 137 stamps, which is 27 stamps less than her sister. How many stamps did they collect together?
S	<i>I must find the kilograms of potatoes that are still unsold after two days. We know the number of potatoes sold on the first day; we need to find it for the second day and then those left in the warehouse.</i>	<i>I must find the number of stamps collected by Emmy and her sister, knowing the number of stamps collected by Emmy.</i>
I	<i>So, I will draw bars showing the kilograms of potatoes left after being sold on the first day. Then, another bar will be for those sold on the</i>	<i>I will draw the bar showing Ema's stamps first and then add another bar for her sister, which will be longer than the</i>

	second day, and the third one will belong to those still unsold.  Total: 2100 First day: 942 Second day: 118 Result: 334	first one. The third bar will represent the total.  Emmy: 137 Sister: 27 Together: 301
M	Yes, I think I am on the right path and getting closer to the objective since now I know what and how much I must subtract to find the final answer.	I think the bar model is helping me to come to the solution because, based on the bars, I understand that Emmy's sister has 27 more stamps than her.
P	First, we subtract the total number of kilograms sold on the first day. The number of kilograms sold on the second day is determined by subtracting 118 from the first day's total. After that, we also eliminated the ones from the second day, which led us to the solution. Day I: $2100 - 942 = 1158$ Day II: $1158 - (942 - 118) = 1158 - 824$ Left $= 334$	Let x be the number of stamps that Emmy's sister has. Emmy $\rightarrow 137$ $x - 27 = 137$ $x = 164$ Together $\rightarrow 137 + 164 = 301$
L	$334 + 824 + 942 = 2100$ (left + second day + first day) = total	$137 + 27 = 164$ $301 - 137 = 164$
E	The final answer is 334, which makes sense since we arrived at the warehouse's total kilograms of potatoes by working backwards.	The result makes sense. We found the number of stamps Amy's sister has by working backwards.

At the end of the lesson, give pupils one or two word problems as homework and ask them to apply every SIMPLE step within it. You can use the word problems provided below.

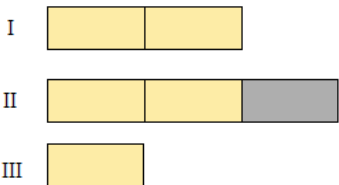
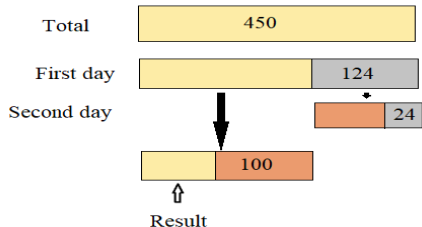
	WP3_H: Omar had three times as many biscuits as Zana. After Omar had eaten 50 biscuits, he had half as many biscuits as Zana. How many biscuits did Omar have left?	WP4_H: Arta is trying to sell cakes. She needs to make at least \$450. Each cake costs \$18 to make. She is selling them for \$30 each. What is the minimum number she can sell to reach her goal?
S	The problem asks to find the number of biscuits Omar left. Although we don't know how many biscuits Omar had at the beginning, we do know that he had three times as many as Zana, and after he had eaten 50, there were left half of what Zana had.	The problem asks to find the minimum number that Arta can sell to make at least \$450. She sells a cake for \$30, while its preparation costs \$18. So the profit is \$12.

I		
M	<i>It is easier now to get the solution since we know what mathematical operation to use based on the model created. So, yes, the bar model method helped me determine the solution process, and now I know I am getting closer to the answer.</i>	<i>It is easier now to get the solution since we know the money Arta needs to make and the profit she gets from selling the cakes.</i>
P	Zana – x Omar - $3x - 50 = \frac{1}{2}x$ Left = $\frac{1}{2}x = \frac{1}{2} * 20 = 10$ $3x - \frac{1}{2}x = 50$ $5x = 100$ $x = 20$	Profit = x; Number of cakes to sell = y $x = 30\\$ - 12\\$ = 18\\$ $y = 450 : x = 450 : 12 = 37.5 \sim 38$
L	$3x - 50 = \frac{1}{2}x$ $3 * 20 - 50 = \frac{1}{2} * 20$ $60 - 50 = 10$ $10 = 10$	$38 * 30 - 38 * 18 = 1140\\$ - 684\\$ $= 456\\$ $38 * 12 = 456\\$ <i>So, the minimum number of cakes she needs to sell to reach the goal is 38.</i>
E	<i>The result makes sense as we found how many cookies Zana and Omar had. Omar initially had 60 cookies, 3 times as many as Zana, so 20. After eating 50 cookies, he had 10 left, half of Zana's cookies left. So we conclude that the solution is correct and that the bar model helped arrive at the solution.</i>	<i>The results make sense since she gets the money needed with the profit she makes by selling 38 cakes.</i>

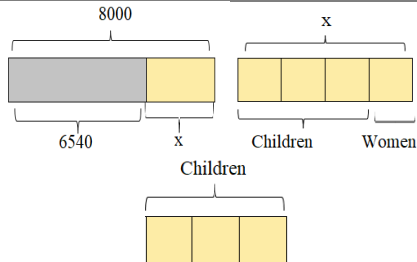
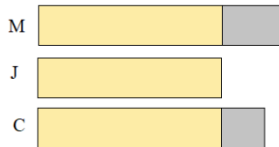
Lesson 2: Practice of the SIMPLE steps

Start the lesson by discussing the homework solution. If needed, provide a solution together with pupils in the blackboard. Use more word problems to practice in the classroom. Form group work to enhance collaborative learning. You can start by asking pupils to work in pairs and then in a larger group so they can learn from each other.

WP9_I: Sarah needed 37 hours and 30 minutes to complete a painting. On the first day, she worked 13 hours, which is 5 hours	WP10_I: Out of 450 books by one author, 124 were sold to a publishing house on the first day, which is 24 more sold books than
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	less than the second day and twice as much as the third day. How many hours did she work on the second and third days?	on the second day. How many of the author's books have not yet been sold?
S	<i>The problem asks us to find Sarah's hours on the second and third days when we know the total number of hours it took her to complete the painting and the work hours on the first day. From the condition given, we know that on the second day, she worked 5 hours more and on the third day, half as much as the first day.</i>	<i>The problem is finding out how many books are still unsold after two days. So, yes, I have to find the authors' books that have not yet been sold.</i>
I		
M	<i>It is easier now to find the solution since we know that on the second day, she worked 5 hours more, and on the third day, half as much as on the first day. So, we have to add and divide to find the answer. The bar model method helped me get closer to the solution.</i>	<i>Since I know what and how much I must subtract to get the final answer, I believe I am on the right path and moving in the correct direction.</i>
P	<i>I – We write as x, the work hours that Sarah did on the first day.</i> <i>II – We write as $x + 5$, the work hours that Sarah did on the second day.</i> <i>III – We write as $\frac{x}{2}$, the work hours that Sarah did on the third day.</i> $I + II + III = 37.5$ $x + x + 5 + \frac{x}{2} = 37.5$ $\frac{5x}{2} = 32.5$ $5x = 65$ $x = 13$	First day: $450 - 124 = 326$ Second day: $326 - (124 - 24) = 326 - 100$ Total unsold: 226
L	$I - x = 13$ $II - x + 5 = 13 + 5 = 18$ $III - \frac{x}{2} = \frac{13}{2} = 6.5$	$226 + 100 + 124 = 450$ (unsold + second day' sold + first day' sold = total)

	$\text{Total: } I + II + III = 13 + 18 + 6.5 \\ = 37.5$	
E	<i>The results make sense since after we found the work hours for the second and third days, we added them together with the first one to arrive at the total number of working hours presented at the beginning.</i>	<i>The result makes sense since we arrived at the total number of books the author had at the beginning by working backwards.</i>

	WP11_I: At a basketball game, there were 8,000 spectators. Of them, 6540 were men. The rest of the audience consisted of women and children, with three times more children than women. Find the number of children in the hall.	WP12_I: In a basketball game, Marc, Jackson, and Cole scored 106 points altogether. Marc scored 15 more points than Jackson. Jackson scored 7 fewer than Cole. How many points did Cole score?
S	<i>The task is to find first the number of women in the hall and then the number of children based on the given conditions. From the requirements, we know that 8000 were present, 6450 men, and the rest consists of women and children, with three times more children than women.</i>	<i>The task asks to find the number of points of three players only when the total number of points they scored together is known. To find how many points Cole scored, we need to start with Jackson's points, which we denote as unknown.</i>
I	 The diagram shows a bar model for the basketball game problem. A large bar is divided into two parts: a grey part labeled 6540 (men) and a yellow part labeled x (children and women). Above the entire bar is a bracket labeled 8000. Below the yellow part, there is a separate bar model showing three yellow boxes labeled 'Children' and one yellow box labeled 'Women', with a bracket above them labeled x.	 The diagram shows a bar model for the basketball game problem. There are three horizontal bars labeled M, J, and C. Bar M is the longest, followed by bar J, and then bar C. Bar M has a grey segment at the end, and bar C also has a grey segment at the end.
M	<i>The bar model method is helping me to reach the solution. Based on the bars, I can find the number of children and women in the game. After finding the number of women, we find the number of children by adding three similar bars. Therefore, I am very close to the solution, and this strategy suits this task.</i>	<i>The bar model method is helping me solve because, based on the bars, I understand that I have to start from Jackson because the points of others depend on him. Since Marc has 15 points more than Jackson, the bar is also longer. Since Jackson has seven fewer points than Cole, Cole's bar is also longer. Therefore, I think I am very close to the solution, and this strategy is suitable for this task.</i>
P	<i>Let: x be the number of women, $3x$ the number of children,</i>	<i>Let x be the number of Jackson's points.</i>

	$x + 3x = 4x$ the number of children and women together. Since I can easily find the number of children and women from the above equation, we divide that number by 4 to get three parts for children and one for women, assuming the proportion between children and women is 3 to 1. $4x = 8000 - 6450$ $1460 = 4x$ $x = 365$ $3x = 365 * 3 = 1095$	Marc has 15 points more than Jackson, which means $x + 15$, and Cole also has 7 points more than Jackson $x + 7$. $M = x + 15; J = x; C = x + 7$ $M + J + C = 106$ $x + 15 + x + x + 7 = 106$ $3x = 106 - 22$ $3x = 84$ $x = 28$ $M = x + 15 = 28 + 15 = 43,$ $J = x = 28,$ $C = x + 7 = 28 + 7 = 35$
L	$3x + x + 6540 = 8000$ $1095 + 365 + 6540 = 8000$	$M: x + 15 = 28 + 15 = 43$ $J: x = 28$ $C: x + 7 = 28 + 7 = 35$ $M + J + C = 106$ $43 + 28 + 35 = 106$
E	The result makes sense because, by working together and adding the number of children, women, and men, we arrived at the total number of spectators present at the game.	The result makes sense because, by working backwards, we determined each player's points and arrived at the final points scored by the three basketball players.

3.3 Constructivist Principles

Constructivism is a theory with solid implications in education which explains how knowledge is achieved, transmitted by teachers, and received by pupils (Chand, 2022; Shah, 2019). According to the philosophy of constructivism in education, knowledge and critical thinking are actively constructed by involving the learners in the learning process and engaging in classroom activities rather than passively absorbing information from the teacher (Zajda, 2021). The theory of constructivism originates from the work of Jean Piaget and Lev Vygotsky, who initiated the idea that traditional learning is passive learning (Chand, 2022; Fosnot & Perry, 1996; Piaget, 1973). Piaget emphasises that knowledge about the world is constructed rather than received in all these stages. Based on Piaget's work, humans assimilate and adapt to new information based on prior knowledge, advocating for an educational approach that encourages problem-solving and critical thinking.

Some of the principles of constructivism are:

- Learning is based on prior knowledge, experience, and beliefs when learning a new topic.
- Knowledge is constructed rather than received.
- The process of learning is both reflective and active.
- The learners regulate and control their own learning on a deeper level.

- Learning is developed through accommodation, assimilation, or adaptation.
- Social interaction is crucial to constructivist learning, including multiple perspectives through reflection, collaboration and negotiation.

To achieve these principles, you can use the examples below in your teaching:

- From small groups of pupils to working together while practising the bar model method and the SIMPLE steps
- Engage pupils by asking questions and developing discussion in the classroom.
- Provide scaffolding techniques when teaching
- Make learning relevant to pupils whenever possible.

Applying these three factors described above facilitates word problem learning and teaching. To see this approach's impact on word problem solutions, it is preferable to use it often, in the early phases of learning word problems, and for a longer period of time.

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Appendix 2: Teachers' Semi-Structured Interview Questions

A semi-structured interview concerning the SIMPLE Approach

Hi, my name is Qendresa Morina. I am happy you have agreed to participate in this study. I organised this interview to learn your thoughts about the SIMPLE Approach after its application. Please answer the questions as honestly as possible, as your response will greatly help my study. I will record and transcribe this interview. However, the information you discuss with me will be kept confidential.

If you have any questions, please feel free to ask me. Thank you!

Date: _____ Place: _____ Type of interview: _____

Starting time: _____ Ending Time: _____

1. Can you describe how a lesson is usually conducted in your classroom?
2. Do you use word problems in mathematics lessons?
3. Have you used any specific method or strategy in solving them? If yes, can you describe how you did it?
4. Is there any strategy presented to pupils in mathematics textbooks?
5. Have you used visualisation to solve word problems before your lessons?
6. What do you think about the teaching approach, and how do you perceive its usefulness?
7. Do you think this strategy is suitable for solving word problems?
8. Did the teaching approach help the pupils to solve word problems?
9. How confident are you with adopting it in your teaching during word problem solutions?
10. Can you describe how you felt when using it?
11. Which SIMPLE steps do you think help pupils the most?
12. Do you think any steps need to be changed, as they hinder pupils from obtaining the solution?
13. Would you use it in the future solution of word problems? If not, why not?
14. Do you think pupils will use this approach in the future?
15. Do you want to add any suggestions to improve the intervention?

Appendix 3: Pupils' Semi-Structured Interview Questions

A semi-structured interview concerning the SIMPLE Approach

Hi, my name is Qendresa Morina. I am happy you have agreed to participate in this study. I organised this interview to learn your thoughts about the SIMPLE Approach after its application. Please answer the questions as honestly as possible, as your response will greatly help my study. I will record and transcribe this interview. However, the information you discuss with me will be kept confidential.

If you have any questions, please feel free to ask me. Thank you!

Date: _____ Place: _____ Type of interview: _____

Starting time: _____ Ending Time: _____

1. Can you describe how you conduct a normal mathematics lesson?
2. Can you describe what a word problem is for you?
3. Do you use these problems with your teacher in mathematics classes? If so, how often?
4. How have you solved these tasks so far?
5. Have you used any specific method or strategy in solving them? If yes, can you describe how you did it?
6. What do you think about the SIMPLE strategy? How does this strategy look to you?
7. Do you think this approach is suitable for solving word problems?
8. Did the BM method and the SIMPLE steps help you solve word problems?
9. How do you perceive the approach's usefulness?
10. How confident are you with adopting it in your learning during word problem solutions?
11. Can you describe how you felt when teaching it?
12. Would you use it in the future solution of word problems? If not, why not?
13. Which of the SIMPLE steps seems the most beneficial to you?
14. Are any steps hindering you or not helping you solve word problems?
15. Would you use it in future word problem solutions? If not, why not?
16. Do you have anything to add about this strategy or anything else?

Appendix 4: Teachers' Diary Format

Date: _____	Lesson taught: _____
Describe the lesson taught	
How did the intervention went / Describe the process	
Any suggestions / critics / opinions / new ideas	

Appendix 5: WPs Used in All Phases of the Study with the Arithmetic, Algebraic, and Bar Model Method Solution

Pre-test WPs

WP1_E. Of the 450 books by an author, 124 were sold to one publishing house on the first day, which is 24 more books sold than on the second day. How many of the author's books have not been sold yet?

Arithmetic	Algebraic	Bar Model
$450 - 124 = 326$ $326 - (124 - 24)$ $326 - 100 = 226$	m - the number of unsold books. t - the total number of books at the beginning. $f1$ - the number of books sold on the first day. $f2$ - the number of books sold on the second day (unknown). $m = t - f1 - f2$ $t = 450; f1 = 124$ $f2 = (124 - 24) = 100$ $m = 450 - 124 - 100 = 226$	

WP2_E. During the running competitions, on the first day, the pupils ran 0.7 km; on the second day, 0.4 km more than the first day; and on the third day, 0.3 km less than the second day. How many km did they run in total?

Arithmetic	Algebraic	Bar Model
$0.7km + 0.4km = 1.1km$ $1.1km - 0.3km = 0.8km$ $0.7km + 1.1km + 0.8km = 2.6 km$	<u>First day</u> $x = 0.7$ <u>Second day</u> $y = x + 0.4 = 0.7 + 0.4 = 1.1$ <u>Third day</u> $z = y - 0.3 = 1.1 - 0.3 = 0.8$ <u>Total</u> $x + y + z = 0.7 + 1.1 + 0.8 = 2.6 km$	

WP3_E. Three groups of pupils in one class bought 35 pizzas to celebrate their success. The first group got 16 pizzas, which is 5 pizzas more than the second group and two times more than the third group. How many pizzas do the second and third groups get?

Arithmetic	Algebraic	Bar Model
$16 - 5 = 11$ $16 : 2 = 8$ $16 + 11 + 8 = 35$	Total: $T = 35$ Group I: $x = 16$ Group II: $y = x - 5 = 16 - 5 = 11$ Group III: $z = \frac{x}{2} = \frac{16}{2} = 8$ $T = x + y + z$ $35 = 16 + 11 + 8$	

WP4_E. On the second day of the trip, a family decides to go to a game park. They buy 4 adult tickets for \$49 each and 8 student tickets for \$31 each. How much money do they get back if they pay with five \$100 bills?

Arithmetic	Algebraic	Bar Model
$5 * 100\$ = 500\$$ $4 * 49\$ = 196\$$ $8 * 31 = 248\$$ $500\$ - 196\$ - 248\$ = 56\$$	Adult: x Student: y Money: $M = 5 * 100\$ = 500\$$ $x = 4 * 49\$ = 196\$$ $y = 8 * 31\$ = 248\$$ Left: $L = M - (x + y)$ $= 500\$ - (196\$ + 248\$)$ $= 500\$ - 444\$ = 56\$$	

WP5_E. Andi, Era and Ari wanted to buy a bicycle that cost 101 euros. Andi had 20 euros more than Era. Era had 6 less than Ari. How much money did Ari have?

Arithmetic	Algebraic	Bar Model
Start with all the money (101 euros). Give 6 euros to Era, leaving 95 euros. Give Andi his extra 20 euros, leaving 75 euros. Split 75 euros between Ari and Era and Andi, ensuring Era has 6 euros less. This leads to: $Ari = 31 \text{ euros}$ $Era = 25 \text{ euros}$ $Andi = 45 \text{ euros}$	Ari: x Era: $y = x - 6$ Andi: $z = y + 20 = x - 6 + 20$ Total: $T = 101$ $T = x + y + z$ $101 = x + x - 6 + (x - 6 + 20)$ $101 = 3x + 8$ $3x = 101 - 8$ $3x = 93$ $x = 31$	

WP6_E. Emma bought 22 children's toys, which is 5 more toys than Andra bought. They donated 22 of their toys. How many toys are left?

Arithmetic	Algebraic	Bar Model
$22 - 5 = 17$ $22 + 17 = 39$ $39 - 22 = 17$	Andra: x Ema: $x + 5 = 22$ $x = 17$ $Andra + Ema =$ $= x + (x + 5)$ $= 17 + (17 + 5) = 39$ Left: $39 - 22 = 17$	

Post-test WPs

WP1_O. Roni had 3 times as much money as Kiara. Greta had 2 times as much money as Kiara. If they have a total of 294\$, how much money does each have?

Arithmetic	Algebraic	Bar Model
$\frac{294}{6} = 49$ $Kiara = 49$ $Greta = 2 * 49 = 98$ $Roni = 3 * 49 = 147$	Kiara: x Greta: $y = 2x$ Roni: $z = 3x$ Total: $T = 294\\$ $T = x + y + z$ $294 = x + 2x + 3x$ $294 = 6x$ $x = 49$ Kiara: $x = 49$ Greta: $y = 2x = 2 * 49 = 98$ Roni: $z = 3x = 3 * 49 = 147$	R:  G:  K: 

WP2_O. Era, Hana, and Andy had 64 books to read this summer. Era read 16 books, which is 10 books more than Hana and twice less than Andy. How many books are left unread?

Arithmetic	Algebraic	Bar Model
$64 - 16 - (16 - 6)$ $= 48 - 6 - 32$ $= 10$	Ema: $x = 16$ Ana: $y = x - 10 = 16 - 10 = 6$ Andy: $z = x * 2 = 16 * 2 = 32$ Total: $T = 64$ $Left = T - (x + y + z)$ $Left = 64 - (16 + 6 + 32)$ $Left = 64 - 54$ $Left = 10$	E:  H:  A: 

WP3_O. Four owners of a sports field must contribute to the purchase of new equipment. Its price is 550 dollars. If the first person gives \$35, the second person twice as much as the first, the third \$30 more than the second, and the fourth the total of the first three, how much money is left ungiven?

Arithmetic	Algebraic	Bar Model
$550 - 35 - (35 * 2) -$ $-(35 * 2 + 30) -$ $(35 + 35 * 2 + 35 * 2$ $+ 30)$ $= 550 - 35 - 70 - 100$ $- 205$ $= 550 - 410 = 140$	Price: $P = 550$ P1: $I = 35$ P2: $II = 2 * I$ P3: $III = II + 30$ P4: $IV = I + II + III$ $L = P - (I + II + III + IV)$ $L = 550 - (35 + 70 + 100 + 205)$ $L = 550 - 410$ $L = 140$	P  I  II  III  IV 

WP4_O. Era read 25 books this year, which is five books more than Hana and 12 books less than Andi. How many books did they read together?

Arithmetic	Algebraic	Bar Model
$25 + (25 - 5) + (25 + 12)$ $= 25 + 20 + 37$ $= 82$	Era: $x = 25$ Hana: $y = x - 5 = 25 - 5 = 20$ Andi: $z = x + 12 = 25 + 12 = 37$ Total: $T = ?$ $T = 25 + 20 + 37 = 82$	E:  H:  A: 

WP5_O. Drena Primary School has 280 pupils, which is 89 fewer pupils than Dielli Primary School and 62 fewer pupils than Hana School. How many pupils are there in total?

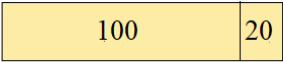
Arithmetic	Algebraic	Bar Model
$280 + (280 + 89) + (280 + 62)$ $= 280 + 369 + 342$ $= 991$	Drena: x Dielli: $y = x + 89$ Hana: $z = x + 62$ $x = 280$; $y = 280 + 89 = 369$ $z = 280 + 62 = 342$ $T = x + y + z$ $T = 280 + 369 + 342$ $T = 991$	Dr:  Di:  H: 

WP6_O. A shepherd has 90 sheep. He has 3.5 times less black sheep than white ones. How many white and black sheep does he have?

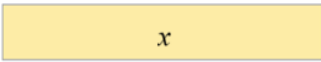
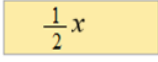
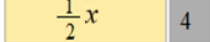
Arithmetic	Algebraic	Bar Model
$\frac{90}{1 + 3.5} = \frac{90}{4.5} = 20$ $90 - 20 = 70$	Black: x White: $y = 3.5 * x$ $x + y = 90$ $x + 3.5x = 90$ $4.5x = 90$ $x = 20$ $y = 3.5x = 3.5 * 20 = 70$	White  Black  Total 

Intervention WPs

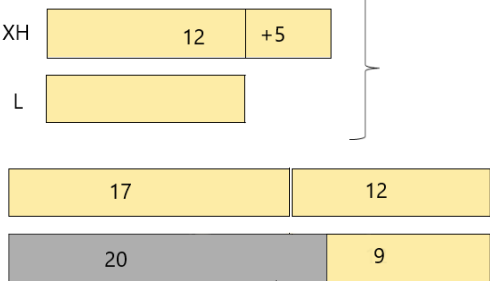
WP1_I. There were 120 visitors to Clara's art exhibit on the first day, which is 20 more than on the second day. How many visitors came to the exhibition on the two days?

Arithmetic	Algebraic	Bar Model
$120 + (120 - 20)$ $= 220$	$I - x = 120$ $II - y = x - 20 = 120 - 20$ $= 100$ $T = I + II = 120 + 100 = 220$	I  II 

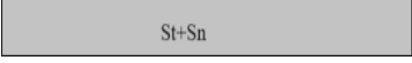
WP2_I. Klara gives half of her paintings to her sister, Suzanne, and then buys herself four more. If she now has 12 paintings, how many did Klara have at the beginning?

Arithmetic	Algebraic	Bar Model
$(12 - 4) * 2 = 16$	$\frac{1}{2}x + 4 = 12$ $\frac{1}{2}x = 8$ $x = 16$	I  II  III 

WP3_I. Xhamie picked 17 flowers, which is five more flowers than Linda picked. They donated 20 of the flowers. How many flowers were left?

Arithmetic	Algebraic	Bar Model
<i>Xhamie(Xh):</i> 17 <i>Linda(L):</i> $17 - 5 = 12$ $(17 + 12) - 20 = 9$	$L = x$ $L + Xh = x + x + 5$ $= 12 + 17$ $= 29$ $Xh = x + 5 = 17 \rightarrow$ $x = 12$ <i>Left: 29 - 20 = 9</i>	

WP4_I. 305 children went to a zoo on Saturday. The number of children who visited the Zoo on Saturday was 278 less than on the Sunday visit. How many children went to the zoo on Sunday?

Arithmetic	Algebraic	Bar Model
<i>Saturday(St)</i> = 305 <i>Sunday(Sn)</i> = $= 305 + 278$ $= 583$	$St = 305$ $Sn = St + 278$ $Sn = 305 + 278 = 583$	St  Sn  T 

WP5_I. Andi, Era, and Ari scored 106 points in a volleyball match. Andi scored 15 points more than Era. Era scored seven less than Ari. How many points did Ari score?

Arithmetic	Algebraic	Bar Model
$106 - 15 - 7 = 84$ Era: $\frac{84}{3} = 28$ Ari = $28 + 7 = 35$	$x + 15 + x + x + 7 = 106$ $3x + 22 = 106$ $3x = 106 - 22$ $3x = 84$ $x = 28$ Ar = $28 + 7 = 35$	A E Ar

WP6_I. Brenda is an art teacher. To prepare for tomorrow's project, she opens a box of 80 crayons. She removes eight broken crayons and divides the rest equally into six boxes. Find the number of crayons in each box.

Arithmetic	Algebraic	Bar Model
I: $80 - 8 = 72$ II: $72 : 6 = 12$	Crayons - $x = 80$ $\frac{x - 8}{6} = \frac{80 - 8}{6} = \frac{72}{6} = 12$	

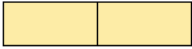
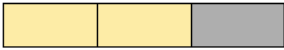
WP7_I. Of the 2100 kilograms of potatoes in the warehouse, 942 kilograms were sold on the first day, while on the second day, 118 kilograms less than on the first day. How many kilograms of potatoes are left in the warehouse?

Arithmetic	Algebraic	Bar Model
$2100 - 942 - (942 - 118)$ $= 334$	Total: $x = 2100$ Day I: $y = 942$ Day II: $z = y - 118 = (942 - 118) = 824$ Left = $x - y - z$ $= 2100 - 942 - 824$ $= 334$	

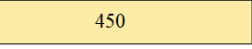
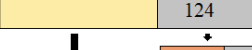
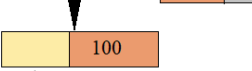
WP8_I. Emmy collected 137 stamps, which is 27 stamps less than her sister's. How many stamps did they collect together?

Arithmetic	Algebraic	Bar Model
$137 + (137 + 27)$ $= 137 + 164 = 301$	Let x be the number of stamps that Emmy's sister has. Emy $\rightarrow 137$ $x - 27 = 137$ $x = 164$ Together $\rightarrow 137 + 164 = 301$	Emmy Sister Together

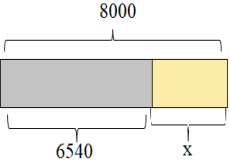
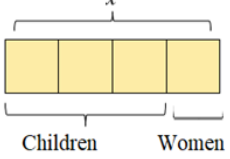
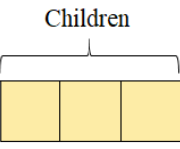
WP9_I. Sarah needed 37 hours and 30 minutes to complete a painting. On the first day, she worked 13 hours, which is 5 hours less than the second day and twice as much as the third day. How many hours did she work on the second and third days?

Arithmetic	Algebraic	Bar Model
13 $13 + 5 = 18$ $\frac{13}{2} = 6.5$ $13 + 18 + 6.5 = 37.5$	<i>I:</i> We write as x, the work hours that Sarah did on the first day. <i>II:</i> We write as $x + 5$, the work hours that Sarah did on the second day. <i>III:</i> We write as $\frac{x}{2}$, the work hours that Sarah did on the third day. $I + II + III = 37.5$ $x + x + 5 + \frac{x}{2} = 37.5$ $\frac{5x}{2} = 32.5$ $5x = 65$ $x = 13$	I  II  III 

WP10_I. Out of 450 books by one author, 124 were sold to a publishing house on the first day, which is 24 more sold books than on the second day. How many of the author's books have not yet been sold?

Arithmetic	Algebraic	Bar Model
First day: $450 - 124 = 326$ Second day: $326 - (124 - 24) = 326 - 100$ Total unsold: 226	$I - x = 124$ $II - y = x - 24$ $Total - T$ $L = T - x - y$ $L = 450 - 124 - 100$ $L = 226$	Total  First day  Second day  Result 

WP11_I. At a basketball game, there were 8,000 spectators. Of them, 6540 were men. The rest of the audience consisted of women and children, with three times more children than women. Find the number of children in the hall.

Arithmetic	Algebraic	Bar Model
$8000 - 6540 = 1460$ $\frac{1460}{4} = 365$ $365 * 3 = 1095$	Let: x be the number of women, $3x$ the number of children, $x + 3x = 4x$ the number of children and women together. Since I can easily find the number of children and women from the above equation, we divide that number by 4 to get three parts for children and one for women, assuming the proportion between children and women is 3 to 1. $4x = 8000 - 6450$ $1460 = 4x$	  

	$x = 365$ $3x = 365 * 3 = 1095$	
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WP12_I. In a basketball game, Marc, Jackson, and Cole scored 106 points altogether. Marc scored 15 more points than Jackson. Jackson scored 7 fewer than Cole. How many points did Cole score?

Arithmetic	Algebraic	Bar Model
$106 - 15 - 7 = 84$ $J: \frac{84}{3} = 28$ $Cole = 28 + 7 = 35$	Let x be the number of Jackson's points. Marc has 15 points more than Jackson, which means $x + 15$, and Cole also has 7 points more than Jackson $x + 7$. $M = x + 15; \quad J = x; \quad C = x + 7$ $M + J + C = 106$ $x + 15 + x + x + 7 = 106$ $3x = 106 - 22$ $3x = 84$ $x = 28$ $M = x + 15 = 28 + 15 = 43,$ $J = x = 28,$ $C = x + 7 = 28 + 7 = 35$	M  J  C 