

Univerzita Karlova
Pedagogická fakulta

DISERTAČNÍ PRÁCE

2023

Miroslava Brožová

Univerzita Karlova
Pedagogická fakulta

DISERTAČNÍ PRÁCE

Individualization, differentiation, and personalization
in mathematics in the Czech Republic through three different
approaches: Scheme-oriented, Child-centred, and Undesignated

Individualizace, diferenciacce a personalizace ve třech přístupech
vyučování matematice na 1. stupni ZŠ v České republice
– metoda budování schémat, pedocentricky orientovaná metoda
a nevyhraněná metoda

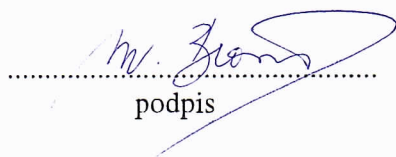
Miroslava Brožová

Školitelka: doc. PaedDr. Jana Coufalová, CSc.
Studijní program: Pedagogika
Studijní obor: Didaktika matematiky

2023

Prohlašuji, že jsem disertační práci na téma *Individualization, differentiation, and personalization in mathematics in three different approaches – scheme-oriented, child-centred, and undesignated approach in the Czech Republic* vypracovala pod vedením školitelky samostatně za použití v práci uvedených pramenů a literatury. Dále prohlašuji, že tato práce nebyla využita k získání jiného nebo stejného titulu.

V Praze dne 19. 6. 2023


.....
podpis

Tímto chci vyjádřit srdečné poděkování své školitelce, doc. PaedDr. Janě Coufalové, CSc., za její vedení, empatii a důvěru při realizaci práce. Za cenné rady a připomínky upřímně děkuji také prof. RNDr. Jarmile Novotné, CSc, bez jejíž podpory by práce nemohla být dokončena. Mé díky patří všem, kteří mě k sepsání této práce motivovali, a zejména účastníkům mého výzkumu. V neposlední řadě děkuji těm, kteří mi neúnavně umožňovali věnovat se práci a poskytovali mi zázemí a prostředí vhodné pro tvůrčí činnost.

I would like to express my cordial gratitude to my supervisor, doc. PaedDr. Jana Coufalová, CSc., for her leadership, empathy, and faith in the realization of the work. Further, I would like to sincerely thank Prof. RNDr. Jarmila Novotná, CSc, for valuable advice and comments; the work could not have been completed without her support. I express my gratitude to all those who motivated me to write this work and especially to the participants in my research. Last but not least, I thank those who tirelessly allowed me to work and provided me with a background and environment suitable for creative activities.

ABSTRAKT

Cílem předkládané práce bylo popsat, zda a jak jsou uplatňována individualizace, diferenciaci a personalizace ve vybraných programech, jejich principech, učitelských přesvědčeních a praktikách ve výuce matematiky na prvním stupni základní školy. Za účelem sběru dat bylo vybráno šest učitelů se třemi odlišnými přístupy k výuce – výuka orientovaná na budování schémat, pedocentricky orientovaná metoda a nevyhraněný přístup hlavního vzdělávacího proudu v České republice.

V teoretické části práce byly definovány základní pojmy a vymezeny všechny tři vybrané přístupy k výuce. V experimentální části byly na pozadí charakteristik individualizace, diferenciaci a personalizace interpretovány epizody z rozhovorů o přesvědčeních a praktikách sledovaných při výuce. Z praktik bylo konkrétně popsáno užití učebnic a zadávání domácích úkolů. Ukázalo se, že učitelé ve výuce orientované na budování schémat i učitelé pedocentricky orientované metody využívají při výuce prvky individualizace, diferenciaci i personalizace. U učitelů v nevyhraněném přístupu hlavního proudu se objevuje individualizace. Analýza ve všech sledovaných oblastech u vybraných učitelů naznačuje, které z charakteristik převažují u jednotlivých učitelů a přístupů (v Hejného metodě, montessoriovské metodě a nevyhraněném přístupu).

KLÍČOVÁ SLOVA

individualizace, diferenciaci, personalizace, přístup k výuce matematiky, první stupeň ZŠ

ABSTRACT

The goal of the presented work was to determine whether and how individualization, differentiation, and personalization are utilized in the principles of the selected programs and in the teachers' beliefs and practices in the teaching of mathematics at elementary school. In order to collect data, six teachers in the Czech Republic were selected with three different approaches to teaching– Scheme oriented approach, child-centred educational approach, and an undesignated mainstream approach.

The basic concepts and all three selected approaches to teaching were defined in the theoretical part of the work. In the experimental part, episodes from interviews about beliefs and practices identified in teaching were interpreted within the context of characteristics of individualization, differentiation, and personalization. Practices such as the use of textbooks and homework assignments were specifically described. The study shows that teachers of a scheme-oriented approach and a child-centred educational approach use elements of individualization, differentiation, and personalization in their practices. Individualization appears in the teaching of the undesignated mainstream approach, but this approach does not contain aspects of differentiation and personalization. The analysis in all monitored areas of selected teachers indicates which of the characteristics prevail in individual teachers' work and the approaches. The research confirmed that selected teachers in all three approaches (Hejny method, Montessori method and an undesignated) individualize, differentiate, or personalize to some extent.

KEYWORDS

individualization, differentiation, personalization, approach to teaching mathematics, elementary level

1 Content

Introduction	9
1 Theoretical Framework.....	12
1.1 Individualization, differentiation, and personalization	12
1.1.1 Individualization.....	13
1.1.2 Differentiation	16
1.1.3 Personalization	21
1.2 IDP in specific parts of the learning process	24
1.3 IDP in teachers' beliefs.....	28
1.4 IDP in teaching and learning – related research	29
1.5 Mathematics education programs in the Czech Republic and IDP	30
1.5.1 Scheme-oriented approach - Hejny Method.....	31
1.5.2 Child-centred educational approach - Montessori method.....	34
1.5.3 Shared principles of Hejny and Montessori mathematics	37
1.5.4 Undesignated mainstream approach.....	39
1.6 Research questions.....	41
2 Methodology.....	42
2.1 Methods of research.....	42
2.2 Participants.....	42
2.3 Data collection	44
2.4 Data analysis	45
3 Findings	48
3.1 Characteristics of IDP stated in the approach principles (by the author or in literature)	48
3.2 Espoused mathematics educational beliefs.....	50

Characteristics of IDP which prevailed in teachers' beliefs.....	59
3.3 Teachers' practices – textbooks and homework	60
3.3.1 Use of textbooks	61
3.3.2 Assigning homework.....	63
3.3.3 Characteristics of IDP in use of textbooks and homework	65
3.4 IDP identified in teachers' practices and their statements collected during videorecording phases	68
3.4.1 Presence of individualization characteristics.....	68
3.4.2 Presence of differentiation characteristics.....	77
3.4.3 Presence of personalization characteristics	92
3.4.4 Characteristics of IDP identified in teachers' practices in classrooms (including interviews a priori and a posteriori)	103
3.5 Overall prevailing characteristics for each teacher and method	108
4 Discussion.....	111
4.1 Limitations of my research	114
5 Conclusion.....	116
References	118
Appendix list	129
Appendix A	130
Appendix B.....	131

Introduction

Working in an elementary classroom with learners¹ aged 6-12 years old and reviewing literature (for example Dewey, 1963; Vygotsky, 1978) caused me to think about individual readiness for further development of mathematical thinking and how widely it differs amongst children. There are less motivated children who are not able to work independently, as well as learners with inner motivation and skills well beyond their current grade. The question is how to divide a teacher's time and energy between these learners in academically diverse classrooms so that teachers of elementary mathematics can help all learners maximize their potential.

In growing numbers, educators have been realising for decades that their theory and practice must be founded upon the principle of individuality (Fynne, 1924). People understand that each human being has not only talents and natural abilities but also weaknesses. Even within mathematics, we do not have equal knowledge and skills and therefore, learners should have the opportunity to show and develop all their educational desires. As Tomlinson (2014) states: "Children seem to accept a world in which we are not alike. They do not quest for sameness, but they search for the sense of triumph that comes when they are respected, valued, nurtured, and even cajoled into accomplishing things they believed beyond their grasp" (p. 10).

Large scale international surveys, such as TIMSS which aims to assess '*what students know*' and PISA which seeks to find '*what students can do with their knowledge*' (EACEA, 2011, p. 13), show that the output of education does not only depend on the learners themselves, their abilities, and family background, but also on the quality of instruction and the organization of the educational system. Individualized or differentiated teaching is implemented in many countries. It usually involves teaching the same content but at different levels of difficulty (EACEA, 2011). It is one of the educational trends encouraged and promoted by the Framework Educational Programme for Basic Education in the Czech

¹ The term "learner" was adopted from Bray and McClaskey (2015) who determined that it was the most accurate term and fits more than "students" or "pupils" in all learning environments.

Republic (Jeřábek and Tupý et al., 2007) to endorse teachers to consider learners' needs and potential.

Strong et al. (2004) stated that “mathematics seems like a kind of worst-case scenario for differentiation. Both the nature of its content and its largely sequential organization make considerations of student differences seem peripheral” (p. 78). Therefore, if we are to ensure the best of education for every learner, we should familiarize ourselves with the programs, processes and practices that could be beneficial to all learners. We should become acquainted with each learner as an individual person, understand their preferred learning styles, learn their interests, and discover their cultural background as much as possible.

The present work is focused on the utilization of individualization, differentiation, and personalization (IDP) in the elementary classrooms (learner age 6-12). The main goal of this work is to determine and understand how IDP are utilized in selected programs, their principles, teacher's beliefs, and their practices.

Firstly, I was interested if there is actually a real need to individualize, differentiate or personalize. The preliminary research was conducted in 2009 (Coufalová and Chmelová, 2010) with the goal of mapping out elementary learners' individual capabilities for further development of mathematical thinking. Since the goal of the study was to determine the potential of each learner, the problems were chosen in such a way that they would be able to be solved through various means. It was found that learners were able to solve even more difficult problems than would correspond to the common curriculum in each grade and its implementation during the school year. Important variations were also recorded in the time range that students needed for success in particular problem solving. It was shown that the differences in elementary learners' individual capabilities are significant at entry to first grade and remain throughout the elementary period.

The consequent survey was conducted in 2011 (Brožová and Coufalová, 2012) with the aim of understanding cultural differences pertinent to teaching primary mathematics. The respondents, representatives from 18 countries both within and outside of Europe, were asked to describe the techniques and procedures used in their countries to teach primary mathematics. The study found differences in writing numbers, writing operations, and in

the processes that are used in teaching mathematics. The authors were confident that cultural differences may influence mathematics instruction.

There were three different school programs selected for this work to get an insight into the process of IDP in mathematics. Namely: a. Scheme-oriented approach (program based on Theory of generic models by Hejny, 2012), b. Child-centred educational approach (program based on Montessori pedagogy), c. Undesignated mainstream approach (teaching program with no set or specified methodology following other than Hejny textbooks, recommended by heads of schools as typical model). We analysed teachers' beliefs and practices in the mathematics classrooms to get an insight into the way individualization, differentiation and personalization are enacted in them.

1 Theoretical Framework

First, we explain the terms IDP using definitions and interpretations of various authors. Additionally, we describe what principles teachers follow in classrooms using different teaching approaches when teaching mathematics and which part of the process of teaching or learning can be individualized, differentiated, or personalized. Finally, we describe what the beliefs are, and how we can use them to determine what causes and influences different kinds of teaching processes. It is specified how I understand these terms, and by which means I use them throughout my work.

1.1 Individualization, differentiation, and personalization

Pupil learning and achievement can be affected by diverse issues such as culture, language, learning style, and personality. Because diversity influences learner behaviour, educators need to consider instructional strategies that meet the diverse characteristics of pupils (Johnsen, VanTassel-Baska and Robinson, 2008). “Elementary school children should enjoy school and should be able to make some choices about learning. Furthermore, all pupils should experience some success in school and should be able to move toward independence” (Jones, 1975, p. 21). It is believed that the teacher needs to assist children to learn for themselves rather than from fear of rejection or desire for praise.

Many of the proposals for changing the traditional system are referred to as individualized programs. Others exhibit the unmistakable hallmarks of individualization even though they do not bear the title, and to these may be added still others, programs not necessarily associated with the school but clearly individualized.

Many different definitions of individualized instruction are offered which describe what should happen in the education of children as well as criticism of current practices in education. It is apparent that much thinking and research has been taking place with the goal of improving the education of children (Jones, 1975, p. 22).

They are based on different interpretations of individualization. They are inspired by different philosophies and theories, influenced by different technologies and expertise, and confounded by the ambiguity of their label. It has been already stated over 50 years ago by

Gibbons (1970) and others, for example Doll and Ronald (1964) or Shane (1962), that programs that accept differences among students have been proposed and developed.

However, IDP should not be taken as buzzwords because, although such terms have been around in teaching learning systems for many years (O'Brien and Guiney, 2001), little agreement exists on what exactly they mean beyond the broad concept that each is an alternative to the one-size-fits-all model of teaching and learning. For example, some education professionals use personalization to mean that students are given the choice of what and how they learn according to their interests, and others use it to suggest that instruction is paced differently for different students. Understandably, state standards summarized in curriculum represent the knowledge to be taught but differentiated (individualized or personalized) instruction gives a meaningful way to reach those required standards (Protheroe, 2007).

In the following sections, I describe the three terms in detail and specify which definitions lay the foundation for my research. For better understanding of each term, I chose various sources. The major differences in terms of IDP I find in the description of Office of Educational Technology, U.S. Department of Education (2010) and work of Bray and McClaskey (2015). Their explanations and tables (see appendix A) inspired me in my research, and I use them in my work.

1.1.1 Individualization

According to the Framework Educational Programme for Basic Education in the Czech Republic (Jeřábek and Tupý et al., 2007), teachers are supposed to apply variable organizational patterns and individualization of the educational process. Teachers usually have some experience with individualization (alternatively formulated as individual approach) to address the needs of special education learners as one of the strategies supporting inclusion (Kratochvílová and Havel, 2012). The Individualized Education Programs (IEPs) are implemented for a limited number of students and generate a lot of additional administration work. In the following definitions, I show that beyond IEPs, individualization can have more complex meaning and use.

I also show that some serious conflicts can be found concerning definitions or descriptions of individualized instruction.

Individualization means adapting the instruction to the individual rather than the individual having to adjust to the instruction.

It means utilizing all the techniques of modern education, communication, and technology to assist the individual towards self-development and self-fulfilment. (National School Public Relations Association, 1971, p. 1)

Individualization refers to the individual interaction between a teacher and a learner, including planning and conducting a general program of study and day-to-day lessons that are tailored to the learning needs and characteristics of the learner (Heathers, 1971). Although the term individualization applies to both teaching and learning, a teacher drives instruction and that might be a reason why, in the literature, individualization is often connected with a teacher's instruction.

McCarthy (1980) emphasized learners' differences in "aptitude, learning rate, culture, and motivation, as well as other variables." (p. 59) By individualized instruction, teachers attempt to improve learning by tailoring instruction to differences. Individualized instruction is paced by the learning needs, skills, and motivation of the individual learner (Office of Educational Technology, U.S. Department of Education, 2010). In accordance with individualized instruction, learners work on materials at their own level and rate (Slavin and Karweit, 1985). According to Larsson (1973), "instructions are given to one pupil at a time. They are shaped by what the teacher knows about the pupil's ability and modified according to the pupil's questions and answers." (p. 34)

Contrary to the previously described definitions, "Howes (1970, p. 67) called individualized instruction self-selection, self-direction with responsibility, and self-actualizing autonomous individuals." (pp. 1-5, cited in Jones, 1975, p. 23).

If this definition is accurate, and individualized instruction is about the responsibility of a learner, it raises the question whether instruction from the teacher is always necessary.

Teachers working with individualized instruction have to give up traditional educator roles such as discipline providers and information distributors in the teaching learning process. The realization of individualized instruction includes planning of instruction, organizing of learners and the educational environment, and motivating learners. Teachers serve as guides

to facilitate learning, mentors, or consultants in individualized instruction (Jonassen and Grabowski, 1993). In this context, teachers have an important role during the implementing and evaluating of teaching-learning activities.

It is the teacher who chooses tasks for one learner at a time. The number of tasks and their degree of difficulty varies from learner to learner. All learners work at their own rate and with methods and materials that suit their ability (Larsson, 1973).

Office of Educational Technology, U.S. Department of Education (2010) states that in individualization

learning goals are the same for all students, but students can progress through the material at different speeds according to their learning needs. For example, students might take longer to progress through a given topic, skip topics that cover information they already know, or repeat topics they need more help on (p. 12).

Similarly, according to Kasíková, Dittrich and Valenta (2007), there are two basic principles of individualization: Principle-managed learning and the principle of continuous learning progress. The first principle can be interpreted so that each learner has the chance to achieve his educational goal independently and in a different way. For instance, a slower learner can have more time to reach the goal. Teachers use alternative means of teaching, adapted to the learner's abilities and learning style. Correction is offered, if necessary. The second principle, the principle of continuous learning progress, can be interpreted so that every learner should still move to the new curriculum requirements in order to achieve all that he is able to achieve in a certain time and under certain conditions. For example, we would not expect faster and more motivated learners to wait for others, and learners should not lose time repeating tasks already mastered. Two learners of the same age can work on different tasks and solve problems at different levels of difficulty. Moreover, the students can work in different subjects at the same time. The teacher respects the needs and interests of each learner and encourages him/her in his/her overall development to reach his/her potential (p. 155).

The teacher is responsible for the characteristics of the instruction. The work of each learner is directed at achieving a goal that is adapted to a particular ability and interest. Thus, each

pupil follows his own course, which develops gradually as the teacher plans for each pupil individually. In setting up goals and planning, the teacher and pupil confer.

The teacher evaluates the work of each learner on the basis of the learner's ability. This means that the learners' achievements are compared with the goals on which the teacher and the learner have agreed. Any given grades refer to the goals. Learners are given a chance to evaluate their work themselves. There is no comparison between learners (Larsson, 1973).

The concept of individualization as utilized in this study corresponds with the definition of Dunn and Dunn (1972) who define that individualized instruction means "a creative learning approach to the teaching-learning process" (Dunn and Dunn, 1972, p. 47-50, cited by Jones, 1975, p. 10). According to them, individualization occurs when

the child (1) assumes some responsibility for his own learning to become an independent learner, (2) learns at a pace comfortable for him, (3) learns through materials related to his perceptual strength, (4) learns in accord with his own learning style, (5) is graded in terms of his own achievement, (6) feels a sense of achievement, and (7) selects options from among alternatives (Dunn and Dunn, 1972, pp. 47-50, cited by Jones, 1975, p. 11).

Furthermore, individualization is understood as teaching and learning with features (2) – (7) in this study. For individualization, I consider the teachers' responsibility for the instructions according to a learner's individual needs as essential.

1.1.2 Differentiation

Like the application of individualization, differentiation is also promoted by the Framework Educational Programme for Basic Education in the Czech Republic (Jeřábek and Tupý et al., 2007). The core characteristics (for example, designing an instruction) of differentiation overlap with some of the characteristics of individualization. When defining differentiation, researchers as, for example (Tomlinson, 1999), likewise talk about customizing learning to address the needs of all learners within a single classroom, in this case organized into groups.

Cash (2017) wrote about differentiation

It aims to meet individual needs through a quality curriculum and effective and efficient instructional practices that target groups of students based on learning readiness, individual interests, and preferred ways of learning (p. 9).

The instruction is, same as with individualization, driven by the teacher. Differentiated instruction is an instructional method that takes individual differences into consideration (Yenmez and Ozpinar, 2017). Tomlinson (1999) defines a differentiated classroom as the classroom where “teachers begin where students are not at the front of a curriculum guide. They accept and build upon the premise that learners differ in important ways” (p. 2). The definition of differentiated instruction is depicted in Figure 1.

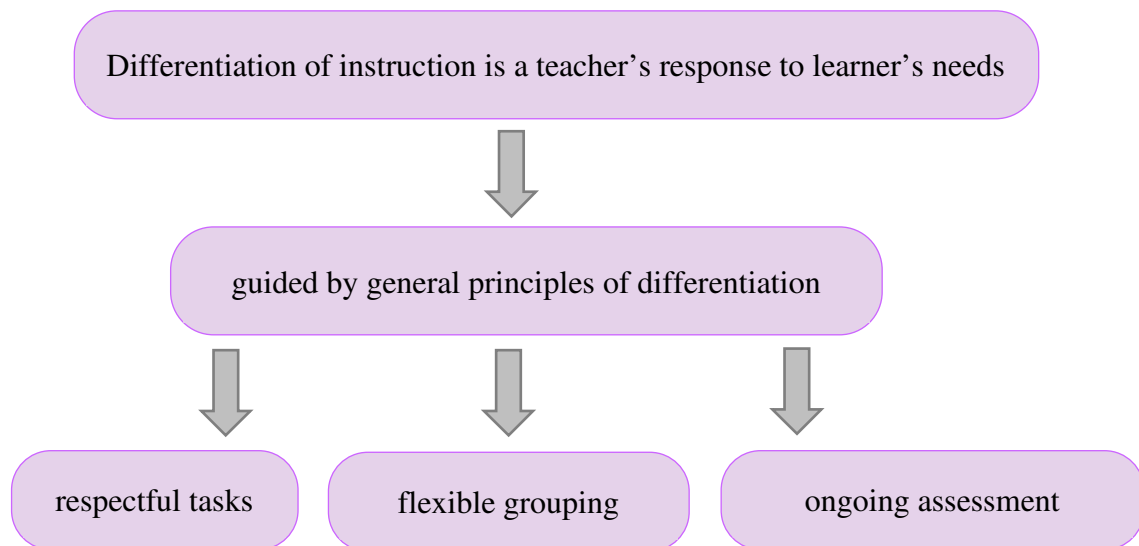


Figure 1 Differentiation of Instruction (Tomlinson, 1999, p. 15)

Bray and McClaskey (2015) emphasized that in differentiation, learning goals are the same for all learners, but the method of instruction varies.

Tomlinson (1999) identified three components that should be differentiated: content (elements and materials used in reaching learning goals and in teaching concepts, principles, and skills that students will learn), process (how to teach the content, flexible groups or whole-group discussion of content or small groups or paired groups; groups are not fixed), and products (students are allowed choices in products or final assessments which should offer a variety of ways for expression, degree of difficulty, and types of evaluation). Teachers

systematically modify the content, process, or product based on learners' readiness for the particular topic, materials, skills, personal interests and learning profiles as in Figure 2.

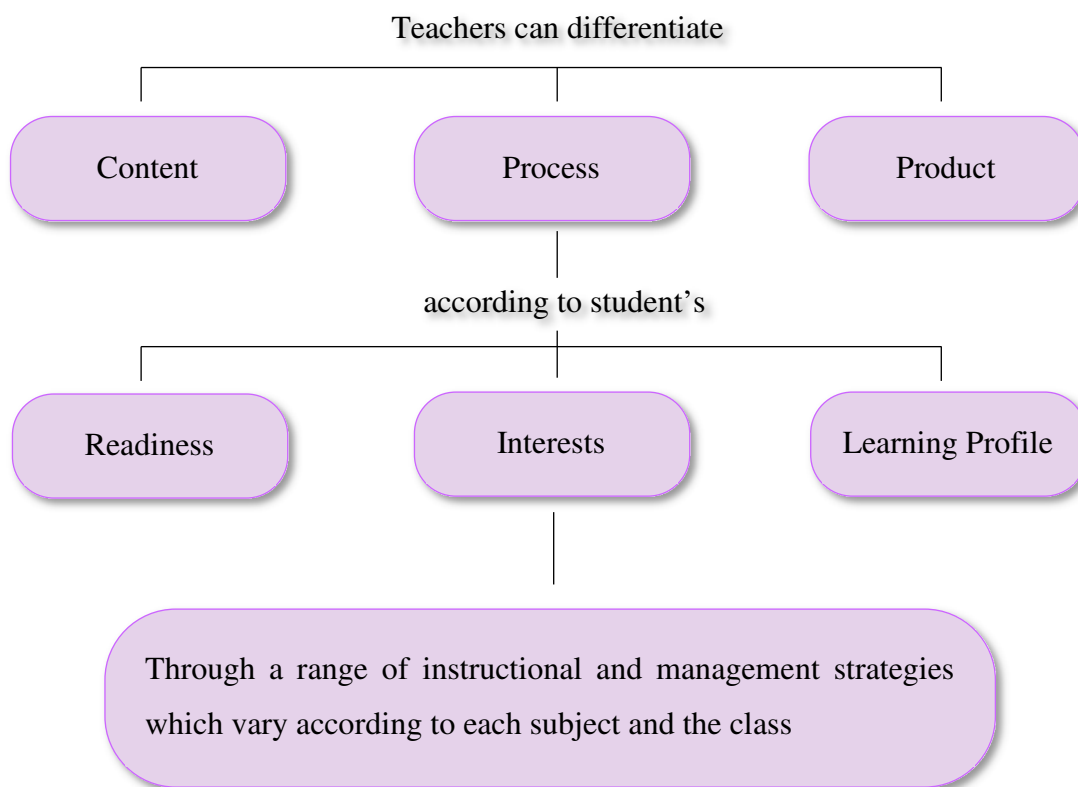


Figure 2 What can be differentiated (adopted from Tomlinson, 1999, p. 15)

Various learner characteristics may serve as a ground for differentiation. For example, Tomlinson et al. (2003) distinguish between differentiation by learner readiness (representing the current level of knowledge and skills in the subject area), learning profile (a learner's preferred way of learning, such as a preference for visual input) and interest (topics about which the learner wants to learn more). Teachers might argue with content requirements in the form of set curriculum. Tomlinson (1999) suggests that teachers "must be ready to engage students in instruction through different learning modalities, by appealing to differing interests, and by using varied rates of instruction along with varied degrees of complexity." (p. 2). Teachers must ensure that learners compete against themselves as they grow and develop. Teachers need to modify curriculum and instruction so that each learner comes away with knowledge, understanding, and skills necessary to take on the next stage of his/her learning (Tomlinson et al., 2003).

Algozzine and Anderson (2007) insisted on demands on the learning environment and opportunities that exclude no child: Differentiation is based

on allowing learners' choices in working independently, with partners, or as a team; or providing varied workspaces that are conducive to various learning preferences (i.e., quiet workspaces, workspaces with tables instead of desks) (p. 50).

It has been suggested that teachers create learning profiles, which are simple profiles of each learner containing pertinent information specific to learning preferences, family structure if compliant with general data protection regulations², favourite hobbies and interests, and other aspects of interest. One of the designs using differentiated instruction can be a choice board in which learners select activities out of several options to demonstrate levels of skills toward the lesson objective. A teacher can introduce requirements and criteria for assessment. There can be

important options, flexibility in how they demonstrate the knowledge and skills they have learned after direct instruction has occurred, as well as affording them the opportunity to make decisions and actively participate in their own learning. (Algozzine and Anderson, 2007, p. 51)

Strong et al. (2004, p. 76) listed strategies (adopted from Marzano, Gaddy and Dean, 2000) which teachers can use to differentiate learning: A. Identify learning goals, and then deliberately use multiple strategies during a unit. B. Use flexible grouping: identify a common purpose, such as developing accurate explanations in a unit of time, rate, and distance problems, and then divide students into different groups that use alternate strategies.

To present how differentiation works in a mathematics class, Strong et al. (2004) determined the principles to which we should make a commitment if preparing differentiated classes:

include all four dimensions of mathematical learning – computation, explanation, application, and problem solving – in every unit we teach, help students to recognize their own mathematical learning styles, use a variety of teaching strategies to explore

² Regulation (EU) 2016/679 of the European Parliament and of the Council of 27 April 2016 on the protection of natural persons with regard to the processing of personal data and on the free movement of such data, and repealing Directive 95/46/EC (General Data Protection Regulation)

mathematical topics, and create or revise our assessments to reflect all four dimensions of mathematical learning and all four learning styles (p. 78).

Maintaining these principles, they adapted a lesson plan from Tomlinson (2001) where they first provided opportunities for learners to learn about the topic in ways that work best for them. Learners should number their choices from 1 to 6 (1-easiest, 6-most difficult) before the study of division:

- Using manipulatives
- Observing demonstrations
- Sketching out the maths situation
- Reading
- Comparing the work with a partner
- Solving complex problems in a team (p. 75)

It was also important for them to follow up on the knowledge and experience of learners. Therefore, they asked learners what they already knew about division.

1. What does 'division' mean?
2. How would you explain division to someone in a younger grade?
3. What are some examples of division that you know how to do?
4. Where do people use division? And why? (p. 75)

The teacher should prepare the data by using pre-assessments or diagnostic assessments such as checklists, interviews, surveys, observations at the outset to collect data on student interests, thinking styles, and readiness for teaching content and skills. The assessment data should be gathered from each child during the beginning of the school year. However, the teacher should continue to collect data daily. He/she should use formative assessments during the instruction process utilizing questioning, quizzes, probing, learning logs, work samples, or think alouds. The use of a summative assessment is also valuable after instruction, for example, the use of unit or chapter tests, projects, or portfolios. The goal of the assessment is "to provide teachers day-to-day data on students' readiness for particular ideas and skills, their interests and their learning profiles" (Tomlinson, 1999, p. 10). Such

assessment provides understanding of how to modify the next instruction or lesson. It does not necessarily come at the end of a lesson.

Consistently with other authors (Tomlinson, 1999; Bray and McClaskey, 2015), the definition of The U. S. Department of Education (Office of Educational Technology, 2010, p. 12) expresses the idea that in differentiation, teachers tailor instructions “according to the preferences of each learner or what works best for learners like them”. For purposes of my work, I will adopt this definition and in the following section, I will describe how the terms individualization and differentiation vary to and yet associate with term of personalization.

1.1.3 Personalization

The term personalization means different things to different people. To see the similarities and distinctions and moreover dissimilarities with the previously defined terms, I will cover a few viewpoints below to show how we can better understand this term.

The Office of Educational Technology (2010) defines personalization as referring to

instruction that is paced to learning needs, tailored to learning preferences, and specific interests of different learners. In an environment that is fully personalized, the learning objectives and content as well as the method and pace may all vary. Personalization encompasses differentiation and individualization (p. 12).

The assumption that personalization encompasses differentiation and individualization (Grant and Basye, 2014) was explained that individualization and differentiation have been combined and focus was shifted to the learner. Bray and McClaskey (2013) see IDP as three distinct strategies. Keefe and Jenkins (2002) see personalization as

students develop personal learning plans, assist in diagnosing their cognitive strengths and weaknesses and other style characteristics, help adapt the learning environment and instruction to learners’ needs and interests, and mentor authentic and reflective learning experiences for their students (p. 441).

Both above definitions of individualization and differentiation mention personalized instruction. However, this is not in line with how personalization is understood in the stated viewpoints. One could say that individualization and personalization evince similar characteristics, particularly in considering individual learners’ needs when tailoring an

instruction. When individualization or differentiation happen, learners receive content and directions on what to learn individually or by groups. Nevertheless, a method of instruction should have the objective of leading the child to discover for himself/herself.

In contrast to the above definitions, five key elements which Miliband (2006) wrote for personalized learning can be noted:

1. Assessment for learning³.
2. Teaching and learning strategies that build on individual needs and actively engage and stretch all students, that creatively deploy teachers, support staff, and new technologies to extend learning opportunities, and that accommodate different paces and styles of learning.
3. Curriculum choice engages and respects students.
4. School organisation is based around student progress and community.
5. Local institutions and social services support schools to drive forward progress in the classroom. (pp. 24-25)

If we consider demands which Järvelä (2006) places on skills that should be taught in schools, we can attach another value to personalization. The skills developed at school should be: “analytical skills, thinking skills, and learning strategies. Teaching analytical thinking means encouraging students to analyse, critique, judge, compare and contrast, evaluate, and assess – but also to continue creative and practical thinking.” (p. 33) Besides that, she points out that “care must be taken to remain realistic in terms of the individual’s ability to be a competent, adaptive, active, goal-oriented, and motivated learner” (p. 32).

Intrinsically motivated people engage in activities that interest them, and they do so freely, with a full sense of volition and without the necessity of material rewards or constraints (Deci and Ryan, 1985). Learners’ search for understanding motivates them to learn with efficiency. When learners want to know more about an idea, a topic, or an entire discipline, they put more cognitive energy into their investigation and discussion and study more on their own (Brooks and Brooks, 1999, cited in Bautista, 2012).

Once learners have emerging or well-developed individual interests, and such topics are connected to academic content topics in school, for which they may have a lower level of interest, the connection between these two areas might support them in developing

³ The term *Assessment for learning* will be explained in section 1.2

situational interest based on their enjoyment, ultimately leading to the development of individual interest in a topic at school. As an example, Bernacky and Walkington (2014) consider a learner who has a well-developed individual interest in music but is not interested in algebra. Nonetheless, in an algebra class, learners may engage with a variety of problems and projects that explore the mathematics behind musical pieces. The authors concluded that “personalization can impact the development of learners’ interest in their math course, the domain of mathematics, and ultimately their long-term learning” (p. 7).

Personalization can be defined as “embedding students’ past experiences and interests into the educational content. It is a method in which familiar people and stories from their own past experiences are used to construct a bridge between new information and existing ones” (Şimşek and Çakır, 2009, p. 799).

Miller and Kulhavy (1991) as cited in (López and Sullivan, 1992), hypothesize that personalization improves memory by increasing the associative strength of the personalized material and closely related content. They found out that incorporating personalized representations during encoding led to significantly greater recall of related information. The meaningfulness of the problem context may increase when its context is personalized; thereby enabling learners to place themselves mentally more readily in the problem situation. Personalization may not only build stronger associations related to the task, but in doing so it may ease the cognitive demands imposed by the problem-solving process.

It is crucial for my following investigation to distinguish between individualization, differentiation and personalization and unify the understanding of all terms. While individualization and differentiation focus on how to tailor an instruction effectively, the following main tenets characterize how I see personalization differently. First, learners have a choice in terms of what, how, where and when they should learn; second, a work-based and skills-focused learning is supported as a more authentic form of education; and third, personal autonomy, responsibility, and flexibility are paramount (Meighan and Toogood, 1992). These understandings of personalization will be used throughout the following text.

1.2 IDP in specific parts of the learning process

While section 1.1 was devoted to the fundamental terms and their definitions, in the following chapters, I endeavour to specify what I will focus on in the practical part of my thesis, and to provide foundations for my research questions.

To study different perspectives of teaching and learning, Roy, Guay and Valois (2013) look at didactical situation from several points of view: contribution of teachers in planning, identification of educational needs and setting goals, giving instruction, practice, and evaluation of progress and process. Progress monitoring is captured by the steps of identification of educational needs and evaluation of progress and process. The component of instructional adaptations is represented by the steps of differentiated goals, instruction, and practice where I include use of textbooks and homework assignments specifically.

Planning and identification of educational needs

Teachers spend time preparing and designing activities for students. The way of such preparation, commonly referred to as planning, depends on each teacher. “From tasks and activities to instructional practices employed during lessons, teachers need to consider a variety of aspects of their instruction before students even enter the classroom” (Superfine, 2008, p. 11). In her work, Danielson (2011) pointed out the attributes of teachers’ planning. Teachers must be familiar with content knowledge and pedagogical approaches they want to use. Above all, teachers need to think of concepts and skills, which are prerequisite to the understanding of others. They should also be aware of typical student misconceptions in the discipline and work to dispel them. She described one of the important stages in planning: identifying resources that are needed for teaching.

One of the first steps in planning is often the question of where to begin. “One starting point is the detailed specification of each child’s attributes and needs.” (Warnock, 1978, p. 206) Teachers use and analyse data to identify the different learners in their classroom. From this data, they can use, adapt, or create different lessons or resources. Teachers discover how learners learn best, and how they access and engage with content, as well as express what they know.

Setting goals

A learner may have the same objectives as another group of learners or even as all learners in the class or there can be different objectives for each learner. However, there can be specific objectives for learners who may need one-on-one support. Teachers or assistants then support and provide accommodations for individual learners to meet the specific objectives. Teachers and learners can co-design objectives based on each learner's learning goals. In some cases, with a high level of independence, the learners drive their learning and own their learning. The learners follow the objectives, monitor their progress in meeting the objectives, and reflect on their progress (Bray and McClaskey, 2015).

Giving an instruction

A crucial part of teaching and learning might be giving instruction, more precisely knowing if and how to talk in different situations. Wood (1997) characterized lessons in mathematics classrooms by interaction patterns and ways of communicating. She studied both the form of discourse (knowing how to talk) and the content (knowing what to say) as a means of describing how the participants communicate with one another. In traditional recitation patterns of interaction, a teacher would simply tell the students that their answer was wrong and seek another child to give the correct response. In the funnel pattern (Bauersfeld, 1980), a teacher attempts to resolve the situation by leading a child through a series of explicit questions until they provide the correct answer. In focusing patterns, the situations allow children to reason on their mathematical ideas and to reflect on their own thinking. Children are encouraged to discuss their ideas with others.

According to Bray and McClaskey (2015) in an individualized learning environment, teachers deliver instruction to individual learners. The teacher determines and designs the instruction and chooses what the learners learn in the environment. The instruction is based on each learner's needs. In a differentiated environment, teachers use direct instruction based upon the learning needs of different groups of learners.

Textbooks

In many classrooms, the instruction is based on textbooks and workbooks. Takahashi (2016) put stress on teachers' knowledge: "In order to provide a better learning experience for their students, all teachers should be able to use the textbook to teach mathematics effectively

using their knowledge and expertise for teaching mathematics.” (Takahashi, 2016, p. 318) The study (Brožová, 2017) showed that the textbooks influence what kinds of concepts of mathematics are introduced and how they are introduced. Some textbooks and workbooks are written purposefully for the teaching method and the textbook plays a key role in the teaching method. There is a good deal of evidence that many teachers depend upon textbooks (Love and Pimm, 1996).

Practicing

The teaching practice can be determined by the amount of time that a teacher is in front of a classroom. Even when the instruction is delivered to the individual or divided into groups, learners can still be passive participants in their learning. Some teachers may set up learning stations by flipping the groups of a class with multiple ways of showing the same content to different types of learners, and yet, these learners can still be considered as learners as they are still only passively receiving content and directions on what to learn and how they can demonstrate understanding. It is only when learners have choices to interact with the content, discuss what they watched, read, or learned, that they can be considered as actively participating learners. In individualized and differentiated learning environments, the teacher selects tools and resources for individuals or groups of learners. The teacher also considers how appropriate a tool, or a particular resource is. In personalized learning, learners actively participate and drive their learning, own and co-design their learning. The learner may be able to choose content, or a resource based upon their skill level. They then have a voice in what they are learning based on how they learn best. Such learners have a choice in how they demonstrate evidence of their learning. The teacher is their guide to their personal journey. In a personalized learning environment, learners can access appropriate tools to support their learning. Their critical thinking skills have been developed so they can self-select the tools they need to support any learning task, whether at school or home. Information and communication technology literacy would be an essential skill; learners collaborate, share, and learn with their peers, experts, and other learners around the world (Bray and McClaskey, 2015).

Homework

A number of studies have looked into the topic of homework, for example Holte (2016); Costa et al. (2016). All parts of an educational process (students, parents, and teachers) seem to have some concerns about assigning extra work for home. Parents usually ask if the assignments can be shorter or longer, harder, or easier or can be more ambiguous (Kralovec and Buell, 2003;). Teachers need significant support from parents, call for training in how to construct good assignments, and look for time to prepare effective assignments (Farkas et al., 1999). Learners protest about the time that they need to spend working at home (Coutts, 2004). Many students consider homework the chief source of stress in their lives (Kouzma and Kennedy, 2002).

Math homework is usually assigned daily. It is chosen from problem sets following the section of the topic that was addressed that day and the entire class can get the same assignment. The study conducted in 2019 (Brožová, 2019) focused on finding out if the teachers, using the different teaching methods, assign homework in teaching mathematics, for what purposes they assign it and if they find it effective.

Evaluation of progress and process

Black and Willian (1998) consider assessment to be

all those activities undertaken by teacher–sand by their students in assessing themselves—that provide information to be used as feedback to modify teaching and learning activities. Such assessment becomes a formative assessment when the evidence is actually used to adapt teaching to meet student needs (p. 140).

Concerning the types of assessment in Manitoba Education (2006), they distinguish among assessment of learning, assessment for learning, and assessment as learning. They explain assessment of learning as “strategies designed to confirm what learners know, demonstrate whether or not they have met curriculum outcomes or the goals of their individualized programs, or to certify proficiency and make decisions about learners’ future programs or placements” (p. 55). Assessment for learning then “provides information about what learners already know and can do, so that teachers can design the most appropriate next steps in instruction,” while assessment as learning should describe “how learning happens and is characterized by learners reflecting on their own learning and making adjustments so that

they achieve a deeper understanding” (p. 34). The learners show their responsibility and must learn to articulate and defend the nature and quality of their learning. “When learners reflect on their own learning and must communicate it to others, they are intensifying their understanding about a topic, their own learning strengths, and the areas in which they need to develop further.” (p. 49)

1.3 IDP in teachers’ beliefs

In this section, the emphasis is on the term *beliefs* in educational context. Teachers’ individual perceptions about instruction are fundamental contributions influencing the way they teach, and how they motivate and engage their students. Therefore, a marker of successful teaching applications is determining the teacher efficacy perception about the instruction (Karadag, 2010).

According to Bosco (1971), the teachers are influenced by the idea of individualization, but it is still hard for them to devote more time to individual instruction. “Teachers may be influenced by the belief that the children who are working with the teacher at any given moment are receiving the maximum value from the program, while children who are working in small groups or as individuals are "marking time" (p. 130). In section 1.1.3 I suggest that personalization is largely about personal autonomy, responsibility, and flexibility of a learner. If this suggestion is indeed correct, then the time and attention spent by the teacher does not matter as much nor does it translate to receiving maximum value from the program.

Teachers who differentiate believe that every child is unique, with differing learning styles and preferences. They may differentiate based on students’ readiness by varying the levels of difficulty of the material covered in class. Teachers may opt to differentiate key skills and material to be understood by aligning them with particular students’ affinities and topics of interest (Algozzine and Anderson, 2007)

Differentiation is a philosophy or mindset that enables educators to plan strategically to reach the needs of the diverse learners in classrooms today so that they can achieve targeted standards. Differentiation is not a set of tools, but a belief system that educators embrace to meet the unique needs of every learner (Gregory and Chapman, 2007).

Considering the definitions of individualization, differentiation, and personalization, the last might be the most demanding on teachers' beliefs.

Teachers who are concerned about personalizing the learning process believe in teaching through genuine experience and thoughtful reflection. Authentic academic achievement, then, must be concerned with accomplishments that are significant, worthwhile, and meaningful for learners preparing for adulthood (Keef and Jenkins, 2002, p. 446).

I will adopt beliefs which fit in Goldin's subcategories of teachers' beliefs: "Beliefs about the learning of mathematics, the teaching of mathematics, and the psychology of mathematics; beliefs about elementary mathematics" (Goldin, 2002, p. 67). The beliefs exposed will be taken into consideration.

1.4 IDP in teaching and learning – related research

It has been decades since the need for breaking down the massive uniformity of instruction emerged. Gibbons (1970, p. 28) summarized, that "it has become clear that students differ not only in intelligence but in creativity". Wallach and Kogan (1965) confirmed that they differ "in at least 80 elements of intellect" (cited by Guilford, 1967, p. 65).

Teachers' beliefs, attitudes and practices have been examined by the Teaching and Learning International survey (TALIS). It was clear to researchers that good instruction is not determined just by the teacher's background, beliefs, and attitudes, but it should also be responsive to students' needs and various student, classroom, and school background factors. TALIS looked at whether teaching practices "adapt" to students' social and language background, grade level, achievement level, and class size. For example, "studies on aptitude-treatment interactions suggest that students with low intellectual abilities profit more from structured, teacher-centred instruction, while students with high intellectual abilities may gain more from less structured and more complex instruction" (Snow and Lohman, 1984 cited in OECD, 2009). Thus, research suggests that there are significant relations between teachers' beliefs, attitudes, and practices.

The ideas of individualization, differentiation and personalization have been the subject of analysis in a number of studies. In as far back as the nineteen seventies, Jones (1975) studied

an implementation of individualization and teachers' attitudes towards individualizing instruction.

Their degree of success of giving such instruction depended upon many variables including experience, attitude, frustration level, commitment, and personal involvement among others. Personal observation would support the written accounts in the conclusion that some level of individualized instruction occurred in each of the eleven classrooms and more teachers exhibited a positive attitude towards this philosophy than a negative attitude. (p. 98)

Teachers often report that fine-sounding theories frequently cannot be implemented because of the realities of the classroom. If we are to improve the situation, we must know more about the gap between what "ought to be" and what "is." All teachers know that, at times, they are unable to function as they would like to function. In the study of Karadag (2010) teachers were highly in favour of individualized goals. Fifty-one per cent of the teachers desired a program oriented predominately or totally toward individualized goals. Only one teacher in the sample expressed a desire for a program that would have the same set of goals for all pupils. However, it was seen that there is a need to determine the efficacy perceptions of elementary school teachers regarding individualized instruction.

Prast et al. (2015) in their study claimed that "effective readiness-based differentiation is consistent with the principles of general good teaching, with the addition that each stage of teaching needs to be differentiated" (p. 109). They also recommended the model and strategies for differentiation of mathematics at the practical level. It showed, for example,

The general guideline that advanced learners should be adequately challenged was made ready for use by providing achievement criteria for selecting high-performing students, suggestions for increasing task difficulty, guidelines for compacting, and a list of supplemental enrichment curricula (p. 109).

1.5 Mathematics education programs in the Czech Republic and IDP

With respect to curriculum documents, namely the Framework Educational Programme for Basic Education in the Czech Republic, which "promotes a comprehensive approach to the implementation of educational content, including the possibility of its interlinking as

appropriate, and presumes the choice of a variety of teaching procedures, different methods and formats of teaching suiting individual pupils' needs" (Jeřábek and Tupý et al., 2007, p. 7), Czech elementary teachers teach mathematics in various ways. As part of the mathematics curriculum revision, the European Commission emphasized the importance of the methods being used. They stated that:

appropriate teaching methods can improve students' level of understanding and help them master mathematical rules and procedures. The methods used also influence how students engage with and enjoy their learning, which, in turn, also impacts indirectly on how much and how well they learn (Parveva et al. 2011, p. 51).

Specific characteristics of individualization, differentiation or personalization can be included in the teaching method base and its principles, which the teacher uses. In the following sections, I look at three different teaching approaches used currently in the Czech schools. The three teaching approaches were chosen for the following reasons. The first of the approaches, the scheme-oriented approach, has been adopted by more than 750 of the 4100 Czech schools at the elementary and lower secondary level, the second method, one of the child-centred methods, has a century long tradition, and the schools working according to this method are spread all over the world. It is not extensively used in the Czech Republic (currently we can find about 65 elementary schools working according to this method). The third method was chosen as a sample of a standard mainstream teaching approach based on regular use of textbooks.

1.5.1 Scheme-oriented approach - Hejny Method

The scheme-oriented approach is commonly known as the Hejny method in the Czech Republic; therefore, I use this term throughout the text.

The Hejny method is described as a "coherent approach enabling children to discover mathematics by themselves and for enjoyment" (www.h-mat.cz). It strives to maximize autonomy in the learner's learning process. The method is based on the Theory of Generic Models, a specific concept development theory which consists of five stages. The learning process

starts with motivation and has as its core two mental shifts: the first leads from concrete knowledge (isolated models) to generalized knowledge (generic knowledge) and the second from generic to abstract knowledge. The permanent part of this process of gaining knowledge is that of crystallization, which involves integrating new knowledge into the already existing mathematical structure. (Hejny, 2012, p. 45).

Twelve main principles are at the core of the Hejny method (www.h-mat.cz/en): 1. Building schemata; 2. Working in environments; 3. Interlinking topics; 4. Character development; 5. True motivation; 6. Real-life experience; 7. Enjoying mathematics; 8. Personal knowledge; 9. The teacher's role; 10. Working with error; 11. Appropriate challenge; 12. Supporting collaboration.

The method strengthens and interlinks schemata which have been defined as “a collection of interrelated pieces of knowledge that we have about a familiar environment” and infers general patterns from them.

Hejny and his co-authors implemented 25 environments, which were described as substantial learning environments and could be referred as a set of linked situations (Wittmann, 2001) from the child's surroundings. They are introduced as, for example, Family, a Bus, Playground.

The environment-based system drawn on the personal experience considers the learners' different learning styles and the workings of a learner's mind. The child is then motivated to experiment further through repeated visits of the environment which each lead to the specific mathematical phenomena. Motivation is understood as a need to learn. It is believed that children are naturally curious. “They have a strong desire to learn about the things that surround them. They ask about anything they come across. They have a never-ending need to acquire new experiences, even if they bring pain: Motivation to learn is an innate characteristic of every child” (www.h-mat.cz). It is emphasized that isolated information is not presented to learners and mathematical phenomena and concepts apart are not set, but different strategies are adopted for solving them.

Hejny emphasizes the need of a learner's intellectual autonomy “in the sense that he/she discovers new ideas or gets to them by communicating with classmates or takes them over

from the classmates. It follows that a teacher's role is indispensable for scheme-oriented education..." (Hejny, 2012, p. 47)

The teacher in this approach presents adequate problems and organises discussions of students in such a way that they reach the required knowledge. His/her role is guiding and mediating discussion while collaborating. There are three additional requirements for teachers to be met: connection to a learner's life experience; a long-term nature, which means that it is useable for learners of different ages; and a differentiated nature which enables them to pose problems to cater to the needs of individual learners (Hejny, 2012). The teacher chooses the tasks according to each child's needs. Errors are explained as a means of learning. Learners are encouraged to identify their own mistakes and explain why they were made.

Within twelve principles character development is also discussed. The child's independent thinking is supported.

Teachers using the method do not pass on ready-made knowledge in their lessons, but rather teach children how to reason, discuss, and evaluate. The children learn to know themselves what is right for them; they respect each other, know how to make decisions, and accept the consequences of their actions. Along with mathematics, they naturally discover the principles of social behaviour and grow morally (www.h-mat.cz).

Hejny and his collaborators' edition of textbooks for elementary schools has been approved by the Czech Ministry of Education in 2007 (www.h-mat.cz). The textbooks written specifically for this method were according to the described principles intended for learners to discover mathematics themselves. It is believed that the pathway of discovery takes the learner from a personal experience to a concept. From the perspective of IDP in this approach, the textbooks present problems with varying levels of difficulty. It always allows weaker learners to solve some problems successfully and provides the other learners with further challenges. This type of textbook may be a solution to the problem of managing nonhomogeneous group of learners so that each individual learner can work according to his/her skills and needs. If the problems are graded by level of difficulty, it facilitates the individual work and progress of the learners (Johansson, 2006).

In Hejny method, learners learn primarily at school; therefore, homework can be voluntary, and discussion with those who did it, follows. It should help children take responsibility for their learning and planning.

1.5.2 Child-centred educational approach - Montessori method

As a pertinent example of the child-centred approach, I chose the Montessori method. It is often described as a holistic approach to learning with the ambition to educate the learner as a whole, including physical, spiritual, social, mental and emotional education. This means that one might find a Montessori child meditating or doing yoga while another is practicing subtraction nearby. Each of these components are considered equally important.

Montessori is not entirely original. The one-hundred year-old method is in effect the history of the work of Pereira, Itard and Seguin (Fynne, 1924). Initially, Montessori shared the principle of individuality with Locke and Rousseau. Locke argued that the individual student was the main concern of the teacher in thoughts concerning education. Moreover, Rousseau thought it was imperative that a teacher should know the nature of a student, because education should be “suited to the nature of the child.” (Wasson and Boyles 1998).

According to Ryniker and Shoho (2001), the Montessori approach is based on the tenet that children learn most effectively when information is developmentally appropriate. Her basic claim was that one must study each child and his or her own unique personality before attempting to educate him or her, because all persons will manifest and realize their own possibilities by following their own impulses.

Montessori education is characterized by multi-age classrooms, a special set of educational materials, student-chosen work in long time blocks, collaboration, absence of grades and tests, and individual and small group instruction in both academic and social skills (Lillard, Else-Quest, 2006). “In summary there are two aspects of Montessori classrooms that are very different to conventional classrooms: the learning materials themselves, and the individual self-directed nature of the learning under the teacher’s expert guidance” (Marshall, 2017, p.3).

All learners are at a different academic level that is observed and tracked by the teacher and one or two assistants assigned to the class. More often than giving group lessons, Montessori

teachers give one-on-one lessons to learners depending on their specific level and needs. This is possible because learners largely work independently; spending much of the day practising and perfecting work they have already been given a lesson on.

Montessori classrooms are referred to as a “prepared environment.” This means that they are designed with everything children need to explore and learn. They are filled with low shelves and materials to entice children to want to learn and work. Montessori teachers observe the children and decide what work to place on the shelves to meet the children’s interests and needs at the time. Montessori classrooms are also minimalist. They are full of muted colours and natural light to foster concentration. Everything in the classroom has a specific spot on a shelf where it belongs, and the work is carefully organized to help children develop a sense of order.

In this approach, children learn at their own pace through manipulation of objects. As such, personal independence, self-discipline, and initiative are essential for learning and motivation, with motivation purportedly fostered through interactions with the environment (Kendall, 1992, cited by Lopata, Wallace and Finn, 2005). Harris and Callender (1995, cited by Lopata et al., 2005) contend that the emphasis on these aspects leads to “inner discipline.”

Italian-born physician and educator, Maria Montessori believed that knowledge acquisition was best facilitated by isolated experimentation. Drawing on the groundbreaking works of Etienne Codillac, Jean-Marc Gaspard Itard, and her mentor, Edward Seguin, Montessori maintained the notion that knowledge is obtained through the senses rather than through the recognition and conceptualisation of ideas (cited by Zuckerman, 2010, p.126).

Individualized pacing and instruction are the features which pronouncedly characterize Montessori programs (Diamond and Lee, 2011).

Hiles (2018) in his meta-analysis listed Montessori principles. 1. Respect for the child where learners should be supported in doing and learning things for themselves without unnecessary help or interruption, choosing their own pace. 2. Support for self-education which means avoiding external rewards or punishment. 3. Protection of the child’s spirit to learn. McFarland and McFarland (2013) in their analysis described the following principles:

1. Support for an inquiring mind when learners can reason as well as to consciously select and remember what they are learning. 2. Allowance of sensitive periods when learner's attention is intently focused on self-mastery, sometimes to the exclusion of other things.

Learners in Montessori schools learn by working with specially designed materials. They use little objects, move, and learn through experience. Montessori classrooms include mixed age and mixed skill levels, generally divided into three-year groups. Peer learning is encouraged as the younger students learn from observing their older friends, and older children solidify their knowledge and gain valuable leadership skills through giving lessons to younger children.

All authentic Montessori schools have long, uninterrupted work periods (generally 2-3 hours depending on age). This long time period allows children to engage with the materials deeply and reach intense concentration. In addition to maths, language, and science, Montessori schools include two other academic areas: practical life and sensorial. Practical life consists of exercises to help children learn skills used in everyday life. For older children, this includes things like budgeting and starting a small business. Sensorial activities help a child develop and refine their five senses.

Montessori teachers are sometimes referred to as guides. The teacher's job is to observe the children and introduce them to the academic materials at just the right time. The teachers are often hard to find in the classroom. Maria Montessori saw the role of the teacher as providing children with tools for learning. The teacher gives a child a lesson on a material he/she hasn't used before, but the child can then independently choose to work on it when he/she pleases. Children in a Montessori classroom choose where to sit and what to work on, with guidance from a teacher. A child will not be allowed to dance around the room and distract others or only draw for the day, but he/she can choose, or whether to work on maths or language, for example, or whether to sit at a table or on the floor.

Maria Montessori did not want to dictate the work of the child at school or at home. In such an environment, homework matches each learner's need whenever possible; trying to ensure that the practice is meaningful for everyone. The children may choose in-class mini-workshops that will help them clarify areas of confusion. The children may voluntarily choose to continue their "work" at home or select homework assignments as an extension of

their educational exploration. A task that one group completes in one day may become homework for a less advanced group within the next few days. As homework, Montessori suggested constructive activities which should be adapted to the pupil's interests and abilities, provide challenges and opportunities for mastery, and include exciting investigative work. Children are encouraged to spend time, for example, figuring the total price at the grocery store, keeping statistics, graphing when they go to bed, and how many pages they read or how far they walk each day. They can measure things and calculate their surface area, divide a pizza into equal pieces, or invent patterns for curve stitching.

“Montessori claimed that she had identified resources in children which teachers had not noticed. Perhaps some of these resources will become more visible if homework practices are based on children's perspectives.” (Holte, 2016, p.30).

The important principle is the principle of freedom. For Montessori,

Freedom originates from the first principle, in that a child matures in his or her development in the absence of restrictions on his or her individuality. Thus, the third principle of child development also supports the claim that Montessori's practices have an affinity for democratic education (Wasson and Boyles, 1998, p. 467).

Maria Montessori lived during a time of world wars and global upheaval. Perhaps for this reason, she placed great emphasis on peace education. She believed that the future of the world depended on us teaching our children the importance of peace and this belief is still reflected in Montessori schools today. There is a great emphasis on community, both the classroom community and the wider global community we are part of. Children learn about the world and also learn tools for calming themselves and conducting peaceful conflict resolution.

1.5.3 Shared principles of Hejny and Montessori mathematics

In this section, I will compare Hejny's and Montessori's approaches which share some important characteristics (see Table 1).

Table 1 Shared and distinctive principles

Hejny – only	Shared Principles	Montessori-only
Work in environment – set of linked situations Real-life experience draws on the child’s personal experience	Prepared environment	Environment – a special set of educational materials, multi-age classrooms
Interlinking topics (No isolation of mathematical patterns), Discovering formulas through personal knowledge	Appropriate challenge tasks for each child at their level	Isolation of difficulty separate ‘variables’ to promote interaction with a particular property or concept and its essential significance
Teacher as a mediator of discussion	Teacher as a guide	Student-chosen work in long time blocks, inner discipline
Character development – supporting the child’s independent thinking	Supporting collaboration acquiring knowledge through discussion	Individual and small group work in both academic and social skills
True motivation Enjoying mathematics	Importance of inner motivation	Absence of grades, tests, punishments, or rewards

From Table 1, we can see that both programs share the importance of the environment principle. For each program, the environment has a different meaning. While for Hejny method, it means a set of linked situations, for example in Montessori it means well-prepared surroundings in or out of a classroom. Appropriate challenge tasks for each child at their level seem to be important for both programs. While in Hejny method the task gradation allows teachers or learners to choose the level of task, in Montessori, teachers or learners have choice of the topic (for example, each learner can work on a different mathematical operation at the same time). While it is claimed in Hejny method that there is no isolation of mathematical patterns, which means that all topics are connected with learners’ experience, it is emphasized that in Montessori the isolation of difficulty is preferred. “The difficulty

that the child must discover and understand must be isolated in a single piece of material. The isolation simplifies the child's task for him and enables him to perceive the problem more readily." (Lillard, 1972, p. 61). In both programs, the teacher is described as a guide, but may play a different role. In Hejny method, teachers, besides other things, lead a debate or discussion while in Montessori, they step back and let learners express their needs or wait until learners ask for help. Both programs put emphasis on social and emotional aspects of education.

1.5.4 Undesignated mainstream approach

The majority of teachers in the Czech Republic do not ascribe to any comprehensive approach to teaching mathematics but use a variety of teaching approaches and concepts, using, for instance, projects and game-based activities. However, we can take it for granted that they should comply with what the national curriculum prescribes. Teachers work towards reaching the level of key competencies that the learners should have attained when finishing their basic education. It is the teacher's responsibility to teach students all the algorithms and strategies. When students lack opportunities to discover connections and relations in mathematics, their creativity and independent thinking can be limited.

Teachers using an undesignated mainstream approach look upon the class or group as an entity. Each child is presumed to have relatively equal learning needs, abilities, and responses. Learners taught by such methods are generally given the same assignments, regardless of individual capabilities or progress. In those cases where individual assignments are made, they must usually be completed within a specified time period (National School Public Relations Association, 1971, p. 2, cited by Jones 1975).

This approach, including its modifications, is used in many of the schools in the Czech Republic on a long-term basis. Teachers are supposed to be effective in this educational style which according to Minor et al. (2002, p. 117) means that they are

subject specialists who are able to select, organize, and deliver content; are efficient and effective in the use of instructional time; and are able to vary their teaching strategies according to student needs. Effective teachers are creative, encourage active student participation, make relevant assignments, arrange for plenty of successful engaged time, are skilful in using questions, promote critical and creative thinking, and

use wait time when seeking student response. In addition, they provide feedback, monitor programs and student progress, use both traditional and alternative assessment, and are fair in assessment and grading procedures.

This approach relies on the teachers where the responsibility for the learners' achievement always falls on them. Teachers attend many teaching development courses where they are introduced to different methods, but it does not always mean that they would adjust their fundamental beliefs affecting true change. The new methods they pick up include projects and explorative activities may appear non-traditional, but the essence of their work remains the same – they are the authority and thus the solving method they introduced (or let the children explore) is often the only correct one.

Harris described this program as “age-graded lock-step system in which all students, regardless of differences among them, are constrained to study the same materials in the same way for the same length of time” (Harris and Liba, 1960, p.222 cited by Gibbons, 1970, p. 28). Teachers following this program choose the textbook and workbooks from a variety of textbooks approved by the Czech Ministry of Education, according to their teaching style or the purpose of using it. They are not usually governed or limited in the choice of textbooks (the commercial criteria can be one of the limitations) but many times we can find the same textbook used throughout the whole school. It is the responsibility of each teacher to choose the right textbook.

Teachers take average learners or use the one-size-fits-all approach (Rock et al. 2008). The fast solvers can repeat and practice within the topic or help others and therefore wait for the time and opportunity to learn further. Usually, a lecture accompanied by clarifying questions and answers is reinforced with homework assignments. Teachers assign the homework choosing an exercise from the textbook, make worksheets, or design projects with deadlines.

Since the principles of the undesignated mainstream approach for teaching mathematics are not officially specified in any publication available in the Czech Republic, I summarize my understanding of the principles based on my own experience and observations: 1. the choice of a variety of teaching procedures, different methods, and formats of teaching 2. the age-graded lock-step system in which all students, regardless of differences among them, were constrained to study the same materials in the same way for the same length of time 3. relies

on the teachers where the responsibility for the pupils' achievement always falls on them 4. integration of projects 5. competition and game based activities 6. explorative activities, solving method is often the only one 7. time limitation of work 8. Evaluation based on grade system (1-5, where 1 means excellent and 5 not acceptable).

1.6 Research questions

To fulfil the goal of this work, that is, to understand how individualization, differentiation, and personalization are utilized in selected programs, four research questions have been formulated:

1. How do principles of the scheme oriented approach, the Montessori method, and the undesignated approach correspond to different characteristics of individualization, differentiation, and personalization?
2. What characteristics of individualization, differentiation, and personalization are included in beliefs and practices of the teachers using one of the three approaches?
3. Do the actual use of textbooks and the way of assigning homework in the classrooms of the three approaches correspond to characteristics of individualization, differentiation, and personalization?
4. Which characteristics of individualization, differentiation, or personalization prevail in the teaching practice of the teachers? Do the characteristics correspond with the Hejny method, the Montessori method, or the undesignated approach?

2 Methodology

2.1 Methods of research

The analysis method used within each situation and across situations (Yin, 2003) is a directed qualitative content analysis (Mayring, 2000). Data material includes written narratives, where the chosen analytical procedures and coding are in an inductive-deductive way.

2.2 Participants

Six teachers (Table 2), teaching mathematics at the elementary level (first through fifth grade where learners are 6-12 years of age), were identified for the method they used while teaching as it was specified in School Education Programs⁴ and confirmed by the head of each school. Two of the respondents (Helen, Henry)⁵ were intentionally chosen for using Hejny method. Two respondents (Maggie, Miriam) were chosen from the schools working according to Montessori method and the final two (Ursula, Ulla) did not use any specific method of teaching mathematics (undesignated approach). The teachers worked in Prague (Helen, Henry, Miriam), and in other cities outside of Prague (Maggie, Ursula, and Ulla), in the Czech Republic. The teachers were different ages and with different length of experience (3,5 – 20 years).

Table 2 Respondents' data 1

Teacher	School Location	Type of School	Teaching Mathematics Approach	Years of Experience	Grades taught in the classroom	Number of Students in the Class	Assistant
Helen	capital	public	Scheme-oriented approach	8 years	5	24	no
Henry	capital	public	Scheme-oriented approach	8 years	2	27	no

⁴ In the system of curricular documents in the Czech Republic, state level is represented by the National Education Programme and Framework Education Programmes. The school level is represented by School Education Programmes, on the basis of which education is implemented in individual schools (Jeřábek and Tupý et al., 2007)

⁵ All names were changed purposely for names with the same initials as the methods which they were selected for (for example Helen for Hejny method).

Ursula	non-capital	public	Undesignated Mainstream Approach	3, 5 years	4	23	no
Ulla	non-capital	public	Undesignated Mainstream Approach	20 years	1	24	no
Maggie	non-capital	private	Child-Centred Educational Approach	11 years	4-5	27	one
Miriam	capital	private	Child-Centred Educational Approach	8 years	1-3	20	one

The teachers chosen for each method (Table 2) were interviewed. Each interview took approximately half an hour. Unfortunately, later one of them changed her position and the others could not be included in the next phase of the research (the videorecording with interviews a priori and a posteriori) due to technical aspects and physical travel distance.

Since only two teachers from Table 2 were able to continue with the study and agreed to being videorecorded, four additional teachers were added to maintain the number of teachers in each approach (Table 3). The lessons were videorecorded during spring 2019 in three public schools with the agreement of heads of school and all participants (teachers, learners, and their parents).

Table 3 Respondents' data 2

Teacher	School Location	Type of School	Teaching Mathematics Approach	Years of Experience	Grades in the Classroom	Number of Learners in the Class	Assistant
Ulrica	Capital	public	Undesignated Mainstream Approach	35 years	4	23	no
Una	Capital	public	Undesignated Mainstream Approach	5.5 years	2	24	one (entitled for a girl)
Helen	Capital	public	Scheme-oriented approach	8 years	5	25	no
Henry	Capital	public	Scheme-oriented approach	8 years	2	25	no
Maria	Capital	public	Child-Centred Educational Approach	6 years	4-6	27	one (entitled for twelve-

							year-old girl)
Michelle	Capital	public	Child-Centered Educational Approach	10 years	1-3	25	Two

2.3 Data collection

For initial data collection in spring 2017, I used a semi structured interview (Fylan, 2005) with a paper-based interview guide. The guide included a list of open-ended and closed-ended questions and topics that needed to be covered during the conversation, in a particular order, where possible. All participants (Table 2) were informed about the background and purpose of the interview and its use before the interviews started. Two pilot interviews with other elementary teachers were carried out in the first phase. The interviewer followed the guide but needed to be able to follow topical trajectories in the conversation. Questions were focused on teachers' beliefs and practices in the approach which they use, for example: what is specific for mathematics lessons in their classes, how they plan and prepare their lessons, how they use textbooks, how they assign homework, etc. The interviews were recorded with the approval of the teachers using a voice recorder. Transcribed audio recordings of the interviews were then analysed. Additionally, initial interviews were complemented with all videorecorded teachers (Table 3).

The videorecording of each set of each teacher's lessons followed in spring 2019. Six teachers (Table 3) were interviewed (5-15 minutes) a priori on the topic which they were going to teach and the techniques they were going to use and a posteriori each videorecorded lesson. It was intended to take recordings in a relaxed atmosphere. Some teachers (Henry, Helen, Maria, Michelle) willingly shared their ideas and were explicitly thankful for the opportunity for self-reflection and feedback, one of the teachers (Ulrica) said several times that she did not have anything to say or share and declined to comment on the lessons. That confirmed the assumption that the best way to gain relevant data would be the observation of real situations and that videorecording must be obtained. The interviews and recordings were transcribed and formed by written descriptive narratives.

The elementary teachers and lessons were selected with the purpose of videorecording the lessons of mathematics, but it was not possible to find similar topics of lessons or the same

grade because of differences of the methods (for example specific environments – topics in Hejny method or heterogenous classes – three grades in one classroom – in Montessori method). Therefore, the results could vary for other classes and topics of the lessons. The teachers were videorecorded during the introduction of a new topic or following the practice where different procedures, according to different school programs, are used to familiarize the children with the specific topic.

Three lessons (*L1*, *L2*, *L3*) from each teacher (*Ulrica*, *Una*, *Helen*, *Henry*, *Maria*, and *Michelle*) and corresponding interviews a priori (*A*) and a posteriori (*P*) as shown in Table 4 were analysed. The numbers indicate the sequence of the answers in interviews a priori or a posteriori, or sequence of statements in the lessons (for example *Henry 1L 28* indicates first videorecorded lesson and statement number 28 in the sequence of the statements during the lesson).

Table 4 Indication of video recorded lessons and corresponding interviews

Teacher/ lesson	Interview a priori	First lesson	Interview a posteriori	Interview a priori	Second lesson	Interview a posteriori	Interview a priori	Third lesson	Interview a posteriori
Indication	<i>L1A</i>	<i>1L</i>	<i>L1P</i>	<i>L2A</i>	<i>2L</i>	<i>L2P</i>	<i>L3A</i>	<i>3L</i>	<i>L3P</i>

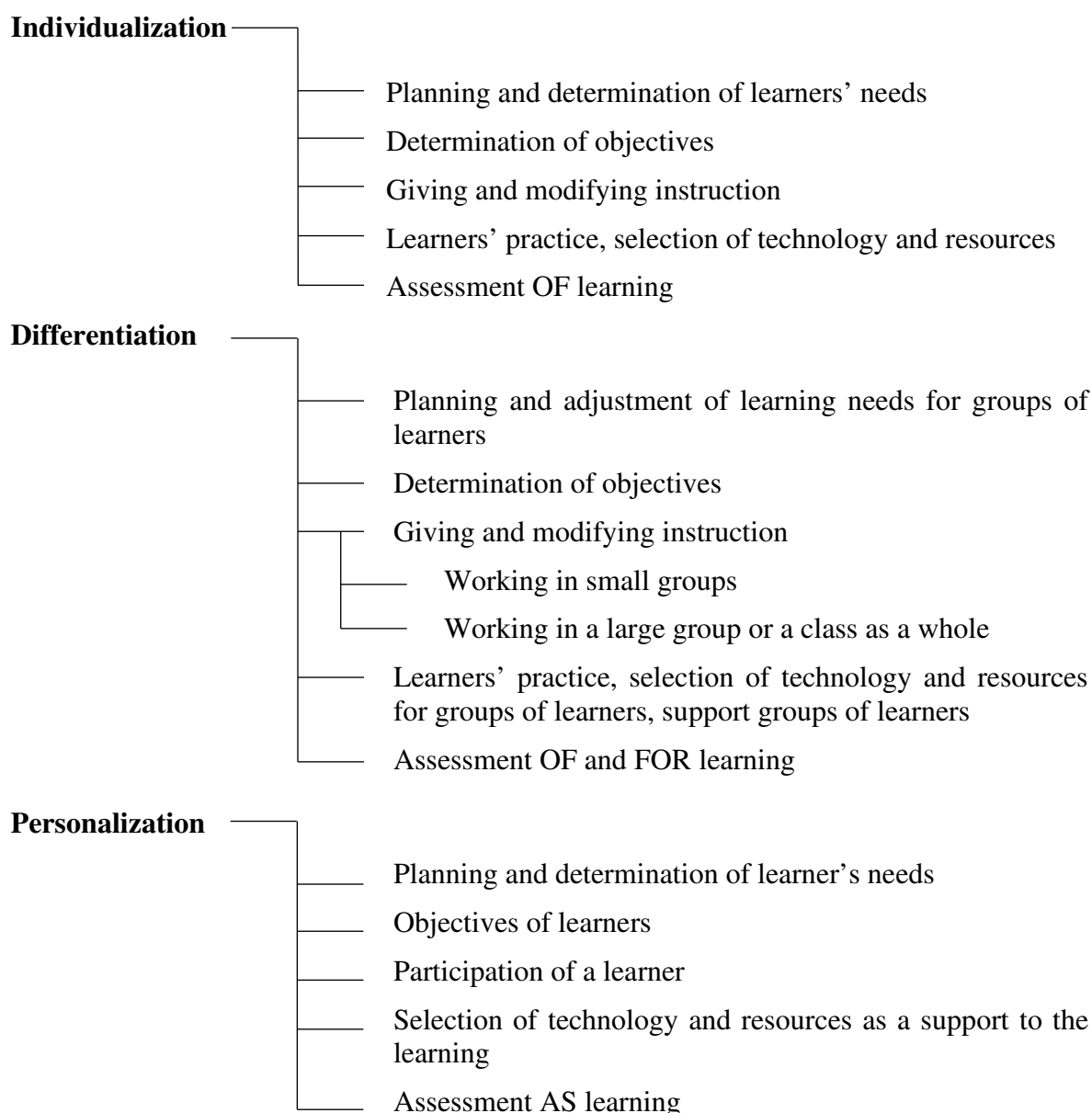
2.4 Data analysis

Interviews and video recordings were transcribed verbatim and translated by the author of this work. All the descriptive narratives were used as a source for analysis.

In the first phase of analysis, the attention was focused primarily on statements related to the teachers' beliefs and teaching style and the other comments obtained from the initial interviews. I identified statements which I considered as connected with IDP and my research questions and formed them in analytical units. I specified categories for use of textbooks and assignment of homework as long term teachers' practices to show that these practices are unified for the method but can still differ in detail between the selected teachers within a method.

In the second phase I analysed situations from each videorecording and I identified 171 situations according to the characteristics of IDP and organized them in three categories of

IDP. Using axial coding I gathered related concepts in meaningful units and specified 30 codes such as “*accommodates learning needs of the individual, adjusts to learning needs of groups of learners, or a learner actively participates in the design of their learning*” (adopted from Bray and McClaskey, 2015)⁶. Using selective coding of the data I linked the categories from axial coding into 5 core categories for each of IDP: planning, setting goals, giving an instruction, practising and evaluation of progress and process. For coding I used the following scheme:



⁶ The table is part of the appendix A of this work.

I take the following particular steps to answer the research questions:

1. Determine if any and how many principles of programs correspond to the characteristics of individualization, differentiation, or personalization.
2. Analyse teachers' beliefs about the program and the way they are teaching.
3. Determine if any and how many of teachers' beliefs and practices correspond to the characteristics of individualization, differentiation, or personalization.
4. Analyse teachers' regular practices, namely use of textbooks and assigning of homework.
5. Analyse teachers' practices - situations videorecorded in the classrooms including:
 - a. interviews a priori, where teachers describe what is going to happen, their plans, or plans for individuals;
 - b. the interviews a posteriori, where teachers are asked about what has happened and give feedback.
6. Determine if any of the identified characteristics prevail for each teacher and approach and count the incidence of situations for each category.

3 Findings

In this chapter, firstly I determine if any and how many principles of programs correspond to the characteristics of individualization, differentiation, or personalization. In the following sections, I use descriptive narratives where I explore and conceptualize teachers' beliefs and the collected data and represent them in textual form. I endeavour to describe the sample statements or situations gathered during the initial interviews or during the videorecording phase of my research.

To analyse the data, I then considered several factors: use of textbooks and workbooks, how teachers assign homework, and situations that I videorecorded in the classrooms. In the paragraphs below, I endeavour to describe which characteristics of individualization, differentiation, and personalization I found within each part of my work. Each section is focussed on separately, and the results are summarized at the end.

3.1 Characteristics of IDP stated in the approach principles (by the author or in literature)

To answer the first of my research questions, how principles of the scheme oriented approach, the Montessori method, and the undesignated approach corresponded with individualization, differentiation, or personalization, I was analysing the principles which are presented by authors of the methods or by literature. Although I tried to list the principles objectively, the principles vary in literature particularly in the Montessori and undesignated approaches. Therefore, I selected the principles which were stated repeatedly by more authors (for example respect for the child, role of the teacher as a guide, and educating the whole child) or seemed the most relevant according to my experience (for example choice from variety of teaching procedures, and the age-graded lock-step system). Table 5 was used as a basis for statements, which summarize the results of the analysis of principles presented in sections 1.5.1 – 1.5.4. The colours indicate the prevailing characteristics of individualization (blue), differentiation (green) and personalization (grey) within the principles listed in each cell. The sections without colours indicate principles where I did not find any characteristics of IDP.

Table 5 Principles of selected method with characteristics of IDP

	Hejny	Montessori	Undesignated
Principles of the approach	Building schemata	Respect for the child	Choice from a variety of teaching procedures
	Working in environments	Support for inquiring mind	Integration of projects
	Interlinking topics	Allowance of sensitive periods	Competition and game based activities
	Character development	Support for auto-education	Only one presented solving method
	True motivation	Protection of child's spirit to learn	Evaluation based on grade system (1-5)
	Real-life experience	Experiential learning	The age-graded lock-step system
	Enjoying mathematics	Mixed-age classrooms	Time limitation of work
	Personal knowledge	Uninterrupted work period	Relies on the teachers
	The teacher's role	Role of the teacher as a guide	
	Working with error	Freedom within limits	
	Appropriate challenge	Educating the whole child	
	Supporting collaboration	Prepared environment as teaching aids	

We can see from Table 5 that all IDP were found in the principles of Hejny method. The principles listed in Section 1.5.1 evince two characteristics of individualization, three of the principles evince the characteristics of differentiation, and five of the principles evince characteristics of personalization. Although the characteristics of the principles prevailing in the findings of this thesis correspond with characteristics of personalization.

As we can see from the Montessori column, all (except for one – mixed-age classrooms) principles of the Montessori method from Hiles (2018), and (McFarland and McFarland, 2013) listed in section 1.5.2 evince characteristics of personalization.

From the last column of Table 5 we can see that one of the principles of the undesignated approach evince the characteristic of individualization. Specifically, choice from a variety of teaching procedures, different methods, and formats of teaching suit individual pupils' needs. That said, we cannot find any additional characteristics of individualization, differentiation, or personalization in the principles.

3.2 Espoused mathematics educational beliefs

In this section, I describe each teacher's espoused beliefs about the approach and educational process in descriptive narratives. The data was collected during the initial interviews. My questions were focused on the specifics of each approach and how they are applied so to gain a broad understanding of each teacher's methods. Additionally, I inserted some of the teachers' previous statements that were collected during the interviews a priori and a posteriori or during the lessons in the videorecording phase of research. The answers identified during the initial interviews are labelled (*name, number of answer, for example: Henry 11*), and interviews a priori (A) or a posterior (P) are indicated as it has been described in section 2.3.

Henry's beliefs about the method

Henry has been teaching at school with a program using Hejny method for teaching mathematics for eight years. He answers the question, what he believes is crucial for the method.

Henry 11: They have 12 principles which make sense to me. I consider the constructivist approach as the most important. Children explore and explore and then based on the experience, they create the abstract knowledge. Children enjoy learning, not in a sense that it is colourful but because the problems are tricky, always different and children have a chance to explore. When they explore something, they experience an "Aha" moment, and are motivated to learn more.

Henry continues describing how understanding is important for this method. He explains (Henry 11) that there is no urgency for speed, no urgency to know the maths facts in the third grade very fast, but it is about their overall understanding. Then teachers address if students

know it quickly or slowly. However, if they do not understand, then it is not only about if they know it quickly but that they do not understand it yet. Teachers implicitly say that learners should turn back and try to understand, try it again. It is focused on logical thinking a lot, so the logic is developed.

To show Henry's enthusiasm for the method and teaching overall I include here his expressions from an interview a priori (Henry L1A) where he is thinking about the lesson. He is willing to speak about his teaching, as in any interview, and describes the creation of the plan for a lesson. It is evident that within the overall framework the teacher uses the experience and improvises while talking about the lesson before the lesson starts and during the lesson, as well.

Henry L1A 12: Hm. (...) I looked at what is next, yesterday. (...) then I just... I often, when I have just a little time, I just load in the head what is next.

Henry L1A 13: Mmh. I go home and think of how I could use it.

Henry L1A 14: Ehm, when I have the idea, I add it in my written preparation.

Henry L1A 16: I often prepare a goal in a written way, what I want in the lesson. Ehm (...) eventually I write an introduction, something like motivation, evocation to get into it faster. Here (he is showing the task in the workbook) I will do it ... I have not said it, I am quickly thinking of it in my head. (...) I will probably write on the board something plus something plus something equals for example, 12, such a small number and we will look for variations.

Henry L1A 36: Well, this is super, if I only could do it before every lesson.

Helen's beliefs about the method

Helen can see the extraordinariness of the school where she teaches and describes that the personality and understanding of the method is important for each class and teacher. She describes (Helen 3) that teachers at her school try to interconnect different methods like the Hejny method but the most important is critical thinking, group work and projects. She thinks that there is a lot of integration of grades and both levels of elementary. It is not specifically one method or some special way of thinking at her school. She is convinced that every teacher finds their own way.

Eventually, Helen describes what she can see special about the teaching mathematics method she uses in the classroom.

Helen 5: It is a discovery method. Mathematics is taught the way that children discover..., I often hear: “Aha, yes, it is that” ...there is a feeling that I found out something. It is an open approach for discussion, we discuss very often during the whole lesson (45minutes). Not all children achieve the same goals. The problems are made to look for more solutions and call for discussion. The method of trial and error is essential. The children enjoy it, they look forward to it. It is not that everyone needs to be at the same level. Somebody will like working with Grandad Lesoň⁷ and somebody will rather step and invent the equations and so on. There is also individualization... It is alright that all learners do not reach the same goal, the process is important.

The word individualization attracted the attention of the interviewer; therefore, Helen was asked how individualization is enacted and if she can describe how it works in the class. She explains (Helen 9) that there is an effort of individualization in a sense that each child is considered individually. Another important thing for her is a prepared environment where we can find a prepared teacher and didactical material which is very specific.

When she works in the lessons, she practices individualization by saying to the learners: Here you have some exercises (she chooses some from the workbook, she writes them on the board), she lets learners choose whatever they want to do, what they want to work on and, whatever they feel like doing because she has it all connected with a joy of mathematics. So, whatever they feel like doing that day they can work on at their pace, she summarizes. When they are finished, they can get extra work which is scaled, and is called task gradation. She can see individualization where the learners have the choice if they want to solve easier tasks, or to work on more complicated ones. She does not include such work as task gradation, in every lesson, she confessed, but she strives to encourage learners. She considers some of children as very good learners, calls them arithmeticians or solvers, or those with

⁷ The „Grandad Lesoň“ is one the environments in Hejny’s method. Learners solve tasks with animals as a preparation for understanding of equations.

an insight in mathematical thinking, a talent, and they work promptly, and with desire to work on the more complicated tasks from the textbook where they have hard things. Therefore, she believes, that she moves them forward. On the other hand, those who do not want to, or for whom it does not work that well, are where they need to be.

Helen talks (Helen 8) about tasks, how she assigns them, and what learners do when they finish the tasks. When she sets work, she strives to present various tasks. It means that they can start with one exercise where she needs them to refresh the classic counting, addition, subtraction, or multiplication, and another exercise which she wants all of them to finish and they can choose the rest they want to do. There she integrates the tasks which she wants and those which are more complicated and the trial-and-error kind. Those learners who need more time to find the solution and do not like to work on such tasks, for them she includes “Fairground⁸” environment.

Helen thinks that everyone can find something, and different kinds of thinking are developed, where all learners catch on. At least within some environments, all learners seem to understand, even those who do not have any insight in simple mathematical operations.

Helen 11: I usually start a new topic with a solved problem, and they should find out how it was solved. Sometimes they should find out what we could do and try to find the idea of how we start, what it means...

The role of a teacher in this method according to Helen is described in the following words.

Helen 10: I try to be the guide, who leads the debate or discussion, because the learners can find a lot of things by themselves. Sometimes I find myself in the situation when I need them to grow, so I intervene. A lot of tasks are prepared to let learners find the solutions themselves. I am lucky to have good arithmeticians who bring the idea and develop it and we gather the solution together.

⁸ The „Fairground“ is one the environments in Hejny’s method. Learners work with number line.

Michelle's beliefs about the method

Michelle described what she considers as the most important and why she uses Montessori method.

Michelle 7: I like working with a three grade heterogeneous class. The children are not at the same level, they choose themselves what they want to do at their age. Therefore, I love first graders working on what the third graders usually do and at the same time the other third graders can do what the first graders usually do.

She complements her words with the example from the current class. She has a first grader who can multiply multi-digit numbers and work on long division material, and it is evident that he understands and knows what he is doing. It is enough to quickly introduce a material and he understands and transfers it in abstract thinking and written form. He likes problems for the development of logical thinking, he likes challenges and thinking. He was not motivated by problems intended for first grade children and he developed problems for himself. On the other hand, she has children with mental disabilities. One of them is the age of a fourth grader and is much taller than the other children. The girl usually works with first graders and thanks to materials she develops her mathematical thinking. She is independent and can say what she wants to work on. The other children accept her differences.

Michelle 14: I also like when children love mathematics because it is concrete and they go through the problems and we can see what M. Montessori said, that the hand is an instrument of the mind. We can refer to the manipulation with materials and the concrete act: "Do you remember, you went to the bank and exchanged."

Maria's beliefs about the method

Maria describes what it means to her to teach in Montessori. She is convinced that it is a part of education which is linked to personal development and that, according to her, work can be interesting and beautiful when you use something different than a textbook or workbook, and paper and a pencil. She says (Maria 4) that the school should not be a place where schoolwork is passed on but the place where we live the process of learning. According to her teachers develop the learning capacity of the student and his personality. The personality

matures and the children become aware of themselves and their own mistakes and learn better, sometimes stressed, but with joy. The child can find what he/she likes to do and the way he/she learns easier just within a system, which allows them to do variable learning with variable methods. Montessori is concrete and visual. The child is able to see how it works in geometric problems. The child can become a number, he/she can use the objects around him, and he can work with the real environment. The task for a teacher is to prepare such standardized problems and tasks to let him/her choose what his/her heart beats for. The freedom of choice is not limitless, but it is structured in the choices which a teacher offers. At the same time, we teach children to be able to choose and decide. That is the key, and it can help the child to direct him/her how he/she wants to do it in the future.

Maria 6. Children are at different levels, and they have variable abilities to reach “aha moment,” which the material offers.

Maria describes the role of the teacher while giving a presentation, where the presentation is described as a lesson with presenting a new topic or material (educational aid). She considers the process of understanding and thinks of immediate reactions of herself as a teacher. She admits that this ability might depend on teacher's experience and competency to observe because problems occur repetitively.

She, as a teacher, follows the course of the presentation and the work with material. At the same time, she needs to observe how far the children understand the process. For example, an algorithm is a sequence of the steps, and each child can take a step back during the long presentation in a different place and stop understanding at different moments. The role of the teacher is to communicate with each child continuously during the presentation. The teacher individualizes the instruction during the process in the bases, and the children are basically divided into two groups. After the first presentation, there are children who are enthusiastic, they understand it and can start to practice the topic immediately and get better at it. They play with the material and through repetition of the work they deepen their knowledge. Then there are children who determine that they do not really understand, and the teacher presents additionally for one or two pupils at a time where they repeat the process and the teacher watches for which moment they stop understanding and identifies the biggest problem - if it is some mathematical step or operation which proceeded. It means if he/she

can do it or it is too complicated, they need to go back and practice the previous topic. When the teacher can see the child struggling and starting to be fearful, then his/her role is, and that is how Maria perceives it, to eliminate or reduce, so offer him/her something different, safely at a lower level to calm him/her down because for the calm child who can say to himself/herself that he/she can make it, this actually works for her, is able to raise the level of difficulty of schoolwork again.

Maria talks about the flexibility of a teacher. The presentation is prepared but can go in unexpected ways.

Maria 15: I plan according to the topics. I have a year plan, a month plan.... When I present something, I have the idea, how I will lead the presentation, which means, where I aim at how many children, I want there, which activities we will do, if they are going to work individually or in groups and there is the opportunity for an alternative plan when children react in unexpected ways. I always count on the possibility that the presentation would go ideal or not so I would need to react to the children, and this is natural in Montessori, and it is convenient for me.

Maria 20: All children daily attend at least two presentations. Every day we also have one activity together, usually it is a creative work. In mathematics the presentations are often more individual and ten to twelve children from the same grade can be separated according to the pace of presentation they need.

Maria 22: I like teaching. Each of us does it differently but the main idea which links us together is that we are here together with the children, and we want to enjoy the process of learning and reach our maximum.

Ulrica's beliefs about the approach

The teacher's expressions about an undesignated approach were concise. The school uses the School Education Program without any specification. Ulrica says that she sometimes uses elements, methods, or activities of the Hejny method because she can bring them from the other class at school which uses the Hejny method. At first, she said that she does not assign homework from mathematics but then she admitted that learners have often trouble

with word problems. If the new topic is introduced and the class seems that it does not work sometimes, therefore an individual learner “needs to sit down at home” as well. The textbook is not essential, and it is not used every day. It is used when presenting a new topic. Learners use workbooks and write in exercise books or just regular paper. She claimed that she uses group work very often. She makes a written week plan, looks at where she wants to go, the goals, and what she wants to teach.

Ulrica 21: I have a boy with special needs here (the second level of support)⁹. Sometimes he has specifications in the plan. He is very good though, we have even weaker children, so they have fewer problems or fewer word problems or on the other hand, I let the top pupils explain or clarify or if somebody has a mistake, the others can ask for advice. I do it all intuitively after thirty-five years of work. I do not need it in written form anymore.

Ulrica uses grades for evaluation of mostly written work. She remembers all other information and does not need to write down any notes. The children have choices. In mathematics, they can choose what they want to do today (meaning as a class). She gives them at least two choices (sometimes three), sometimes they vote, sometimes somebody makes a choice, another a different choice. When they have choices; according to Ulrica, they like it.

Una’s beliefs about the approach

Una says that she teaches according to a standard teaching program, it is not Hejny method or Montessori. She strives to use a lot of activation elements. She claims that she herself does not like the frontal method and either does not have the desks in the classroom as it is in regular schools. She likes the setting which she calls nests and does a lot of group work. According to her, she strives to let the fast children help the slow ones and uses a lot of movement; learners go outside of the classroom to the hall, an opportunity for which she is grateful. She says that she strives to have some excitement for them, to have children who like mathematics and to have them not just sitting at their desks. The situation described in

⁹ The levels of support are decided according to Decree on Education of Pupils with Special Educational Needs and of Gifted Pupils.

footnote 9 shows one of the videorecorded group works, during which the learners use just one algorithm for subtraction.¹⁰ This is included due to a lack of relevant information from the initial interviews. The teacher uses activation methods and the praise for all learners comes after the work. Additionally, Una describes (L1P 5) the peg system and another praise system as a motivation for learners.

Una L1P 5: ..., I give them the pegs, the points system works here. They get pegs up if they are successful in something or did extra work at home; when they practice something at home and bring it to school. It is also a negative motivation, of course, so when they do silly things, the pegs go down then.

¹⁰ *Una L1 35: Work in the "nest." You will invent two hard word problems with dynamic counting to hundreds, for example, 56+7, counting over ten, we need to calculate to ten. Try addition and subtraction separately at first, and then addition and subtraction in one problem. Let's begin; the record keeper starts. (The teacher walks around, repeats the instruction, the learners talk, discuss, and cooperate loudly).*

Learner L1 27: ... (reading a word problem describing the traditional Easter in the Czech Republic) Pepíček went around the village and trick or treated 12 households. His friend Tonda trick and treated 10 girls. Every girl gave them 3 eggs. How many eggs did the girls give all the boys together?

Una L1 47: Does anybody know it? (Una is going to write on the board). Thus, who went at first? (She answers the question) Pepíček (Writing on the board). How many girls did they whip together? Then it was 3 eggs for each girl. There were 22 girls.

Learner L1 28: I know! 66

Una L1 48: How did you count it? 20 times 3 is.... How much? How did you say it?

Learner L1 29: I thought that 3x20 is 60 and added 2x3.

Una L1 49: Super, you can go to the third grade, you do not need to sit here. Read another one.

Learner L1 34: ...Alenka had Easter chocolate eggs. Her dad gave her 10 eggs. How many eggs did she have?

Una L1 54: How many? Once again...

Learner L1 35: Alenka had Easter chocolate eggs. Her dad gave her 10 eggs. How many eggs did she have?

Learner L1 36: 64

Learner L1 37: A mom dyed 28 eggs. Her daughter dyed 6 eggs. How many eggs did she dye altogether?

Una L1 56: Great, who does know?

Learner L1 38: 34

Una L1 57: What did you count now? What did you count? Mom 28 and a daughter 6 more. How many did they dye altogether?

(Assistant teacher points out that it was 6 eggs, not 6 more).

Una L1 58: Oh Jesus, I'm sorry. Well, you were not taken in. I say, kids, it is enough for me.

Una L1 59: I praise you; you have surprised me. All of you managed to invent word problems, so great, kids....

We also have another form of praise. A big preserve jar is in front of the class. When the learners get above ten points, they put some beans in the jar. There are three levels in the jar. When they get first level, they can choose a subject for one hour. The second level means a pizza and for the third level we went to the cinema. I strive to motivate them this way.

Characteristics of IDP which prevailed in teachers' beliefs

The second step in my investigation was to look at teachers' beliefs and what characteristics of individualization, differentiation, and personalization are included in teachers' statements and beliefs. Since I am working with a relatively small sample of respondents, I will be focusing on qualitative analysis of information obtained from individual respondents without generalization of this data in relation to educational programs and their methods.

In Hejny method, the characteristic of individualization was identified a statement concerning additional work via task gradation. Characteristics of differentiation were identified in statements, which focused on identification of the same objectives for groups of learners and the process of learning including discussion. When Henry and Helen answered the question, *what teaching and learning look like*, we could find the characteristics of personalization in their statements. Both teachers claim that learners have a chance to discover and explore and find their own way of learning. Learners have, according to Helen, a choice of work. They can choose the kind of exercise according to their mood or desire. Particularly, the learners who have the insight into mathematical thinking and are talented, finish the work and continue with more difficult tasks in their textbooks. For Helen, the choice of work is connected with the joy of mathematics.

Interviews with Maria and Michelle show that their beliefs comply with the characteristics of individualization in the following aspects. Teacher Michelle says that each child is considered individually and with their special educational needs and differences. The other learners accept them. She considers the prepared environment as important, where we can find a didactical material that is very specific and ready for learners according to their needs. Maria says that learners are at different levels, and they have variable abilities to reach an "aha moment," which the material offers. According to her, teacher needs to observe until which point the children still understand the process. Maria believes that when the teacher

can see the children struggle and start being fearful, then the teacher's role is to eliminate or reduce that anxiety, so they offer them something different.

Some of the beliefs of Maria and Michelle in Montessori correspond with personalization. Michelle says that learners can find what they like to do and the way they learn is easier within a system, which allows them to do variable learning with variable methods. Maria says that this approach is not just about studying, but it is a part of education which is linked to personal development. Maria says that the Montessori aid serves as a tool for exploration. According to Maria, while all children are invited to the presentation (but do not need to come), sometimes it is just a repetition for some of them. Sometimes the plan is dependent upon a situation. Michelle says that the children can decide what they want to work on or continue with, or to work independently. They can take any material from the shelves at all times according to their preferences. It is Maria's belief that freedom of choice is not limitless, but it is structured in the choices that are offered. At the same time, learners are taught to choose and decide for themselves.

Only isolated signs of characteristics involving individualization were identified in exposed teachers' beliefs using the mainstream undesignated approach. During all interviews teachers talk about the whole group of learners and specify only a few distinctions or dissimilarities of individuals concerning IEPs. Although there are learners with special educational needs in each of the classes, teachers do not plan systematically for them. One of the teachers, Ulrica, assigns fewer problems or fewer word problems or lets "top learners" explain or help the others. The teacher believes that she can do so intuitively after thirty-five years of experience. Una in her plans follows the textbook with "the slower ones" and has a folder with extra maths work for those who finish their work earlier. She concludes that there are not many learners in her classroom who do not have any problems with mathematics. The plans, activities, and progress of a child with learning difficulties are managed by the assistant.

3.3 Teachers' practices – textbooks and homework

In this section, I describe if teachers using the Hejny method, the Montessori method, and the mainstream undesignated approach, use textbooks while teaching mathematics. I also examine whether textbooks play a key role in their classrooms or not. The data for this

section was collected during the initial interviews with Maggie, Miriam, Helen, Henry, Ursula, and Ulla as described in Table 2 in section 2.2. For this section, I consider their opinions as the relevant sample of a teacher using the approach. Some of the statements in the section 3.3.2 were added from interviews with Una, Ulrica, and Michelle. I focussed on the teachers' specific beliefs to gain a clearer view of how each approach uses textbooks and homework assignments to achieve results. I consider the use of textbooks and assigning homework as long-term practices which give me data to identify characteristics of IDP in each approach.

3.3.1 Use of textbooks

Helen, Henry, and Ursula thought that they needed textbooks or workbooks as a key source for their teaching. Helen, Henry, Ursula, and Ulla confessed that they followed the textbooks.

It is pleasant for Helen (Helen 13) to have a workbook because she knows the sequence and the work is prepared. Henry expressed himself similarly:

Henry 21: It is a kind of relief if the book is good, and the teacher has experience with it. It is pleasant because he doesn't need to have a pile of books and look for other resources. It is also easier for the parents because they have it if the child is unable to attend school.

Henry says that he uses the textbook fully. The other teachers do not use the textbooks in the same way. Ursula talks about not using textbook in the beginning.

Ursula 13: I do not need the books in the first and second grade. Subject matter is more complicated when the children are older, and I also verify if the method I use is appropriate. I also use the exercises.

Una describes their system of prizes. When learners finish their tasks, they go and take additional work copied beforehand from different workbooks.

Una 2: ... Here in the back of the classroom we have extra mathematical work because every child works differently, so they are used to prevent disturbing the others... it is enough to remind them, or they are used to take something else automatically in the back.

Teachers use different parts of the textbooks. Helen, Henry, and Ursula say that they use all the parts and pupils complete almost all the tasks in the workbooks. Henry sometimes uses additional sources if they miss any important topics. For him, textbooks and the method include open-ended questions for guiding students to develop discussion and understanding, instead of explanations of the concepts and procedures. Helen says (Helen 28, 32) that sometimes, she includes technical drawings from other sources because she cannot find it until the upper grades of the elementary program in their textbooks and workbooks.

Both Montessori teachers claim that they teach without a textbook and use it only as a source of inspiration. Miriam (Miriam 14) says that they do not use the textbooks in their method the way they are used in the traditional schools. The pupils never use them. The only book the pupils in her classroom had, was a little workbook where they practiced math facts.

Similarly, Maggie (Maggie 18) also argues that textbooks do not exist in Montessori method because every child works on something different at the same time; everyone works according to what he/she needs and does not have to go through everything. One child understands when he/she just looks at it and the other child needs to spend weeks before he/she realizes the principle. She adds that every child needs something different, and textbooks basically do not serve it. It is not working. It is fundamentally impossible. One of the situations where Miriam uses textbooks is when assigning homework.

Miriam 26: Homework is not assigned on a regular basis. However, when I do give assignments, students are supposed to work with tasks from a chosen textbook or a copied page.

For Maggie, the textbook is used when looking for information.

Maggie 10, 11: Textbooks are not necessary as in the traditional schools. They are excellent to have one or two copies in the school library because it is a kind of summary, it is a short and simple and understandable source of information especially for younger pupils...

Miriam adds that she needs to concede (Miriam 44) that sometimes she is very critical. Her biggest complaints are about cartoon-like characters which she thinks move the attention

away from providing clear explanations of the concepts and procedures or from simple practicing.

3.3.2 Assigning homework

In this section I examined if teachers, following any of the three selected teaching approaches, assign homework. Helen, Ursula, Ulla assign homework and demand the same tasks from all learners.

Helen 2: All of them have the same homework. We check the homework every working day and I note if somebody does not have it. They need to finish it anyway. The exception is when the children did not understand it. Parents asked for more homework when the children were in the third grade.

She explains further (L2P 5) that they have implemented the daily checked homework, and she tries to see daily who has and who has not brought it. She assigned this regulation when somebody has not brought it, she notes it, and they must do it additionally. For two unfinished homework assignments they have the note in their record book. They discussed it in the beginning of the school year, and she strives to be fair. Fairness is very important to her and thinks that the learners perceive it. The parents thought that it was not enough homework. She takes it as homework is something, in which they should assure themselves that they know. It is not anything new. One parent has said that we are short of homework compared with other schools. It was a parent of a learner with an individual plan.

Helen L2P 15, 16: Children grumble, of course. We have day-to-day homework and long-time projects when they know a deadline...

Helen (L2P 40) and Ursula (16, 26) say that they have textbooks for verifying knowledge and for homework and that the homework is usually an exercise from a textbook/workbook. Ulla occasionally assigns homework with the purpose of teaching children to study the text because she is convinced that they will need it later with the other teachers and to inform parents regarding their child's performance in school mathematics. They had homework weekly. She did it for the parents to see what to study and practice with the children.

Helen counts on homework when working at school and Una checks if everyone does the homework and assigns it in the following way.

Una 10: *Well, kids, you must count at home! Let's see. Who counted during the holidays every day?* (4 learners raise their hands).

Una 65: *Ok, page number 80, 81. Who does not have it, page number 80 is not finished, circle it as homework.*

When learners did not prepare for the lesson and did not do their homework, Ulrica's instruction is to get the book and do the task. The work is graded.

Ulrica 15: *Kittens, I would be interested in what you do now.* (Towards learners who have not finished their work.)

Learners 3: *We have done it, but I left it at home.*

Ulrica 16: *Great* (sarcastically).

Ulrica 17: *Thus, those who do not have any work, take the exercise book. Take your textbook, page number 70, exercise number 4.*

Ulrica 22: *You get grade five girls for that job, right? Thus, you do not think it will be just as it is.*

On the other hand, Henry leaves a lot of responsibility to pupils and the homework is more or less just finishing the classroom work. He said that children can be excused from homework assignments when their work on the school assignments indicates they have mastered the homework material. This is illustrated in the following example (*Henry 5*). He looks at the workbook and record it in the system. If the work from the lesson is correct, they can play soccer in the afternoon, he says. Sometimes they bring corrected work the very next day, sometimes it is flashing in the system until June, and it is up to them if they leave it as it is. If they have a lot of unfinished work, he just tries to force them, but he does his best to leave the responsibility up to them. He does not want them to think that the man wants something. He wants them to feel that they want to correct their own work. Henry can see the homework as an even counter-productive way to understand the topic.

Henry 14: *When the parents observed that the child struggles with something, they tried to help him at home and explained it the other way. They left out the important steps. Therefore, the child knew it but did not understand it and it could not slowly dawn on him.*

Henry, Maggie, and Michelle do not assign homework in the traditional way. Henry (Henry 7) does not assign any homework. The homework is part of the system where they go back to their mistakes, and they learn from them. About five children and that is what he says he likes, have learned that they correct their work right away. He believes that this kind of work works well. Unfortunately, according to him there are also about five children who never do it. The parents rely on the teacher. Henry feels he needs to communicate more. He wants them to understand that they are in charge. On the other hand, he is aware of the reality. One parent comes home at 7 p.m., the other one at 9 p.m. It must be a question of priority then.

Maggie describes (Maggie 18) that she does not have homework as it is, meaning that she would assign something extra above what children do at school. She uses homework in two cases. It is when she needs children to prepare or bring something for a school activity. It can also be an unfinished work from the school day. Children have independent work when they need to finish an assignment from a previous presentation. If the child decides not to work during his independent work time, he needs to take the work home. She notes if they bring it back to school but cannot force anybody. Similarly, Michelle reasons (Michelle 49) why she usually does not assign any homework.

Michelle 49: The children do not get homework. It is because the wooden manipulative material is the core and we do not lend it to take home. The homework is to practice something they should already know when they go on vacation.

Michelle 51: They have some assignments like counting the steps to reach the waste separation dustbin. The children should be at school when they are at school and should be at home when they are at home.

3.3.3 Characteristics of IDP in use of textbooks and homework

To answer my research question, if use of textbooks and the way of assigning homework correspond to any characteristics of IDP, I identified some differences during the interviews in compliance with the selection of tools and resources. I divided the results in two parts, textbooks, and homework.

The criteria for the decision whether IDP prevailed in textbook use and assigning homework were set up in the following way:

- Individualization:** Teachers use textbooks and workbooks individually (either different textbooks for different learners or choose textbooks for individuals). They tell the individuals what to do (which exercise), or where to find something. Similarly, they assign different homework to different learners according to learners' needs.
- Differentiation:** Teachers tell groups of learners which exercise set from the same textbooks to do or how to find something in the textbook and assign homework according to the groups of learners' needs.
- Personalization:** Learners use various different textbooks (or parts of textbooks) according to their choice with guidance from the teacher. Learners have the voice on whether they do something additional at home or not. The responsibility is put on the learners to decide for themselves whether or not to do homework.

Textbooks

Both teachers of Hejny method, Helen, and Henry, thought that they needed textbooks or workbooks as a key source for their teaching. Helen emphasized that sometimes she assigns several exercises on the blackboard and tells the children to choose what they want to do based on how they feel. They can work at their own pace. When they are ready, they get extra work that is graded by level of difficulty. They choose easier or more difficult work according to their interest. This kind of use of textbooks corresponds with characteristics of personalization.

Another use of the workbook is observed when the solving of an exercise is offered to the learners who do not want to continue thinking of more solutions in the previous problem. In the other three situations, teachers of Hejny method assign the same exercise to all learners. They should solve as many problems as possible. If someone is not finished, they can do it later. Teachers help individually. This kind of work corresponds with characteristics of differentiation.

Both Maggie and Miriam (in compliance with Maria and Michelle) claimed that textbooks and workbooks are not followed, and Maggie specified that textbooks are used only as an

source of inspiration for themselves and learners, or when researching independently from the school library. That corresponds with characteristics of personalization. Contrary to that, one of the teachers said that they use small thematic workbooks, where learners practice, and in the higher grades (the fourth, fifth, and sixth grades), they have such workbooks. Such workbooks are being used for individual learners but there is no evidence if teachers assign the work or learners choose it.

Ursula considered textbooks or workbooks as a key source for her teaching. It means that textbooks and workbooks are thus being used for all learners simultaneously. All the learners solve the same exercise and when some learners are finished, they either wait for the others or help their classmates or get additional work. Ulla sometimes uses additional sources. Ursula says that they use all the parts and learners complete almost all the exercises in the workbooks. No characteristics of IDP were found in the statements related to textbooks and workbooks during the interviews.

Homework

Helen admitted that she assigns homework and demands the same tasks from all pupils. On the other hand, Henry, using the same method, leaves a lot of responsibility to learners and the homework is more or less just finishing the classroom work. He said that children can be excused from homework assignments when their work on the school assignments indicates they have mastered the homework material. More so, in some situations, he sees homework as counter-productive to understanding the topic. This kind of procedure corresponds with the characteristic of personalization. The difference between the two teachers shows that teachers' beliefs and their experience can influence the decision about assigning or not assigning homework and in this case, it does not depend on the teaching method.

The respondents, Maggie, and Miriam, using Montessori method do not assign homework in the traditional way either. It is assigned in special situations such as missing a school day on purpose (a learner works on holidays instead of at school) or as a responsibility to remember to bring something. These kinds of activities are in accordance with individualization and personalization.

Ursula and Ulla admitted that they assign homework and demand the same tasks from all learners. Therefore, there were no identified characteristics of IDP.

3.4 IDP identified in teachers' practices and their statements collected during videorecording phases

In this section I describe some of 168 situations collected during the videorecording phase of the work. I insert some of the situations which illustrate teachers' expressions that were collected during the interviews a priori and a posteriori of each lesson. The situations are labelled (*name, number of lesson, and number of expression, for example: Henry L1 98*), and interviews a priori (*A*) or a posteriori (*P*) are indicated as it has been described in section 2.3.

In the following three sections, I endeavour to describe situations in detail to outline my analysis and to present the most reflective situations. The identified situations were organized according to the characteristics of IDP in three categories of IDP. In each category I divided situations into 5 core categories for each category: planning, setting goals, giving an instruction, practicing and evaluation of progress and process according to scheme, described in section 2.4. The analysis of the situations in context of each teacher and method will follow after that in the next section.

3.4.1 Presence of individualization characteristics

A total of 43 situations with aspects of individualization were recognized in all six teachers' classes. In this section I organize and describe relevant situations for each category.

How teachers plan and determine learners' individual needs

The characteristics of individualization appeared in the context of planning and accommodating learning needs for the individual learner in 8 situations, and were identified in Helen, Henry, Michelle, and Maria's classes. To illustrate the situations, in the following paragraphs, I describe most reflective situations in detail.

Both Helen and Henry plan individual work and consider learners with individual needs such as a slower pace or problems with concentration in their planning. During the interviews, they describe activities for individuals, which are different from the work of the others in the class and are open to different scenarios of the lessons when planning individual but same work for all the learners.

Helen described the procedure of planning a following lesson during the interview a posteriori (L2P). She will look at the workbooks and will see, what they fulfilled and how they managed. Based on that and her observation of learners' previous work, she will plan the following lesson for the next day. She will encourage a few learners to work independently and will work with the rest of the learners slowly. When she would see that some of them did not have a clue or they have different numbers, she would have them work with her and the others would have independent work for example. For her lesson (L2), Helen planned that one learner will work independently on the extra work on the multiplication of fractions. The others in the group will be adding and subtracting fractions.

Individualization appeared when Henry planned (L1A 28, 29) to spend more time with a boy with a problem with concentration. He described that it works in the way that if he goes to him and stays with him, he works, he can actually solve anything. Henry planned that he would probably spend more time with him. According to Henry, there were two other learners who needed additional attention and support. On the contrary, during the interview a posteriori (Henry L1P), Henry admitted a risk of a situation because he identified the child who did not get the individual care and was missed during the lesson. He (L1P 7) divided the class and had to take care of the others.

Henry complemented different point of view, which he described during the interview a priori (L1A), concerning the individual needs of learners who ask for challenges and would be finished faster than the others.

Henry L1A 20, 22, 24: I have some great hopes, learners who I think will manage and will be looking for combinations. ...Ehm but I know that there will be three of them who put zeros everywhere and will be finished. They are usually learners who can count. My experience is that it is not to have it easier, but it is easy for them, and they want the real challenge. They get over it and look for something else.

Before the next lesson (L2), his plan is to set up some individual work and see how it goes. In the beginning of the lesson, Henry gives the learners time to solve the problem individually.

Henry L2 51: In the beginning we will try to solve the “Bus¹¹” independently this time. Everyone on one’s own, really everyone can calmly find the solution without being distracted by somebody else with some advice or ideas, really everyone on one’s own. I think that all of you can manage this Bus fine. I will ring the bell in about five minutes.

Unlike the two previously described situations where teachers planned the same tasks for all the learners and then considered individual needs of learners who finished earlier or needed extra support, in Michelle’s class (L3), it was observed that a specific task was planned individually for a girl. She worked on counting beads in a bead chain. Michelle gave her clear individual instruction. The girl demonstrated her knowledge of the bead chain and the procedure of labelling them while she was working by herself. She counted the beads one by one.

Objectives determined for each learner individually

The characteristics of an individualized environment, where objectives for individuals who receive teachers’ or assistants’ support were specific but other learners had common objectives, were identified in 8 situations, in Helen, Henry, Michelle, Maria and Una’s lessons.

Helen communicated the objectives for herself as the goal of the lesson (L2) in the interview a priori (L2A). The goal of the lesson was to check whether all learners can note an equation in connection with a picture. She prepared options on how to continue for the learners who finished first. During her lesson, a boy with an individual education plan quickly finishes his work. Helen goes closer to him and checks his calculation. The boy becomes a supervisor who can check on other calculations. Another boy becomes a supervisor later as well. During the lesson Helen showed one-on-one support.

Henry set the same objectives for all learners by considering various guesses on the number of answers. In the following described situation (L1), he is aware of learners who would

¹¹ The „Bus“ is one the environments in Hejny’s method. Learners solve the task about the number of people going by bus.

have just a few solutions; the others are expected to have more. During the lesson, Henry supported a few learners individually.

Henry L1 2: The day before yesterday we did... Do you remember?

Learners L1 2: Yes!

Henry L1 3: Something plus something equalled nine and we looked for different solutions. How many solutions did we have?

Learners L1 3: Five!

Henry L1 4: Five. (...) Yes, five. Thus today (...) I will complicate it a little. I am really curious how you cope with it. (...) We will also look for various solutions, something similar, but this time we will have three squares. (Learners look slightly surprised while the teacher is drawing squares on the black board.) (...) Well, before you start to solve, guess how many possibilities we will have this time. (...). Are we going to have more or less of them?

Henry set up specific objectives for learners who needed one-on-one support even in his third videorecorded lesson (L3). It was observed that when learners do not know how to continue or if they finish their work, they wait for Henry's help or following instruction.

Henry L3 160: I am very curious if we all have the same answer. Thus, finish now, please. (Teacher coughs up as a sign till it is silence). My question is: How many men went by the bus today? You, Kubo, take your time and solve it by yourself. Do not disturb our discussion and count it. Eh, Nelo, your opinion?

Individual support provided by Una was identified in 3 situations during the lessons (L1, L2, L3). She gave it when she could see a problem or when learners asked questions. She led the learners to an answer. The extra work was assigned when learners finished their work. Una walked around the classroom and checked each student's work. At the back of the classroom, there were binders with copies, and when learners were fast at solving, they could solve more. The additional problems were at the same level as the problems the learners solved daily. The learners did not work for development, just for practice.

In one of the situations during Una's lesson (L2), the learners had the same numbers on the blackboard. The goal for all of them was to make four problems with addition and subtraction

using those four numbers. Una gave extra attention to a girl when she did not know how to continue. She provided accommodations for her to meet the previously set specific objectives according to individual plan.

Una L2 44: Radka, have you finished (while walking closer to the girl)? This is the answer (circles a number on the board). This and that makes 54. You cannot have 54 here. You need to use all numbers, which you have here. Thus, when you add it together, it is not 60.

In another of Una's lessons (L3), learners were supposed to guess the length of a line, measure it, and then compare their guesses and the real length of the line. One of the girls did not know how to get the difference, so Una went to do it individually with her to help her reach the common goal.

In the interview a posteriori (L2P), Maria emphasises the work with manipulatives and the importance of the joy as the output of the lesson. She is aware of the objective of the lesson for an individual: to count the volume of the object which was built by a learner.

Maria L2P 3: Today it was nice to see how one of the boys came in, he didn't want to build a cube, he wanted to build a garage. You let him, because your goal for them is to count the volume of the object they created. Thus, impression is good, I went with the process and let it be. And when I saw the beginning, I was ready to really stop the presentation if I saw that it was really over the edge, because it is not worth fighting at any cost. Yeah, for that learning to make sense, the students should not leave the presentation with the impression that they are the bad ones, that they did not manage it, because my role as a teacher is to make sure the environment is safe and stimulating for learning, so if there is no peace, or joy and it is more about the nerve and the calming them down, then I prefer not to do it. I am glad it worked out.

Giving and modifying instruction based on the needs of the individual learner

Delivering instruction to individual learners, determining, and designing the instruction based on each learner's needs were identified in 11 situations during Helen, Henry, Maria,

and Ulrica's lessons. The following situations from Henry's lessons illustrate how he gives and modifies instructions.

During an interview a posteriori (L1P), Henry describes that he modified a task for learners based on their individual needs:

Henry L1P 8: Eh, at least I devised that the child should try to write down the next solution and that it could be shifted. It worked for a while and the learner invented three more solutions. Then I advised that child to check if the other child's solutions are correct. It looked like the child liked it more than just looking for other solutions.

During a part of the lesson (L2), learners work independently, and Henry circulates among the desks. He strives for learners to work individually and whispers not to disturb others. He reacts to the individual by taking a step back.

Henry L2 118: Well, originally, I wanted to do one more task with vehicles but finally I think that we do not have enough time to talk about it later. We will do it tomorrow then. Now, we will do just a little thing to rest. You do task number 3 and then, do as much as possible in task number four. We will see how it goes.

At the end of the same lesson, each learner works in the workbook. Henry modifies instruction as he can see how individuals solve the problems. In some cases, he can see that learners would not find as many solutions as needed to finish the task. He advises that one solution would be enough for a task.

In another situation (L2), Henry shows his trust in a learner's skills and talks about the use of time. A learner asks if they could wait for a sign to cooperate to check their own solution. Henry answered:

Henry L2 54: If it does not work, you do not have another choice. Anyways, if you try to solve it by yourself, you can feel good about doing it yourself because I think you can do it... and on top of it, we will save a lot of time, won't we. We will not wait for someone to rest, or I do not know what you want to do. You just invent questions to ask, right?

Learner L2 54: No, I thought that it was a good plan, if I keep it in mind, I can then... with my friend....

Henry L2 55: You can choose your own strategy. Next time, you can try another one and you will see. Anyways, now everyone will try to solve on their own quietly. Somebody still talks and disturbs...

How is the learning supported? How do learners practice? How are technology and resources to support the learning needs of the individual learners selected?

In 9 situations during Helen, Henry, and Maria's lessons, tools, and resources are selected by the teacher or an assistant for individual learners of which correspond with characteristics of individualization. The tools include a special graph paper, and manipulative aids that support the specific individual goals. Use of textbooks and workbooks would also be included in this section. This has been already covered in chapter 3.3. In the following paragraphs I describe if (and how) the tools are used during the analysed situations.

In Helen's lesson (L2), a boy takes a geoboard for solving a task with fractions by himself after Helen had suggested it before the activity started. Helen offers other resources when she can see something is not working. She offers to take the square notebook where they can work with squares and draw the fractions.

Helen comments on it in the interview a posteriori:

Helen L2P 30: We will work on it tomorrow again. The boy worked in the desk at first and I told him to take the geoboard. He did not take it at first but found out that it was difficult for him and went to take it later. He claimed himself that he sweated on the problem.

In Henry's lesson (L2), learners work on one "bus¹²" task. Anyone who wants can take little stones or can take out "the paper bus¹³". During the lesson (L2) only one girl uses an opportunity and works with the little stones. The other learners work in their workbooks and check with their teacher working on an interactive board.

¹² As it has been explained in footnote 7.

¹³ „Paper bus“ is the drawing of the bus which illustrates the real bus situation.

In Maria's class, a girl is dependent on individual assistant's help and spends most of the time with her. Such a situation was identified as characteristics of individualized support of learning. On the other hand, Maria, during the initial interview, is in a quandary about any excessive help from an assistant in the classroom.

Maria 13: A child with specific needs and entitled assistant, must experience moments when he is alone to develop. The child must handle specific situations. When the assistant is with the child, the child loses desire for surpassing himself. Therefore, the assistant supports the child but does not sit with the child all the time. When we started leaving the child working alone and just oversaw the process, explained it deeper and left the child again, it helped the child to independence.

Assessment of learning

The characteristics of assessment of learning were identified in 7 cases during Helen, Henry, Maria, and Michelle's lessons. Helen and Henry use grades and Maria and Michelle use verbal (written) assessment. Similarities can be seen in the processes of the assessment.

During Helen's lesson (L2), learners complete the work and bring in an assignment to confirm what they know and get her feedback. Helen (L2) and Henry (L1, L2, L3) take the written outcomes into consideration. They collect all exercise books and see individually what learners wrote as demonstration of whether or not learners have met curriculum outcomes or the goals. Then they make decisions about learners' future plans. Helen describes in the interview a posteriori (Helen L2P) that she determines how they are doing during the class, whether there is more or less understanding, or something more needs to be practised. She is aware of them all not working all the time. Some of pupils discuss different things. From cooperation with a friend, learners get to work independently. Helen does not grade them right away. She rather looks at their procedures and solutions and if learners are oriented. If they are not oriented, then she calls on expert pupils in the class or those who have already experienced it to help others, or she treats it herself.

Henry indicates learners' mistakes in the workbooks and gives them back to them so that they correct them. Concrete situations from L1, L2, L3 to illustrate such situations are written

in footnote¹⁴. He does not force learners to fix the problems but reports the results to the parents.

Henry L1P 19: Some of them go back; some of them have it fixed, so when they have it back, they will solve the problem and turn it back to me immediately. There are about three boys who have a lot of unfinished work and do not care about it. It is up to them; it is their life actually (laughing).

During Maria's lesson (L2), a learner is asked to hand in his "index"¹⁵ and Maria signs the finished work. Since learners do not use any workbooks, she evidences the topics the learners worked on and signs the learner's index as she described during her initial interview (Maria 16, 17).

She writes a diary but claims that it is still developing. Her ideal idea is that after every presentation she makes a detailed note about the children who are ahead a lot and the children who are behind, but it does not work because, according to her, if she wants to be with the children, she needs to be there. She does the evidence, the topics, and subject matters they went through and with the extreme cases she makes a short note.

Each child receives three kinds of information in the written assessment. The first is a range of topics on which the child concentrated. The second is information about what works well, what the child knows, or in what he/she is interested. The third is a recommendation for the next period. All involved adults can see where the children develop and on the other hand where they do not develop, because they do not work to be better at the recommended fields.

Michelle confesses (L1P) that she does not write any notes about learners' development or any evaluation for the following reasons.

¹⁴ *Henry L1 124: Think about it anyway, in a moment... Anyone else want to say something? No. Good. Thus, in the end, I ask you to give me the maths exercise book.*

Henry L2 119: Eh, ..., go ahead and turn it in.

Henry L3 177: So (claps his hands) it will ring in a minute. Eh, so who has checked or finished counting, will submit it... or counts. Who has submitted it will have a break...

¹⁵ An index can be described as the calendar where learners write the notes about the work that they have done during the particular day and Maria signs it when she checks the work.

Michelle L1P 14: ..., I remember it.

Michelle L1P 15: I do not need it. What I know from my heart is that the little girl is the same one who came to the multiplication. I know that she is no further than up to a hundred and with the support, so I can clearly put it out of my head like this. For others, I know that they are capable of it on some level with some of my support or completely alone with an aid.

She evaluates verbally, so then she describes on the report what she saw in that class. When writing a final report, twice a year, she cooperates with assistants. As she says it is in fact the only time, when the annual evaluation is approaching that she writes down what learners have accomplished. It is a kind of stop condition for her when she has to evaluate but otherwise, she does not feel the need and considers the time as a waste of energy. She is convinced that it is a development. She claims that just because somebody cannot do it now does not mean that he will not be able to do it in two weeks.

3.4.2 Presence of differentiation characteristics

The aspects of differentiation were recognized in 82 situations across all six videorecorded classrooms.

Planning and adjustment of learning needs for groups of learners

Characteristics of differentiation were identified in 12 situations during Helen, Henry, Michelle, and Maria's lessons where teachers proactively planned varied approaches to what different groups of learners needed to learn and how they would learn it. Teachers used the existing curriculum and adapted the lessons to meet the needs of different groups of learners.

During the interview a priori, Henry (L1A) is planning a lesson where learners should come up with a number using addition or subtraction. He predicts groups of learners based on their skills and previous situations, and then describes that sometimes the task would be extended and graded according to the level of difficulty and learners' needs. Although the characteristics of individualization in this lesson were already described in one of previous sections, we can see in the three following statements that Henry differentiates between content and process and is aware of the situation where groups of learners will find one solution and others will find more of them.

Henry L1A 2: Today, we have a task (pointing it out in the textbook) where they have ... use various ways to count up to forty. Once they add up three numbers, then subtract one of them, add one and then inverse. We will touch on commutative law and such.

Henry L1A 31: Then, also next wider range, they will continue and then we have some of them who will solve this immediately and will do that, so....

Henry L1A 32: ... I do not care about them. Ehm. (...) Just here, I might not take care of them. Sometimes I invent a harder problem.

Before the second videorecorded lesson, Henry (L2) expected different learners to separate naturally into groups. He thought of faster learners and prepared extra work for them. If some of the learners finished sooner, they could do different work of their own choice from the back of the workbook.

Henry L2A 2: So, I am expecting someone to start drawing it, for example, to see it, and we will talk about it, we will see who handles it and so on. Thus, there will be some discussion. There are a lot of columns. The goal will be to finish five of them. For the faster ones there will be two more and for the very fastest there will be two extras, they can make up their own. So, someone will insert a zero in there, someone will just insert billions in there again, infinity ... If anyone was really done also with this one, then there are still some “cobwebs”, some “parquets”¹⁶, so it will just be additional to everything.

Maria prepared a presentation (L2) for a group of learners who were identified based upon their challenges in a specific content area and skill level. This time the teacher has a fixed group of learners planned. The teacher does not force learners to come to the presentation and invites other learners to come. The choice is limited, and it indicates the characteristics of differentiation. They have done the topic before. The rest of the class works independently or with an assistant.

¹⁶ The “cobwebs” and “parquets” are the environments in Hejny’s method. Learners work in a structural environment and in a geometric manipulative environment.

The presentation (Maria L2A 1) covers calculating volume, so far, a cube. Maria assumes that enthusiastic students will quickly understand that the same could be calculated with a prism, but Maria will try to hold them with the calculating volume of the cube, because they will work with the cubic material so that the cube and its volume stand in control. The presentation should be short, it is planned for twenty, or twenty-five minutes of the first joint work, where they actually recall what they already know, and what they will build on, meaning what is the difference between perimeter and area. Those are the topics which preceded and which we will follow up. She presents the work with cubic material and how those cubes and blocks are composed and how that volume is calculated. The children would then work on their own for the next twenty to twenty-five minutes. Because this is the second presentation out of three, she knows that there is little material, so she has only five or six children so that they can work alone or in pairs, and yet there was enough material for the work.

The same objectives for different groups of learners identified as they are for the whole class

Characteristics of differentiation were identified in 11 situations during Henry, Maria, and Helen's lessons.

Before his lesson (L1), during the interview a priori, Henry touches the plan where he identifies the same objectives for different groups of learners. Even though the teacher claims in an interview a posteriori (of the same lesson L1P 4) that the main goal was to count, it was evident from the lesson that the objectives for groups of learners were distinct depending on the time they use for counting individually. He hopes that after this lesson, nine is simply a better-known number for them, and they get acquainted with it. Henry also claims that if they found eight solutions or fifty, it does not matter what is important is that they counted. Learners choose their partners when they discuss their answers in pairs. It is somebody sitting next to them, a friend or someone with a similar level to them, explains the teacher. They say to each other what they came up with and discuss it for a while.

As Henry says in an interview a posteriori (L1P), when he analysed the situation in the classroom and identified the groups of learners with more or less motivation, he decided to give them a choice.

Henry L1P 7: I told myself that I just had to go somewhere else in that moment. It was immediately clear to me that there would be a lot of solutions, so it was like... I started thinking about how I would do it during that hour. Finally, it turned out that some of them were really motivated, so I divided them. It occurred that even the others, who decided not to do it, were suddenly glad that they could solve that task then. They were already quite tired at the time, in my opinion, but by being given the opportunity to choose, I think that they quite liked it.

Contrary to the identification goal for a group of learners which could be even a whole class, this is the goal of Helen's first videorecorded lesson (L1).

Helen L1P 12: Well, the goal of the lesson was supposed to be a return to the equations which we started around last Wednesday after a long time. We did a number; I think and repeated writing or rewriting of equations in the environment of "Děda Lesoň"¹⁷ However, for the time reasons, it was not time for everyone to check it out for themselves, uh, in a notebook. Because I thought (gestured) it was even better now...

In her lesson (L2), Helen differentiates a group of "speeders," and those who have already some insight into the topic and chooses the next topic or level of work. She would like to give them, individually or in pairs, more work on sheets, where she wrote several tasks for addition and subtraction plus the challenges, where she included multiplication, but it is really for speeders and especially for those with an insight into it. There were several of them.

Michelle comments on a presentation during the interview a posteriori (L3P 5, 7). According to her, all third graders in the classroom, which is a heterogeneous class with first, second and third grades, will have the presentation within three weeks. The same goal is set for all of them, but a presentation is offered repeatedly for a small group. Today, it was intended for third graders. She actually limited the number of children she wanted. She took extra,

¹⁷ The "Děda Lesoň's" is one of the environments in Hejny's method, which helps students understand equations.

but according to her it did not matter, because the girls were all actually very good, and they got their grasps very quickly. According to her, it is from some basic experience that she knows that she needs a small group here to be able to perceive what is being written. Even though in the meantime the girls did a lot of other things while working on division and the teacher had to ask them for their attention. Therefore, she deliberately had a small group for that, and this is how she will gradually address all third graders. If they have time and they want to go, they can. Michelle will gradually go around them so that everyone is there.

Variety of instruction based upon the learning needs of different groups of learners, designed and provided explicitly

To address the various learning needs that make a classroom differentiated, I identified that all videorecorded teachers built instruction for small groups, large groups, or the class as a whole. Teachers facilitated discussions that fostered students' active analysis of task demands. In the following text, I describe some of the videorecorded situations.

Working in small groups

In small groups, instructional activities generally parallel those in individual instruction. However, one difference arises in terms of how a task is selected.¹⁸ Working in centres has been identified in 19 situations across all six selected teachers' classrooms.

Before the second videorecorded lesson (L2), Maria gives the instruction to different groups of learners. Some of them are asked to sit close to the teacher and take their exercise book, maths book and pen. The others can independently continue with any work in any subject they choose.

Maria L2 1: ...And you, who are going to present cubic, to work with cubic material and volume, please bring a notebook, maths book and pen, it will be enough for you. So that you have a place to write.

¹⁸ Tomlinson (1999) distinguishes between stations and centres. Centres can vary from learning centres which are collections of activities or materials designed to teach, reinforce, or extend a particular skill or concept in a classroom area, to interest centres which are designed to motivate learners' exploration of topics in which they have a particular interest.

On the other hand, stations seem also flexible enough to address variable learning needs. Following definition,¹⁹ stations were recognized in Ulrica's lesson (L1). Ulrica's class worked in pairs around the same topic of financial literacy and each pair found a different problem to solve, although they were supposed to collect similar data.

Ulrica L1 1: We will start with our project which we started about two weeks ago. Concerning financial literacy, you were supposed to determine... or we estimated the price of particular products and then you were supposed to find it out. Make pairs, which you have already had before.

Learners focus on one common task at a time but work in pairs. Ulrica assigns the same task and negotiates a common focus as well as task examples. She facilitates discussions within which learners share ideas and let learners discuss what they see, think, and should do next.

Ulrica L1 31: So, listen, you who have finished. What could we do with the counting or with the data, which we have on the paper? Think about what we could do with them. I will wait until everyone has finished solving, we will say that.

Ulrica L1 34 ...So we will start talking about what we could do with the information that we already have. We have some items, we have some estimation of what we thought we would buy, and we have the real price, and finally we have how our estimation differs. What else could we do?

Learners then offer what they could solve. They suggest for example finding a real price (they determine how much it is all together) of a big purchase when a mom goes grocery shopping on Friday for a weekend. They also compare the estimation, how it differs and if they estimated more or less. Then they suggest multiplication of prices if they need more items. For example, if they do not need just one roll but they need 10 or 15 or 8 of them. Ulrica suggests finding out where is the purchase from. When learners suggest another solution, Ulrica asks if that would be helpful and how.

In contrast to the stations, in Una's lesson (L3), learners are also divided into pairs and do the same task. In pairs, learners choose their role, but the choice is limited by the teacher.

¹⁹ "Stations are different spots in the classroom where students work on various tasks simultaneously." (Tomlinson, 1999, p. 62),

Learners go in the hall to measure the line drawn in the paper. After they go back in the classroom, they dictate to the classmate what they have seen and measured.

Una L3 24: Thus, the left side will stand up. You will prepare a ruler. You know that you only talk and dictate. You do not point, name, do not point, ok? Therefore, there is only paper on each desk. The one who draws has a pencil and a ruler. Close your mouth, all of you now lock and lock, want to see it here in the hall. (Learners pantomime locking the mouth) The one who draws, can sing but not talk.

In her lesson (L1), Una asked learners to work in stations (calling them nests) to differentiate the process. Learners were supposed to write down word problems on the same topic (Easter). The teacher circulates among the stations during the whole lesson.

Una L1 35: You have the nests, take a white board and a pen. ... Now you have a chief recorder, but you will work together. I want you to come up with two-word problems together, so that one will be the addition and one the subtraction. Work in the nest, you will come up with a difficult word problem, there will be computing up to a hundred with exchanging, of course, for example $56 + 7$, we go above ten, we have to calculate it. Try addition and subtraction first, and then you can also use addition and subtraction for a one-word problem...

In Maria's lesson (L1), learners work in a group of eight learners in a teaching station and use manipulatives. They sit in a semicircle near the whiteboard where Maria gives direct instruction based on the learning needs of the group. The other learners in the class work independently, or in other stations with an assistant teacher.

The topic of the presentation was planned for about 20 minutes and is multiplication and division by ten, hundred and thousand. Learners work with a material for a short time, and then practise solving with it. Maria demonstrates what she wants learners to do and writes the topic on the white board. During the following block learners take the exercise books and count. They have a choice which colour they want to work with. They put squares in a row on the carpet.

Maria L1 4: ... You know, when we multiply, divide, or add and subtract as well, we will need the material, our number line of units, tens, hundreds, thousands up to million, please. Each of you will choose and make the place value line from units. Who is...?

Learner L1 2: Up to a million?

Maria L1 5: From units up to million.

Maria L1 6: So, units. (Learner 3: Places the first square. Maria 7: Calling another student, Learner 4: Places another square (tens)

Maria L1 8: Yes, tens are blue. (Calling another student)

Maria L1 9: The red is...?

Learners L1 4: Hundreds.

Maria L1 10: Hundreds, another one, please.

Learner L1 5: Thousands.

Maria L1 11: Thousands, we start another class (Calling another student).

Learner L1 6: Blue, ten thousand.

Maria L1 12: Yes, ten thousand is blue (Calling another student).

Learner L1 7: Hundred thousand.

Maria L1 17: (Taking a marker) We have made the basic place value line, and here we have another material to make our work easier (pointing at it).

During Maria's first videorecorded lesson (L1), learners have choices to interact with the content, discuss what they watched, ask questions, and offer answers. They are actively participating as learners. Maria chooses the topic and controls how learners demonstrate understanding. The students work with manipulatives while the teacher sitting in the circle writes on the whiteboard. They solve the problems for example, 400 times ten. The first thing is to make number 400. They put four wooden beads on red square which represent hundreds on the board. Then they multiply it by number 10. The beads move as many places as they have zeros in the number. It makes four thousands. Maria writes the result on the board. The next, more difficult step follows. Maria again interacts with her students, checks what they do with manipulatives, and writes on the board.

Maria L1 38: We will try something different. 320 times 100 (writing on the board). What will it look like in this case? How many hundreds do I have?

Learners L1 20: Three!

Maria L1 39: Three. Thus, three hundred. (Put them on the material.) Tens?

Learners L1 21: Two!

Maria L1 40: Two (put) and we have 320 (pointing at the material). I multiply it by a hundred, it means...

Maria L1 41: That I multiply each of the place values by a hundred. Thus, here (working with the material) times hundred, times hundred, equals 32

Learners L1 22: ...thousands.

Maria L1 42: (Writing the result on the board.) Do we have? So, how would it be with thousands? Will try, for example, 47, we do not have any zeroes, times thousand. Anybody interested in trying it? (Almost all raised their hands up).

Michelle, in her second videorecorded lesson (L2), works with a small group of learners. They use the manipulatives. Michelle gives the instruction based on the learning needs of the group. The other learners in the class work independently or with an assistant teacher.

Michelle L2 8: You count without any problem with the material. It is for you to learn counting without the material. It will be enough for you to have it in your head. Therefore, we will work the way that the girl will be working with the material, and I will be writing it down. You two are going to be observers and you will be watching me how I write it down and then we will all exchange. I will be in the role of observer and adviser.

Learner L2 6: ...and I will be counting.

Michelle L2 9: ...and somebody will be counting and another one will be taking notes.

Working in a large group or a class as a whole

Characteristics of differentiation were recognized in 16 cases in Helen, Henry, Michelle, and Maria's lessons.

During the lesson (L1), Henry facilitates large-group discussion that fosters students' active analysis of task demands. Learners estimate a number of solutions raising their hands. He wants them to say the concrete numbers. He writes the estimated numbers (5, 8, 10) on the

board. After some time, he is willing to write just prominently different numbers such as 100, or infinity. Then the learners solve the problems individually but can cooperate. Henry emphasizes that it is explorative work, and they should thus do it calmly, and quietly.

A similar situation takes place in Maria's lesson (L2) where she gives the instruction after repeating the topics the group of learners would use during the following work.

Maria L2 90: Your task is, for illustration, we will take some smaller (bases) to do it faster. I recommend you take a smaller one. Can I finish the presentation? (A student playing with cubes).

Learner L2 74: Yes (the student cleaned up the cubes).

Maria L2 91: I am glad. The first thing you will do is you count how much is in the cube. How do you count the volume of the cube, any idea? First, we count..... this down here is? How would you count this?

Learner L2 75: Three times three.

Maria L2 92: Three times three (Writing on the board). But this object is specific because it also has height. What is the height of this cube?

Learner L2 76: Three.

Maria L2 93: Three as well (writing on the board). That it is. Therefore, three times three is...

Learner L2 77: Nine.

Maria L2 94: Times three...

Learner L2 78: Twenty-seven.

Maria L2 95: Twenty-seven... what?

Learner L2 79: Cubic centimetres.

To illustrate the work in Maria's classroom during the described situation, I will paraphrase what can be observed from the videorecording: The teacher gives instructions showing the work on manipulatives, how to put together a building and how to count the volume. Everyone does their job. Some already build random buildings, but all learners still work. There is a quiet atmosphere in the classroom. Learners go to get the material without making any noise, the teacher pays attention to the learner from the presentation and helps her with the calculation. The rest of the learners do their job, some with the help of adults. The boys

work at the board themselves, everyone builds their building. Independent work continues, the boys each have their own building, but they debate together. The boys invent very elaborate and complex buildings. When they are finished, they go to have it checked by the teacher. Other learners go to the teacher to check their work. Learners take turns in the classroom, some come, others leave. Concrete follow-up of the situation from Maria's L2 is written in appendix B.

During lessons (L1 and L2), when learners get instruction, Helen monitors what they do and helps whenever she thinks that it is needed.

Helen L1P 24, 25, 26: ...I walk around the class, so I notice where it really does not work, so we just practise it together. Eh, unfortunately, it will not happen that I can cover all the children like that in one hour. But I am mapping it out a little...

Helen L2P 35: That is what I want to do to find out how many children are involved.

Learners' practice, selection of technology and resources for groups of learners

I identified characteristics of a differentiated learning environment in the context of learners' practice and selection of technology in 10 situations during Helen, Henry, Michelle, and Maria's lessons. The first of Helen's videorecorded lessons (L1) is based on seat time²⁰. She asks questions and leads learners to think about the problem and she designs the instruction according to the needs of the group.

Helen L1 1 We attempt again, to write down the equation I say. We want to uncover the intended number. When I subtract five from the quintuple of intended number, I get thirty. Once again, when I subtract five from the quintuple of the intended number, it equals thirty (some children record something in their notebooks). Could anyone say it in their own words or in a different way from me, it will be maybe more comprehensible for someone. I used a little different form today, a different statement than last time. Some of you might be confused.

²⁰ Seat time is understood as the time spent sitting in one's seat in class engaged in passive rather than active learning.

In Helen's second videorecorded lesson (L2), the teacher is the primary helper of the learners. Tools and resources for the groups are selected and geoboards are prepared. The learners mark the smallest part of the fraction and clarify that they are actually cuts. Helen wants to make them realize that there are twelfths, sixths, and so on. She designed it as a follow-up to the previous lesson. She claims in interview a posteriori (L2P 18) that she should have started with repeating what was said before. The groups of learners then continue with differentiated activities such as individual work with fractions or working with the teacher at a slower pace according to their results in their notebooks. Helen then clarifies the goal of the lesson and notices that learners did not use the suggested resources.

Helen L2P 44: Well, I am curious now how they handled it. Therefore, it will be ... that is the question of what is next. Otherwise, I would say that, uh, the goal of the fifth grade is not to be able to add up all the fractions and convert them to a common denominator, but to meet the topic, to touch it, to get to know it, to have an idea. Finally, I did not notice anyone taking graph paper ...

After the presentation during the first videorecorded lesson, where Maria and the learners work together on the set of problems, eight learners, divided into two groups, make up and solve the five problems using manipulatives. During this lesson, learners use materials flexibly. Most often, learners are one another's common source of help. In some moments everyone uses the same materials, but it seems to be effective to have many materials available. They build a series of the squares and come up with at least five problems either multiplication or division. They have a board, and a pen.

Group 1:

Learner L1 36: We have it inverse! (Making an order of the place value material)

Learner L1 37: How come?

Learner L1 42: Everyone will come up with one...

Learner L1 43: Everyone will write one.

Learner L1 44: Will write one. We will do it together. (Teacher goes to check the work of the other learners in their notebooks and on computers then goes back to the groups).

Group 2:

Maria L1 63: Thus boys, please, you have the board, it is up to you if just some of you think up... but you need five of them together and you need both options, right? If you need me, I am here. From which side will you start?

Learner L1 39: There (pointing side).

Maria L1 64: There, excellent. Therefore, where will you have units?

Learner L1 40: (Pointing at the other side.)

Maria L1 65: There you cannot have units. Why?

Maria L1 66: Exactly, because we need to get a grasp of it. You sit on the right side (Teacher leaves).

Maria L1 67: You make it too difficult. The point is to understand the principle. And of course, the more complicated you make it, the trickier it will be. (Learners cooperate; the teacher leaves and checks the work of the other learner).

Learner L1 45: 80 thousand four hundred... (Writing it in the notebook, the other learners think how to do it, put the bead bars on the squares).

Learner L1 47: It is a division... So, we need to go backwards (moves bead bars).

Learner L1 48: Yes (nodding). Takes the board and notes the result.

Learner L1 68: It is a good big number. It happens. (Learner nods and does the next problem).

During the interview a posteriori after her lesson (L2), Maria comments on the concentration and the will of learners to work with manipulatives. They have a 20 minute lesson when they repeat together what perimeter, area, and volume are. They sit in the semicircle and show their impatience when they go to take wooden manipulatives and the teacher has to hold them back.

Maria L2P 1 ...It was challenging for me to hold back with less concentration. We could see that boys look forward to that work and it has been since this morning and wanted to touch it and want to work with it. Some of them have seen it before and knew the material, but we have it as the repetition this year. It is something that they like every year. They build constructions, they count it, and my goal is for them to work with it a lot, to invent creative

buildings and enjoy the process and because of it to strengthen the facts about volume.

Assessment FOR learning

I identified 14 situations where Helen, Henry, and Maria used assessment for learning techniques. For example, during the lesson (L1), Henry evaluates throughout the learning process and aligns instructions with the targeted outcomes. Immediate feedback and directions to learners are provided.

Henry L1 16: Maybe he actually was doing the same thing the whole time, right? So, you tried it? Therefore, now you actually can tell him what the result was.

Similarly, as in the individualization section, to provide immediate feedback from another person than himself, Henry uses group work to evaluate each other's work and a question to find out about the process of solving a problem.

Henry L2A 2: Then, they will evaluate in groups together, who is right, who is not, and possibly correct.

Henry L2 1: How did you solve this?

Learners walk around the classroom with textbooks during Henry's lesson (L2). They show them to each other or show them to the teacher. There is a lot of noise. Some children stand in line with the teacher and show him their solutions and consults with them quietly.

In the comments from interview a posteriori (L2P 33) Helen says that she would like to indicate where learners should revise what was wrong. She also uses different strategies for evaluation as we can see from her answer (L2P 36) where she says that she uses the method of pairs. Some of the students play the role of teachers. They may choose their "teachers." Whoever they choose should explain it to them. She claims that it already works pretty well. Even though she does not grade the particular work, it will be included in their grades later anyway.

During the lesson (L3), Maria uses the opportunity to give individual support when she is not in the small group lesson. Learners go to the teacher or can ask when she circulates the class and oversees what the individuals are doing. Maria evaluates and modifies the plan of instruction for a group of learners according to their interests and needs.

Learner L3 2: Maria, do you know what seemed weird to me? That I found one problem with decimal numbers in the workbook from which I was supposed to write the problems.

Maria L3 3: And you found it because you wanted to count it? Or did you come across it as you went along?

Learner L3 3: I came across it and it seemed good to me and that it probably would not be for the fifth graders when it is in the notebook for four-graders.

Maria L3 4: Each one has it differently. Someone learns it in the fourth grade, another one in the fifth.

Learner L3 4: However, I understood it a little.

Maria L3 5: We are now at the stage that we have been divided by ten, a hundred, a thousand, and this is just followed by decimal numbers. Someone will have it next year depending on how far you are. It just happened to you that you randomly chose a problem that you accidentally hit on a topic we do not deal with yet.

Maria L3 6: Well, are you happy with yourself?

Learner L3 6: Yes.

During Maria's lesson (L3), they summarized units of length. In the interview a posteriori (L3P 2) Maria says that learners naturally meet centimetres, and metres; when they draw technically, they use units. At the presentation they are supposed to get the context to converse during the practical work without manipulatives. Maria comments on the need for revision and why it is important to understand the bigger picture. She says that even though she works with the older ones (age 9-12), the key is to recall some experiences. She asks to show a metre or how they measured it. It helps when she demonstrates that we can actually measure the length of the class, the length of a pencil, or we measure the trip in kilometres, not in centimetres. All are things from practical life. Sometimes learners do not understand why she does it, so she explains that it is necessary for future work.

During the lesson (L1), Maria notices that the manipulation does not go well and suggests a change quickly to prevent the student from making a mistake.

Maria L1 85: You are on this side and units are there...

Learner L1 60: So, I need to move.

Learner L1 86: Yes, you need to see it from this side. Otherwise, the mistake will happen. I was building it from your point of view, so you need to work from the side to make it correct.

While working with manipulatives, Maria follows the strategy of stepping back and not monitoring all steps of the work. It may happen after a presentation or demonstrating a procedure first. For instance, she asks learners to show her results when they are finished with counting and goes away to continue seeing other learners' work during the second videorecorded lesson.

Helen uses pair evaluation during (L1), which might bring honest thoughts to the learners' assessment when control together is inefficient. She says in interview a posteriori (L1P 8) that she had noticed that she missed checking for clear understanding. She saw that they did not have it correct initially and had corrected each other. She also mentioned (L1P 18) that usually it is one lesson for verification, where they each work alone. Then they work in pairs. Usually, when they start a new topic, she lets them support each other and work together when they submit the work in written form. Then she writes her notes. She evaluates there, but at the same time she admits that when they work in pairs, it is not clear whether both understand it to an equal extent. The control together is not really efficient but from written work she can see what they filled in there and how they coped with it and based on that she will plan the next lesson.

During the lesson (L1), according to the learners' needs, Henry stops the work and leaves it for the following lesson. He does not close the topic.

Henry L1 2: I am sorry for disturbing you from counting and thinking but, ehm, we need to conclude it because you need to have some break. I suggest, I suggest not terminating it.

3.4.3 Presence of personalization characteristics

The characteristics of personalization where responsibility is devolved to learners and teachers believe that learners understand how they learn best so they can become active participants in the process of learning, were recognized in 43 situations in Helen, Henry,

Maria, and Michelle's classrooms. For this section I describe sample situations divided according to the context of the part of the lesson.

Planning and how the learner's needs are determined

11 situations with characteristics of personalization were identified in Henry, Maria, and Michelle's lessons and interviews a priori, and a posteriori.

Henry, for instance describes during his interview a priori (L1A 2,5) that he does not plan in detail because each of his learners can choose his/her work. He also does not need to have his lesson plans prepared for a whole 45 minute lessons because he uses the end of a lesson for individual work as it happened in L1 when for the rest of the lesson, learners can continue according to their choice. They can choose a game, and possibly if someone was very fast, then they can go further in the back of the textbook. Increasingly, often he just tells them the numbers of the tasks and they continue by themselves, and he feels that they enjoy it.

In the interview a posteriori after the lesson (L3), Henry comments on the boys who themselves chose to find and count challenging problems.

Henry L3P 7 ...even there by the window, two boys, I bet exactly on them (laughs) that they would put there two hundred minus... They could have put something simpler there, but they chose two hundred minus, one hundred and fifty, or how it was there, plus ... I do not know exactly how it was there anymore ... Therefore, they made this up there.

Learners share how they would like to acquire information, express what they know, and what ways they like to engage with the content as it was identified in Maria's lesson (L1). She finds out that learners discovered something they have not talked about at school yet. She suggests that she would prepare the following presentation to introduce the topic. She identified and wanted to follow the learners' needs.

Maria L1 73: Boys, how about you, any conflict? Anything was wrong?

Learner L1 54: Something was not correct because we came across exchanging.

Maria L1 74: Exchanging.... It happened, show me, and try it.

Maria L1 75: It happened... you dropped the zero here (shows out of squares). Imagine what you actually did? You have got to the area... you will have it next year.

You got into a topic called a decimal number, because ... and I do not really know if you know it completely, maybe you have seen it with older learners. So far, we have counted integers and started with units ... even mathematicians have figured out that they can divide that unit, right? They can also divide it into smaller pieces. Here behind that unit (points to the green square) is such a line. It is hidden there, now it cannot be seen. The numbers behind it are getting smaller and smaller. That means smaller and smaller place values. Therefore, by inventing it, you discovered another type of problem, a task that can also be counted. Would any of you be interested in what is behind this? (The whole group raises their hands).

Maria L1 76: Hm, well. I will now tell you what the task will be, and I will prepare this presentation for you...

Maria L1 77: ... next week, I'll take note of it, and next week I will show you what a decimal number is.

Michelle describes in interview a priori (L1A) that a number of the presentations varies during the year. They have a lot of presentations in the beginning of the year; give the learners options so they can choose later and can work with the materials independently. It usually is only one a day for each child in the second part of the school year. To explain her planning better, she described their morning routine. They begin together around the round carpet (used to be an ellipse) every day. They greet each other, count themselves, do some morning rituals, and say the day plan. It is also written on the notice board when and which presentations (lessons where teacher introduces a new work) the children can expect. They also discuss if the children can decide what they want to work on, continue with, or if they know that there is a time to finish their independent work. They spend more time working (afternoon club) because they encourage hands-on work, and they want children to figure it all out themselves. As Michelle describes in interview a posteriori (L3A 18) she plans the materials and presentations. She invites all the children, sometimes it is just a repetition for some of them. Sometimes the plan is consequent upon a situation. They can always take any material from the shelves according to their preferences and during the presentation they show them how to use it. Presentations are prepared for different numbers of pupils (1-10 as they want to join a lesson).

Objectives of the learners

Characteristics of personalized objectives were identified in 6 situations during Michelle and Maria's lessons. To personalize learning, Michelle and her learners co-design objectives based on each learner's learning goals. There are different objectives for each learner. Characteristics of personalized objectives described in the following paragraphs were identified in Michelle's first interview a priori and lesson.

Michelle L1A 4: I will not do it with the whole class. Only who will. Someone has already worked with it, seen it before, so I do not know who will come.

Before the work Michelle is empathetic and thinks of other work students might have. Considering their needs and interest, she also thinks that some of them might be busy therefore they are not interested in a lesson.

Michelle L1A 5: Others have independent work. Firstly, they have a deadline for the fish project which we have been doing until tomorrow, so I think they will be dedicated to it. And then they have the sentence of the week, problems of the week plus what they have such a long-term job that they know about.

For her lesson, Michelle then invites all learners who want to have a lesson on multiples. She said that some of the learners have seen this work, and some have not. The work was set as learners should use colour coded squares to indicate multiples of numbers one to nine. They notice different things and the teacher asks some of the initial questions and lets the learners express their thoughts without final comments. They notice that some numbers finish with the same numerals, and they use colour-coding to indicate it in the fleece material. They count the number of coloured squares. There are just a few of one, three, seven, and nine squares which are odd numbers. The two, four, five, six, and eight are even numbers which they count as many of them. The objectives for each learner varied according to their skills.

How do learners participate in their learning?

Characteristics of personalized objectives were identified in 14 situations during Helen, Henry, Michelle, and Maria's lessons.

To show an example of such a situation, I choose a lesson (L1), where Michelle invites a group of learners to join her in the aside around the little rug. The other learners come because they want to see the presentation again. As we can see from the following conversation, the participation is voluntary and discussed.

Learner L1 1: I have not done that...

Michelle L1 1: Have you not been here yet?

Learner L1 2: I have been, but I want to try again.

Michelle L1 2: Kubo, did you say you wanted to go or am I wrong?

Michelle L1 3: Thus, it is related to the multiplication. You all know how multiplication works.

Learner L1 3: Yes. Almost.

Michelle L1 4: Almost. Just watch first.

Michelle comments on the participation later during the interview a posteriori (L1P). It is explained that some of the learners come into the lesson even when they have not been ready. During L1 a boy who has already been there, signed up. He wanted to go for it, he went again, and Michelle had to restrain him. According to her he tended to reveal it to those children, or he wanted to do it quickly for those who had never been there before, so she had to restrain it a bit. Therefore, it happened there. In fact, no one stayed after the presentation to continue working with manipulatives. She thinks that they enjoyed it and some learners discovered there. The boy who has already been at the presentation left right after it was explained and did not continue working. Michelle considered that as interesting, that this boy does not really like working with aids, and when he gets to one, he goes to it repeatedly, but he does not go to them voluntarily. No one stayed with the activity in any way. Michelle comments on the specific children's participation.

Michelle L1P 2: ...In fact, at the very beginning two children appeared there, who multiplied... or more precisely: One girl is not so far away that she could handle a multiplication table, and then a boy appeared there, who knows the principle of multiplication and is very happy at all presentations and he absorbs. This is a first grader and he actually appeared there too.

Michelle L1P 3: The little girl is a second grader, but she went to school immature, very immature. Now she is at the level of a first grader. Therefore, there is such a long way before her. We will see what happens to her. Well, the girls who were there, in fact they were just girls, they were actually discovering something there. They actually found some things there.

During interview a posteriori (L1P 4) Michelle specifies that she planned the lesson for someone who was not there. It is an offer that she repeats from time to time during the school year, but it is not mandatory. She thinks that one can live without it. If someone does not want to figure it out, they will not figure it out. Someone will figure it out without working with it. Michelle is convinced that not everyone enjoys this discovery, and this is more about discovery. She adds that there is always something that she says that they should learn from such an exercise. That the multiplier of two always ends in even numbers, and it keeps repeating itself, so it is just some consciousness. The children often take the paper and go draw it in the table, but this time they are so busy with another work, none of them continue. Michelle, during the second videorecorded lesson (L2), offers a lesson on long division with manipulatives. She asks specific participants but allows them to decide if they take a lesson or not. Anybody from the whole class can participate. In the beginning of the lesson, she says that learners need to split up and exchange their roles in counting, writing notes, and observing. They can decide their order.

Michelle L2 38: And think about which of you will count now and who will write.

Learner L2 20: Jane.

Learner L2 21: I am not going to take notes...

Learner L2 22: Good Suzy will take notes.

Michelle L2 39: ...because you will watch.

Learner L2 23: Suzy writes and then I will write and then Lily will write, and Suzy will count and then I will write, and Suzy will count.

Michelle L2 40: You all should count, write, and watch.

Learner L2 24: Well, yes.

Michelle sets up the task with recommendations for a small group of learners who just participated in the presentation during the lesson (L2). Learners have the voice on when they do it. They are allowed to cooperate but can also do it individually.

Michelle L2 95: ...I recommend you do the three problems as soon as possible while you have it in your head and repeat it, because if there is something new like this, it will quickly blow away from your head again. So, as it went now, and it went really well, so count one for sure today, next one tomorrow and another one the day after tomorrow.

Learner L2 117: Or all of them today.

Michelle L2 U96: Or all of them today. I recommend doing them when you have independent work today. Take it and do another one.

Learner L2 118: And by when do we have to do it?

Learner L2 119: The first one.

Learner L2 120: No, not that.

Michelle L2 97: Three problems by the end of this week. Ok, by the end of this week.

The teacher also comments (L2P 8) on what the other learners did during her presentation and how the work is set up for each week. They have deadlines for another work. There is one assigned work for each week. They meet on Monday, and it is up to them when they do it, but it must be sometime during the week. She reminds them in the morning to have a worksheet ready. Every week they have a sentence and problems for the week. She usually has something that she wants most children to do during the school year. Therefore, in fact, in those problems of the week, she includes what she knows, that the children have already gone through, that everyone can do. For instance, second graders have already gone through written addition, and they can also do it at a level that includes writing it down. That everyone or almost all passed. On the other hand, one little girl does not even do the written addition, or any subtraction. They went through written subtraction, so Michelle actually puts it there on an ongoing basis to practise it or to automate it. A lot of children have a feeling for some reason that when they are at the presentation, they already know how to do it and do not tend to practice it again.

Michelle describes that; therefore, a lot of students concentrated on different work and did not voluntarily participate in the presentation during the L2. They learned to organize their time. They want to have it ready in the beginning of the week, so that they actually have peace or time off for some other things.

Maria oversees the whole class during the lesson (L1), but learners decide what they need to continue with. Almost all of them work on different things or in different pages of their workbooks. The other group of learners gather by a flipchart. A girl asks for help with deciding what kind of exercise she would do in her exercise book while Maria is preparing for another presentation. They look in the book together and find the page. Then the girl decides herself that she can do exercise number three. The help of an assistant teacher is offered in case the girl would need additional support. Then Maria's presentation starts.

In the beginning of the second videorecorded lesson, Maria offers the presentation to the learners who have had the same one before. It is the second presentation of three which means that if some would like to, they can come again if they are interested.

Henry offers time for learners to decide how they use the time at the end of the lesson (L3). They can finish whatever they have not finished during the lesson; furthermore, they can choose any exercise in the final part of their exercise book.

Henry L3 176: Now, just at the end, just look, some of you did not manage to solve task number four yesterday. If you do not have it, you deal with it calmly. And whoever has it, they do some task in the back, something else. Or he can fix back what he does not have yet.

Helen lets the learners write down all solutions, which and how they find during the lesson (L1). They can choose the way they note it.

Helen 1P 2: Personally, when we were actually doing equations, so uh, I left some room for the children to always write down all solutions as they came to it. They have freedom in those records. Today, we could see what they have learned, or such a learned solution procedure has already been shown there, but I think it has manifested itself nicely in the fact that it is actually learned, but

it is not mastered, because the division has actually been exchanged or replaced there.

Selection of technology and resources as a support to the learning

In a personalized learning environment, learners can access appropriate tools to support their learning. They can self-select the tools they need to support any learning task, whether at school or home. Such characteristics were analysed in 5 situations during Helen, Henry, Michelle, and Maria's lessons and interviews.

Helen during her second videorecorded lesson (L2 101) offered use of geoboards with two rubber bands. Alternatively, she thought that if it was too complicated to work with geoboards, and learners could not see the fractions there, they could easily take a square workbook and work with squares. She also offered to take chocolate as a model. On the other hand, there was a challenge, a challenge for speeders, a challenge for those who just liked challenges. Therefore, there was already a merged addition and subtraction, and in the next column was the max challenge. When some of fractions were multiplied, or when they divide those fractions.

Helen L2P 30: But, uh, we will get back to that tomorrow. And Luke, just as he was working at the desk, I told him straight away that he could take the geoboard. He did not get it at first and then realized that it was difficult for him. And he himself said that he was already working hard (was sweating).

This situation shows that learners have a range of learning aids which they can use during the lesson but are not forced to choose any of them if they think they can manage without them.

In his lesson (L1), Henry introduces and encourages learners to cooperate and choose a classmate; learners can talk about the problem. Some learners go across the classroom and find a friend with whom they want to work. To build a network of peers to support learners' own learning is one of the signs of personalized learning. Communication as it is shown in the following paragraph is encouraged.

Henry L1 19: ... So, let us do it. Um, calmly, calmly cooperate. It is quite a research work. Therefore, feel free to whisper, whisper softly.

Learner L1 19: Adame, hey, let us cooperate.

Learner L1 20: Are we going to cooperate?

When manipulating with coloured squares representing numbers, Michelle asks learners (L1 5) in the group if they need any other helping manipulatives. They can decide if they need it or not. In this particular situation, learners decide not to use any of the additional manipulatives.

While talking about the problem during the lesson (L1), Maria asks if the learners used the tool which she had offered to them. Similarly, as in Michelle's case, it is up to the learners if they want to use it. During their work, learners decided that it would be helpful to use a tool and asked about it as it is indicated in the following interview.

Maria L1 6: ...Is there anything you had a problem with?

Learner L1 5: ...with the word problem.

Maria L1 7, 8: With the word problem... What was the problem? Did you use the tool while counting... (indicates a tool with a finger)

Learners L1: Yes.

Maria L1 9: Does it help?

Learners: L1: Yes.

Learner L1 6: I just did not get it (talks about the tool).

Learner L1 7: I did not either.

Maria L1 10: I gave it to those who asked about it. I did not give it to everyone, I offered it and if someone asked about it, they had it. Thus, you two still want it.

Assessment AS learning

In personalization, learners need to monitor their progress toward them using clear criteria of good practice. Learners are guided to provide descriptive feedback and record revisions in their own words and become confident in all aspects of the process of evaluation. Such characteristics were identified during Henry, Maria, and Michelle's lessons.

Henry calls for feedback at the end of the second videorecorded lesson. Learners share ideas about how to revise approaches to build on successes but overcome challenges.

Henry L2 126: Ok then. Eh, would anyone say at the end, I will pick three people to say what you found interesting at that hour (...)? What did you learn today or what did you find out? If that, what we did here ... Philip?

When the learners are finished with the individual work during the first videorecorded lesson, they ask Maria to check the work and explain the word problem they had a problem with. During the teacher's talk, the other learners in the class work on their own. When the teacher repeated the procedure, the girl found out herself where the problem with the word problem was.

Learner L1 28: We should have ... We did not convert it.

Maria L1 31: Aha, it means... you worked with it first... and you forgot the conversion, so it did not work out.

Learner L1 29: But in the end we realized...

Maria L1 32: Well, so you do not need it?

Learners L1: No.

Maria L1 33: OK. In that case, you can leave, I will sign that you have the check, okay? And we are done. Try filling in this (pointing to the task in the textbook) according to the same key. Do you want to look at it? (Asking one)

Learner L1 30: Yes.

During the division lesson with manipulatives (L2), Michelle is observing the whole process and assigning the task for the following days. Learners work together to compare outcomes to task demands. They check their work using an answer key, calculator, or computer.

Michelle L2 93: What is next? You will each solve three problems, but I will probably write them to you separately, because it occurs to me that there are not so many of them here. So, I will write three problems for each of you. Can I...

Learner L2 116: We will count it.

Michelle L2 94: You will count it. It can happen that you have a reminder. Do you feel like you understand it, girls?

Learners L2: Yes.

Michelle L2 95: If it happens that it does not work out, we will sit down together again. From what I have seen, I felt that you knew exactly what you were doing and when you were doing it....

Finally, learners in Michelle's class write a self-evaluation about their work, referring, if necessary, to the sample language. They leave their signed and dated work in the appropriate box at the station.

3.4.4 Characteristics of IDP identified in teachers' practices in classrooms (including interviews a priori and a posteriori)

To answer the research question, which characteristics of individualization, differentiation, or personalization prevail in teachers' practices in the classroom and if the characteristics correspond with the Hejny method, the Montessori method, or the undesignated approach, I will now look closer at the teachers of each method and describe the characteristics which appeared in their practices.

Hejny method

In the interviews a priori, lessons and interviews a posteriori of both teachers using the Hejny approach I coded a total of 99 situations, including 34 situations that contained characteristics of individualization, 50 situations that contained characteristics of differentiation, and 15 situations with characteristics of personalization. It deserves our attention to compare that in Helen's lessons there were 33 situations of IDP identified compared to 66 situations in Henry's lessons who was teaching the same method as Helen. As we can see in table 2, section 2.2, both of the teachers have about the same experience, they both teach in a public school with no assistant, and no significant differences among their beliefs were found. However, we can see differences in their practice, specifically for example during assessment (14 situations in Henry's lessons compare to 6 situations in Helen's lessons with IDP principals) or assigning homework.

Helen

Characteristics of individualization appeared in 7 situations during Helen's lessons (L2, L2P, L3P). The teacher sets objectives for the lesson applicable to all learners. During the lesson, she shows one-on-one support and offers alternate resources when she notices something is

not working as expected. The characteristic of monitoring learners based on seat time and grade level is met. She prepares options for continued learning for those learners who finish a particular activity first. She uses data and learning assessments to measure learning progress in order to decide on the next steps. Assessment of learning was therefore identified which corresponds with individualization.

The characteristics of differentiation appeared in 22 situations during Helen's lessons (L1, L1P, L2A, L2, L2P, L3, L3P). The teacher identified the same objectives for different groups of learners. The instruction was designed according to the needs of the group. During the lessons, learners had a choice to interact with the content, discuss what they think, ask questions, or offer answers. She walked around the classroom and gave a variety of instructions for different groups of learners. She asked questions and led learners to think about the problem. She selected technology and other resources to support the learning needs of different groups of learners. She monitored what learners did and helped whenever she thought that it was needed. The teacher used data and assessment to modify instruction for groups of learners and provided feedback to individual learners to advance learning. In some situations, assessment for learning, which corresponds with differentiation, was also identified.

The characteristic of personalization appeared in 4 situations during the lessons (L1P, L2, L3). Learners had the opportunity to decide what to write down and if they want to work independently or in pairs. They themselves selected appropriate technologies and resources to support their learning. It was clear that it was possible for them to choose some, but they were not given this option again during the videorecorded lessons.

Most of the situations (22 out of 33) identified in all parts of Helen's lessons contained characteristics of differentiation. She took responsibility for most of the lessons, including identification of objectives, giving, and modifying instructions, and providing feedback.

Henry

I identified characteristics of individualization in 27 situations during Henry's lessons (L1A, L1, L1P, L2A, L2, L3). To accommodate learning needs for the individual learners, the teacher identified learners for their strengths or challenges in mathematics and used the identifications for intentionally spending extra time according to the needs of learners. The

teacher was able to observe that the child did not get individual care during the lesson. He showed responsibility for modifying the instruction based on the needs of the individual learner. He sets the same objectives for all learners but gave support to the individuals. Individually, he also selected resources to support the learning needs of the individual learner. The little stones and paper models were offered for use to everyone, but the teacher predicted in the interview a priori which learners would use the tools. The characteristic of monitoring learners based on seat time and grade level was also included.

The following characteristics of differentiation were found in the 28 situations during the lessons of teacher Henry (L1A, L1, L1P, L2A, L2, L3, L3P). During the lesson, the teacher facilitated large-group discussions that foster students' active analysis of task demands. He predicted groups of learners based on their skills and previous situations, and then described that sometimes the exercise would be extended, and the task would be graded according to the level of difficulty according to the learners' needs. In the lesson, he used direct instruction to all with motivation and showed the same content to different types of learners. The learners looked for different solutions. During one of the lessons, according to the learners' needs, the teacher stopped the work and left it for the following lesson without closing the topic. He identified the same objectives for different groups of learners. He used data and assessment to modify instruction for groups of learners and provided feedback to individual learners in order to advance learning.

The following characteristics of personalization were found in 11 situations during Henry's lessons (L1A, L1, L2, L3, L3P). Learners were given the opportunity to choose what they worked on, with whom they worked with and when they should have the work completed. They chose difficult maths problems and, in that way became active participants in designing their learning. At the end of the lesson, the teacher allowed learners to decide how to use their remaining time. They could finish whatever they had not finished during the lesson; furthermore, they could choose any exercise in the final part of their exercise book. He let learners self-evaluate the lesson asking what was interesting for them in the lesson.

Characteristics of IDP were identified in all five parts of the teaching-learning process in Henry's lessons. Although characteristics of individualization and differentiation prevailed, the characteristics of personalization were also found. Personalization was identified less

frequently in the instructional part of the lesson, which I understand is because the teacher took responsibility for giving and modifying instructions based on the needs of the individual learners or groups of learners in all monitored lessons. During my video recordings, learners did not have a voice on what they learned but their choice (for example, of which exercise they would do) was identified in a few cases.

Montessori method

In the interviews a priori, lessons and interviews a posteriori of both teachers using Montessori method a total of 69 situations were found, including 11 situations that contained characteristics of individualization, 25 situations that contained characteristics of differentiation and 32 situations with characteristics of personalization. I will now look at both teachers and describe the characteristics that appeared in their practices more closely.

Maria

The characteristics of individualization appeared in 6 situations in Maria's class (L1, L2, L2P, L3). Maria accommodates all the learners taking the lesson and gives additional support to a female learner who needs more help.

The characteristics of differentiation appeared in 18 situations during the lessons (L1, L2A, L2P, L3, L3P). Before the lesson, the teacher proactively planned the approach for a specific group of learners, including how they would learn, and/or how they would demonstrate what they had learned. She presented a new topic to a group of students, while the rest of the class worked on other activities in any subject they chose independently or with an assistant teacher. The teacher designed the instruction based on the learning needs of the group. During the lessons, learners had choices to interact with the content, discuss what they watched, ask questions, or offer answers. They were actively participating as learners. The teacher chose the topic and controlled how learners demonstrated understanding. The teacher selected technology and resources for demonstration. After the presentation, learners could make up and solve other problems using more manipulatives of the same kind if they wanted to. The teacher commented on the needs for revision and made sure the students had the knowledge required for further work.

The characteristics of personalization were identified in 14 situations during her lessons (L1, L2A, L2, L3). The teacher found out that learners discovered something about which they had not yet talked about at school. She suggested that she would prepare the following presentation to introduce the topic. She identified and wanted to follow the learners' needs. She oversaw the whole class, but ultimately learners decided what they worked on, and almost all worked on distinct activities. When the group was finished with the work, and if they encountered any difficulties with the task, they asked the teacher to explain further.

In Maria's lessons, it was identified that individualization and differentiation were most frequently used when selecting recourses for groups of learners. In presenting a new topic (planning and identifying learners' educational needs) and in practical parts that included participation of learners, characteristics of personalization were observed.

Michelle

There are 4 situations which evince characteristics of individualization in Michelle's lessons (L1P, L2P, L3). She prepared lessons for individuals and repetitively delivered instructions to individual learners based on their needs. She evaluated verbally, and then in written form, she individually described on the report what she had observed in the class.

The characteristics of differentiation were identified in 7 situations during the lessons (L1P, L2, L3A, L3). The teacher worked with a small group of learners in part of the lesson. They used manipulatives. The teacher designed the instruction based on the learning needs of the group. The other learners in the class worked independently or with an assistant teacher.

The characteristics of personalization appeared in 18 situations during the lessons. She gave a presentation and asked specific participants to join her in a huddle. Participation was voluntary and anyone from the entire class could participate. It was explained that some of the learners could participate even if they had not finished the assigned work. The teacher then set up an activity for a small group of learners who just participated in the presentation, and they had the choice of whether or not to participate.

Undesignated approach

A total of 8 episodes captured during Una and Ulrica lessons contained characteristics of individualization or differentiation. In one case, it was repeatedly found that objectives for

an individual with special educational needs, were specific. The characteristics of personalization were not identified in the lessons of teachers using the undesignated approach.

Una

Individual support by the teacher was identified in seven situations. Una offered support when she spotted any problems and led the child to a correct answer by posing questions. An individual learner seemed to be dependent on her even when the learner did the same work as every other learner in the classroom. Una often identified the same objectives for all learners without specific objectives for individuals but that was not considered as individualized learning. There were no other characteristics of IDP identified in Una's lessons.

Ulrica

Ulrica twice customized her instruction in small groups and once individually during the videorecorded lessons. She led the class discussion asking questions of the class as an entity, and not of individual learners. She considered and evaluated the answers, often repeating them, and then summarizing them. In the other cases, the learners worked on the same problems at the same time with the same objectives.

3.5 Overall prevailing characteristics for each teacher and method

In Table 6, we can see an overall outline of the number of characteristics and how I identified them in different categories. For each column I counted the incidences of characteristics of IDP as I described in sections 3.2 and 3.4. Additionally, the criteria for decision on whether IDP prevailed in textbook use and assigning homework were set up in section 3.3.3. Setting such criteria led to the decision not to state the number of incidents in the Textbooks use and Assigning Homework columns. The colour indicates the prevailing characteristics according to the criteria.

Table 6 Overall characteristics of IDP in selected methods

	IDP Incidence/ Teacher	Principles of the Approach	Beliefs	Textbooks use	Assigning Homework	Practices Identified in the Lessons	The Most Frequent IDP Characteristics
Hejny	Helen	I – 2, D – 3, P – 5	I – 1, D – 4, P - 2			I – 7, D – 22, P - 4	Differentiation
	Henry		D - 2, P – 1		Not found	I – 27, D – 28, P – 11	Differentiation
Montessori	Michelle	P – 11	I - 3, P - 4			I – 4, D – 7, P – 18	Personalization
	Maria		I - 2, P - 4			I – 6, D – 18, P – 14	Personalization
Undesignated	Una	I- 1	Not found	Not found	Not found	I- 3, D-1	Individualization
	Ulrica		Not found	Not found	Not found	I-2, D-2	Individualization

The colours indicate the prevailing characteristics of individualization (blue), differentiation (green) and personalization (grey), the numbers indicate how many times the characteristics have been identified.

The principles of Hejny method (as we can see in the first two lines of Principles of the approach column) correspond to the characteristics of personalization. In contrast, most of the prevailing characteristics of both teachers' beliefs correspond to characteristics of differentiation. The reason can be that the principles of the method were set but teachers have different beliefs and therefore do not follow the principles closely. Importantly the same result we can see in teachers' practices according to the observations in the classrooms (see Practices Identified in the Lessons column).

In the Montessori method, we can see prevailing characteristics of personalization in the principles, teachers' beliefs, and most teachers' practices. I identified prevailing characteristics of differentiation in the observation of Maria's lessons.

The characteristics of individualization prevailing in the principles of the undesignated approach correspond to the characteristics identified in practices in the classroom during the lessons. On the other hand, there were no characteristics of IDP found in teachers' beliefs or teachers' practices (use of textbooks or assigning homework) in the undesignated approach. The reasons can vary. One of the reasons can be that even where the principle is set, teachers do not have beliefs about the use of individualization and therefore they do not use it.²¹ Bosco (1971) concluded similarly that elementary school teachers found individualization of goals more acceptable than individualization of teaching procedures. While the majority of teachers favoured individualized objectives for each child, most instruction was presented to the total class as one group or to groups ranging in number from two to six.

Incidence of individualization was not significant with the exception of practices, and it was used mainly with learners with special educational needs in planning (according to the Decree No. 27/2016 Coll., on the education of learners with special educational needs and talented learners).

The last column in Table 6 indicates which of the IDP characteristics were the most frequently identified for each method and teacher. It was counted as a sum of all identified characteristics with an additional overall colour for use of textbooks and assigning homework where I decided not to indicate numbers and use the set criteria.

²¹ The speculation is proceeded from conviction that there is a mutual relationship between beliefs and practice. Teacher's beliefs are viewed as crucial for making decision in teaching process and reciprocally, the practice forms teachers' beliefs. (Raymond, 1997).

4 Discussion

In my investigation, I first looked at different aspects of the three teaching methods and studied their principles. I understood that tendencies to find the features of effective elementary mathematics teaching have been here for decades. My research confirmed that each of the selected methods used in the Czech Republic are beneficial to learners and my findings correspond to the one-year research study of Swan et al. (2008), who arrived at the same conclusion that

it is not possible to identify a single best method but found out that there are many different types of learning and many different methods that should be applied, 'appropriate to the learner and the particular learning outcome required' (Swan, 2008, p. 2, cited in Parveva et al. 2011, p. 51).

Nevertheless, from Section 1.5.3, it is evident that similarities exist in both Hejny, and Montessori approaches and that their principles overlap. Furthermore, our three selected programs have been compared in the Czech Republic. Unlike this dissertation, which concentrates on concrete educational processes and how individualization, differentiation and personalization are enacted in them, the authors Chytrý, Říčan and Živná (2019) concentrated on an empirical level of metacognition and mathematical performance of groups of learners from the three schools following the programs: Hejny, Montessori and undesignated and concluded that learners from a regular elementary school achieved significantly worse results in the didactic tests than learners from the other two programs. Chytrý (2020) validated that within these approaches “pupils achieve different performance in terms of success in didactic tests in mathematics. These differences are significant and therefore generalizable. On the contrary, non-significant differences were found in the case of the pupil's metacognitive knowledge” (p. 204).

To answer the first of my research questions: how principles of the Hejny method, the Montessori method, and the undesignated approach corresponded with individualization, differentiation, or personalization, I was looking at the principles which are presented by authors of these methods. All IDP were found in the principles of Hejny method. It is in accordance with the author of the method who talks about individualization (Hejny, 2015, p. 9) and differentiation (Hejny, 2018, p. 11).

Contrary to Lopata, Wallace and Finn (2005), Bunnag (2000), Murray, (2008), and Lillard (2013) according to whom Montessori promoted individual and small group learning which correspond with individualization and differentiation, all (except for one – mixed-age classrooms) principles of the Montessori method studied in this research evince characteristics of personalization. Only one of the principles of the undesignated approach evince the characteristic of individualization. It has been stated in section 1.1.1 that according to the Framework Educational Programme for Basic Education in the Czech Republic (Jeřábek and Tupý et al., 2007), teachers using an undesignated approach are supposed to apply individualization of the educational process. It is a question of whether teachers know how to individualize and what it actually means. My research shows that teachers are supposed to support individualization where at the same time it is not based on the principles of the approach, they use, and they have few possibilities and tools to do so. The way they implement individualization in this approach is through Individualized Education Programs which are only assigned to a limited number of pupils.

My second research question revealed what characteristics of individualization, differentiation, and personalization are included in beliefs and practices of the teachers using one of the three approaches. The teachers in the study of Bosco (1971) expressed their beliefs that individualization is most desirable in mathematics right after reading. Nevertheless, teachers could not teach mathematics using a method without its predominant elements. That said, teachers use various other elements according to their pedagogical beliefs and their beliefs have impact on the entire method they use, which can be gradually transformed through professional development activities. Even if it has been said that teachers' beliefs are crucial in using IDP regardless of the method, I concluded in my investigation that in the Hejny method, all the characteristics of individualization, differentiation, and personalization were identified in their statements. Beliefs of teachers using the Montessori method complied with the characteristics of individualization and personalization. In the undesignated approach, only isolated signs of characteristics of individualization were identified in the exposed teachers' beliefs. The characteristics were related with inclusion and dissimilarities of individuals and their IEPs.

My research confirmed that the selected teachers in all three approaches individualized, differentiated, or personalized to some extent. It is indisputable that content knowledge is a crucial part of the teacher's job, but this research confirmed my prior beliefs that it is beneficial that teachers consider pedagogical and even psychological aspects in all subjects and look at the teaching situations from an IDP point of view.

Asking the third of my research questions: if the actual use of textbooks and the way of assigning homework in the classrooms of the three approaches correspond to characteristics of individualization, differentiation, and personalization, I was looking at the way teachers use textbooks or workbooks. The different kinds of use of textbooks in Hejny method correspond with characteristics of differentiation and personalization. In Montessori I found characteristics of personalization whereas there were no characteristics of IDP in the statements related to textbooks and workbooks during interviews in the undesignated approach. The procedure of assigning homework by one teacher in Hejny method corresponds with the characteristic of personalization. The difference between the two teachers shows that teachers' beliefs and their experience can influence the decision of assigning or not assigning homework and in this case, it does not depend on the teaching method. The kinds of activities of teachers using Montessori method are in accordance with individualization and personalization whereas, similarly as in use of textbooks and workbooks, there were no identified characteristics of IDP in the undesignated approach.

The final question of my research was which characteristics of individualization, differentiation, or personalization prevail in the teaching practice of the teachers and if these characteristics correspond with the Hejny method, the Montessori method, or the undesignated approach. My study showed that individualization, differentiation, and personalization are not at all limited by any of the three approaches and also that the use of any approach itself does not dictate the use of individualization, differentiation, or personalization. Literature confirms though that IDP elements are beneficial for learners and teachers. For instance, Tieso (2005) in his qualitative study of teachers and students who took part in a three week enhanced unit in mathematics found that the students evidenced positive levels of engagement, motivation, and excitement about learning. Slavin et al. (1984) considered individualization as one of the most controversial issues. They

summarized that individualization requires students to interact with paper rather than people. Learners can feel isolated from one another and the potential for healthy interaction can be reduced. Therefore, they concluded in their study that combining individualization and cooperative learning increases student achievement in elementary mathematics. Considering all my findings and the above mentioned studies, I believe that it is imperative for researchers to continue to pursue this line of inquiry. It is also imperative that all learning needs of all learners are addressed so educators truly leave no child behind.

4.1 Limitations of my research

Firstly, several limitations of my research may reduce the general validity of the findings. Although I consider myself familiar with one of the researched methods – the Montessori method, I am honestly interested in the Hejny method and the differences that can be determined between these two approaches. According to my experience of these methods they evince similarities but seemed very different in instruction and management of the lessons. Despite the maximum effort for objectivity, the researcher's personality is reflected to a certain extent in the selection and evaluation of situations. To minimize this reality, I set criteria for coding which was described in section 2.4.

Furthermore, to determine the characteristics of IDP, it is important to note that I was looking at different factors (such as principles and teachers' beliefs and real situations in the classrooms) but not all the factors could be included (such as learners' individual characteristics and their educational results, teachers' long-term planning or the influence of an assistant). I am aware that all these factors can have different causal interpretations. For example, the teachers' statements during the interviews may not correspond to their true beliefs for various reasons.

Finally, the size of respondents for my research was selected as a sample for each approach. It needs to be said that the number of identified characteristics in interviews and lessons varies according to the approach. The reason could be that some characteristics come from the approach itself or occur throughout the process and therefore were not identified repetitively. The teacher's personality is also clearly manifested in terms of the personality characteristics (temperament, introvert-extrovert, level of empathy, tendency to plan or react spontaneously, etc.).

On the other hand, it was the awareness of these facts that led me to a more comprehensive approach to the IDP issue. I looked at several aspects of the educational process. The goal could not even be generally valid conclusions, but rather studying a phenomena that must be noticed and from which one must start when trying to achieve IDP.

5 Conclusion

The main goal of this work was to determine and understand how individualization, differentiation and personalization are utilized in selected programs, their principles, and teachers' beliefs and their practices in teaching mathematics at the elementary level.

Within the described theoretical background and the study, readers can better understand that individualization means to look at each child individually and start where this learner is. Depending on the homogeneity of the class, there are areas of the mathematics curriculum, where learners may be at similar levels. To give one example we can choose practicing solving division problems. If ten learners have a hard time solving division problems, we can differentiate and prepare a lesson for them while the others would work on something else. Characteristics of personalization (according to my interpretation) then give us tools to develop responsibility and independence in learners' education.

There is no need to mandate the use of only one set of IDP tools using elements of individualization only, the group work of a learner's activity would be missing; if we used only differentiation, individual plans would be missing and not all children can be successful with personalization. The process of learning should instead gradually unfold, and learners should be allowed to proceed step by step and be independent. I see IDP as the whole system, which brings results through application in various stages of the educational process and has a long-term effect on the learner's personality.

It is possible, and, in my opinion, necessary to plan individually in each method. It is important to use analysis a priori for each ability, skill, or topic for an individual child, and to work with individuals and groups. Learners should be engaged in discussion, allowed to choose the best tasks for them, to learn manipulation with specific subjects, and to connect them with their personal experience and ask them what they already learned. Learners should be allowed to talk and should be free to decide for themselves what they learned. Lead them to do it when they want, where they want, and as long as they want. It is possible to evaluate the *work* and not the *children*; to lead them to self-evaluation, and to correct their own mistakes.

It would be worthy for further research to pay attention to the characteristics of individualization, differentiation, and personalization and find out if they appear in other

school subjects and if the same characteristics of individualization, differentiation and personalization dominate for the same teacher or educational approach. The relationships between them have been indicated. It would also be worth researching these relationships on a larger sample of teachers, and furthermore in secondary schools and high schools.

References

- Algozzine, B., & Anderson, K. M. (2007). Tips for teaching: Differentiating instruction to include all students. *Preventing School Failure: Alternative Education for Children and Youth*, 51(3), 49-54. <https://doi.org/10.3200/PSFL.51.3.49-54>
- Bauersfeld, H. (1980). Hidden dimensions in the so-called reality of a mathematics classroom. *Educational Studies in Mathematics*, 11(1), 23-41. <https://doi.org/10.1007/BF00369158>
- Bautista, R. G. (2012). Students' Attitude and Performance towards Algebraic Word Problem Solving through Personalized Instruction. In *Conference Paper: Proceedings of IATED-EDULEARN12, 4th International Conference on Education and New Learning Technologies*. Barcelona, Spain, 3294-3301.
- Bernacki, M., & Walkington, C. (2014). The Impact of a Personalization Intervention for Mathematics on Learning and Non-Cognitive Factors. In Stamper, J., Pardos, Z., Mavrikis, M., McLaren, B., M. (Eds.) *Proceedings of the 7th International Conference on Educational Data Mining*, London: CEUR-WS, 80-87.
- Black, P., & Wiliam, D. (1998). *Inside the black box: Raising standards through classroom assessment*. Phi Delta Kappan International. <https://doi.org/10.1177/003172171009200119>
- Bosco, J. (1971). Individualization - teachers' views. *The Elementary School Journal*, 72(3), 125-131. <https://doi.org/10.1086/460686>
- Bray, B., & McClaskey, K. (2015). Make learning personal: *The what, who, wow, where, and why*. Thousand Oaks, California: Corwin, A SAGE Company.
- Brooks, J. G., & Brooks, M. G. (1999). *In search of understanding: The case for constructivist classrooms*. AscD.
- Brožová, M. (2017). The Use of Textbooks in Different Approaches-Hejny Method, Montessori, and Traditional Approach in the Czech Republic. In: Novotná, J. and Moraová, H. (Eds.) *International Symposium Elementary Maths Teaching*. Equity and diversity in elementary mathematics education, Prague: Charles University 91-100.

Brožová, M. (2019). The Homework in Different Approaches–Scheme-Oriented, Montessori and Undesignated Approach in The Czech Republic. In: Novotná and Moraová (Eds.) *International Symposium Elementary Mathematics Teaching: Opportunities in Learning and Teaching Elementary Mathematics*, Prague: Charles University, 18. – 22. (Vol. 120, p. 88).

Brožová, M. & Coufalová, J. (2012). Kulturní podmíněnost individualizace a diferenciacce práce v matematice na 1. Stupni ZŠ [Cultural Conditionality of Individualization and Differentiation in Primary Mathematics Work]. In: Uhlířová, M. (Ed.) *EME Proceedings*, Olomouc: Pedagogická fakulta UP, 51– 55.

Bunnag, D. (2000). Classroom Adaptation: A Case Study of a Montessori School. Issues in Early Childhood Education: Curriculum, Teacher Education & Dissemination of Information. *Proceedings of the Lilian Kätež Symposium* (Champaign, IL, PS 030-740.)

Cash, R. M. (2017). *Advancing differentiation: Thinking and learning for the 21st century*. Free Spirit Publishing.

Chytrý, V. (2020) Srovnání preferovaných strategií řízení učebních činností v hodinách matematiky na 1. stupni ZŠ z hlediska úspěšnosti v didaktických testech a úrovně dosažených metakognitivních znalostí žáků. [Comparison of preferred strategies of learning activities management in mathematics lessons at primary schools in terms of success in didactic tests and the level of achieved metacognitive knowledge of pupils.] Habilitační práce. Prague: Charles University. <http://hdl.handle.net/20.500.11956/123897>

Chytrý, V., Říčan, J., & Živná, D. (2019). Matematická výkonnost a metakognice žáků základních škol běžných, základních škol Montessori a žáků vyučovaných podle Hejného metody. *Studia Paedagogica*, 24, 107-133. <https://doi.org/10.5817/SP2019-1-5>

Costa, M., Cardoso, A. P., Lacerda, C., Lopes, A., & Gomes, C. (2016). Homework in primary education from the perspective of teachers and pupils. *Procedia-Social and Behavioral Sciences*, 217, 139-148. <https://doi.org/10.1016/j.sbspro.2016.02.047>

Coufalová, J., & Chmelová, M. (2010) Individuální předpoklady žáků prvního stupně pro další rozvoj jejich matematického myšlení [The Individual Predisposition of the Elementary

Child for Further Development of his Mathematical Thinking]. In: Uhlířová, M. (Ed.) *EME Proceedings*, 78. Olomouc: Pedagogická fakulta UP, 78-83.

Coutts P., M. (2004) Meanings of Homework and Implications for Practice, *Theory into Practice*, 43(3), 182-188. https://doi.org/10.1207/s15430421tip4303_3

Danielson, C. (2011). *The Framework for Teaching evaluation instrument, 2011 Edition*.

Deci, E. L., & Ryan, R. M. (1985). The general causality orientations scale: Self-determination in personality. *Journal of research in personality*, 19(2), 109-134.

Diamond, A. & Lee, K. (2011). Interventions shown to aid executive function development in children 4 to 12 years old. *Science*, 333(6045), 959-964.

Doll, R. C. & Ronald, C. (Eds.) (1964). *Individualizing Instruction*. Washington, D.C.: Association for Supervision and Curriculum Development. <https://eric.ed.gov/?id=ED095649>.

Dunn, R. S., & Dunn, K. (1972). Practical questions teachers ask about individualizing instruction: And some of the answers. *Audiovisual Instruction*, 17(1), 47–50.

Farkas, S., Johnson, J., Duffett, A., Aulicino, C., & Mchugh, J.T. (1999). *Playing their Parts: Parents and Teachers Talk about Parental Involvement in Public Schools*. New York: Public Agenda. <https://eric.ed.gov/?id=ED434392>

Fylan, F. (2005). Semi-structured interviewing. *A handbook of research methods for clinical and health psychology*, 5(2), 65-78.

Fynne, R. J. (1924). *Montessori and her Inspirers*. Longmans, Green and Company.

Gibbons, M. (1970). What is individualized instruction? *Interchange*, 1(2), 28-52.

Goldin G.A. (2002) Affect, Meta-Affect, and Mathematical Belief Structures. In: Leder G.C., Pehkonen E., Törner G. (Eds.) *Beliefs: A Hidden Variable in Mathematics Education?* Mathematics Education Library, vol 31. Springer, Dordrecht. https://doi.org/10.1007/0-306-47958-3_4

Grant, P., & Basye, D. (2014). *Personalized Learning: A Guide for Engaging Students with Technology*. Washington, DC: International Society for Technology in Education.

Gregory, G. H., & Chapman, C. (2007). *Differentiated Instructional Strategies*. Thousand Oaks, CA: Corwin Press.

Guilford, J. P. (1967). *The Nature of Human Intelligence*. New York: McGraw-Hill. Print.

Harris, C. W., & Liba, M. R. (Eds.). (1960). *Encyclopedia of Educational Research: A Project of the American Educational Research Association*. Macmillan.

Harris, I., & Callender, A. (1995). Comparative Study of Peace Education Approaches and Their Effectiveness. *NAMTA Journal*, 20(2), 133-44.

Hejny, M. (2012). Exploring the Cognitive Dimension of Teaching Mathematics through Scheme-oriented Approach to Education. *Orbis scholae*, 6(2), 41-55. <https://doi.org/10.14712/23363177.2015.39>

Hejny, M. et al. (2015). *Matematika AB: příručka učitele* [Mathematics AB: Teacher's guide] (1. vydání). H-mat, o.p.s.

Hejny, M. et al. (2018). *Matematika pro 1. ročník: příručka učitele pro první ročník* [Mathematics for first grade: Teacher's guide for the first grade] (1. vydání). H-mat, o.p.s.

Hejny method. (n.d.). Retrieved from <http://www.h-mat.cz/en/hejny-method>.

Heathers, G. (1971). *A Definition of Individualized Education*. Paper presented at AERA annual meeting, conference proceedings, New York: American Educational Research Association.

Hiles, E. (2018). Parents' reasons for sending their child to Montessori schools. *Journal of Montessori Research*, 4(1), 1-13.

Holte, K. L. (2016). Homework in primary school: Could it be made more child-friendly? *Studia paedagogica*, 21(4), 13-33. <https://doi.org/10.5817/SP2016-4-1>

Howes, V. M. (Ed.) (1970). *Individualization of Instruction: A Teaching Strategy*. New York: Macmillan Publishing Company, Inc.

Järvelä, S. (2006). Personalised learning? New insights into fostering learning capacity. *Personalising Education*, (pp. 31–46). OECD. <https://doi.org/10.1787/9789264036604-en>

- Jeřábek, J. & Tupý, J. et al. (2007). *Framework Educational Programme for Basic Education* (with amendments from 1. 9. 2007). Praha: Výzkumný ústav pedagogický. <https://www.msmt.cz/vzdelavani/zakladni-vzdelavani/framework-education-programme-for-basic-education>.
- Johansson, M. (2006). *Teaching Mathematics with Textbooks: a Classroom and Curricular Perspective* (Doctoral dissertation, Luleå tekniska universitet).
- Johnsen, S. K., VanTassel-Baska, J. L., & Robinson, A. (2008). *Using the national gifted education standards for university teacher preparation programs*. Corwin Press.
- Jonassen, D. & Grabowski, B. L. (1993). *Handbook of individual differences, learning and instruction*. Hillsdale, NJ: Lawrence Erlbaum.
- Jones, N.F. (1975). *A Study of the Effects of Individualized Instruction on the Attitudes and Behavior of Teachers and Pupils in the Middle Elementary Grades*. Chicago: Loyola University. A Dissertation.
- Karadag, R. (2010). Teachers' Efficacy Perceptions about Individualized Instruction. *The International Journal of the Humanities*, 8(8), 1-14. <https://doi.org/10.18848/1447-9508/CGP/v08i08/42997>
- Kasíková, H., Dittrich, P., & Valenta, J. (2007). Individualizace a diferenciacie ve škole [Individualization and differentiation at school]. In: Vališová, A., & Kasíková, H. (Eds.), *Pedagogika pro učitele*. Praha: Grada Publishing. 153–164.
- Keefe, J. W., & Jenkins, J. M. (2002). A Special Section on Personalized Instruction. *Phi Delta Kappan*, 83(6), 440-448. <https://doi.org/10.1177/003172170208300609>
- Kendall, S. D. (1992). *The Development of Autonomy in Children: An Examination of the Montessori Educational Model* (Doctoral dissertation, Walden University).
- Kouzma, N. M., & Kennedy, G. A. (2002). Homework, stress, and mood disturbance in senior high school students. *Psychological reports*, 91(1), 193–198. <https://doi.org/10.2466/pr0.2002.91.1.193>

Kralovec, E., & Buell, J. (2003). End homework now. Beyond Class Time, Educational leadership: *Journal of the Department of Supervision and Curriculum Development, N.E.A.* 58(7), 39–42. <https://eric.ed.gov/?id=EJ626293>

Kratochvílová, J., & Havel, J. (2012). Individualizace a diferenciaci ve výuce jako základní podmínka pro strategie podporující inkluzi [Individualization and Differentiation in Teaching as a Basic Requirement for Strategies which Support Inclusion]. In Ježková, V. (Ed.) *Kvalita ve vzdělávání: Sborník příspěvků z XX. výroční konference České asociace pedagogického výzkumu*. Praha: Univerzita Karlova. 358-370.

Larsson, I. (1973). *Individualized Mathematics Teaching. Results from the IMU Project in Sweden*, 53Lund: CWK Gleerup.

Lillard, A. S., & Else-Quest, N. (2006). The Early Years: Evaluating Montessori Education. *Science*, 313(5795), 1893-1894. <https://doi.org/10.1126/science.1132362>

Lillard, A. S. (2013). Playful Learning and Montessori Education. *Namta Journal*, 38(2), 137-174. North American Montessori Teachers' Association.

Lillard, P. P. (1972). *Montessori, a modern approach*. Schocken Books.

Lopata, C., Wallace, N. V., & Finn, K. V. (2005). Comparison of Academic Achievement between Montessori and Traditional Education Programs. *Journal of Research in Childhood Education*, 20(1), 5-13. <https://doi.org/10.1080/02568540509594546>

López, C. L., & Sullivan, H. J. (1992). Effect of Personalization of Instructional Context on the Achievement and Attitudes of Hispanic Students. *Educational Technology Research and Development*, 40(4), 5-14.

Love, E., & Pimm, D. (1996). 'This is so': a text on texts. In Bishop A. J., Clements K., Keitel C., Kilpatrick J. & Laborde C. (Eds.), *International Handbook of Mathematics Education* (Vol. 1, pp. 371-409). Dordrecht: Kluwer.

Manitoba Education, Citizenship and Youth Program Division (2006). Rethinking Classroom Assessment with Purpose in Mind: Assessment for Learning, Assessment as Learning, Assessment of Learning. *Winnipeg, MA: Minister of Education, Citizenship and Youth*.

- Marshall, C. (2017). Montessori Education: a Review of the Evidence Base. *npj Science of Learning*, 2(1), 1-9. <https://doi.org/10.1038/s41539-017-0012-7>
- Mayring, P. (2000). Qualitative Content Analysis. Forum: *Qualitative Social Research*, 1(2), Klagenfurt.105–114. <https://doi.org/10.17169/fqs-1.2.1089>
- Marzano, R. J., Gaddy, B. B., & Dean, C. (2000). *What Works in Classroom Instruction*. Aurora, CO: Mid-continent Research for Education and Learning. <https://eric.ed.gov/?id=ED468434>
- McCarthy, J. F. (1980). Individualized Instruction, Student Achievement, and Dropout in an Urban Elementary Instrumental Music Program. *Journal of Research in Music Education*, 28(1), 59-69. <https://eric.ed.gov/?id=EJ222870>
- McFarland, S., & McFarland, J. (2013). Montessori Parenting. *Montessori Life*, 1, 30-39. <https://eric.ed.gov/?id=EJ1008540>
- Meighan, R., & Toogood, P. (1992). *Anatomy of Choice in Education*. Ticknall: Education Now Publishing Cooperative.
- Miliband, D. (2006) 'Choice and Voice in Personalised Learning', pp. 21-30 in *OECD Schooling for Tomorrow: Personalising Education*. Paris: Organisation for Economic Co-operation and Development Publications. <https://doi.org/10.1787/9789264036604-2-en>
- Miller, D. C., & Kulhavy, R. W. (1991). Personalizing Sentences and Text. *Contemporary Educational Psychology*, 16(3), 287-292. [https://doi.org/10.1016/0361-476X\(91\)90028-J](https://doi.org/10.1016/0361-476X(91)90028-J)
- Minor, L. C., Onwuegbuzie, A. J., Witcher, A. E., & James, T. L. (2002). Preservice Teachers' Educational Beliefs and Their Perceptions of Characteristics of Effective Teachers. *The Journal of Educational Research*, 96(2), 116-127. <https://doi.org/10.1080/00220670209598798>
- Murray, A. K. (2008). *Public Perceptions of Montessori Education* (Doctoral dissertation). Available from ProQuest Dissertations & Theses Global. (UMI No. 3297628).
- National School Public Relations Association, & Education USA. (1971). *Individualization in Schools: The Challenge and the Options* (Vol. 28). Washington, D. C.: National School Public Relations Association.

O'Brien, T., & Guiney, D. (2001). *Differentiation in teaching and learning*. A&C Black.

Office of Educational Technology, U.S. Department of Education. (2010). *Transforming American Education: Powered by Technology*. Retrieved from www.ed.gov/sites/default/files/NETP-2010-final-report.pdf.

Organisation for Economic Co-operation and Development., & Teaching and Learning International Survey. (2009). *Creating effective teaching and learning environments: First results from TALIS*. Paris: OECD, Teaching and Learning International Survey. <https://doi.org/10.1787/9789264068780-en>.

Parveva, T., Noorani, S., Ranguelov, S., Motiejunaite, A., & Kerpanova, V. (2011). *Mathematics Education in Europe: Common Challenges and National Policies*. Education, Audiovisual and Culture Executive Agency, European Commission. Retrieved from <http://eacea.ec.europa.eu/education/eurydice>.

Prast, E., Weijer-Bergsma, E.V., Kroesbergen, E.H., & Luit, J.E. (2015). Readiness-based Differentiation in Primary School Mathematics: Expert Recommendations and Teacher Self-assessment. *Frontline Learning Research*, 3(2), 90–116. <https://doi.org/10.14786/flr.v3i2.163>

Protheroe, N. (2007). What Does Good Math Instruction Look Like? *Principal*, 87(1), 51-54. <https://eric.ed.gov/?id=EJ775964>

Raymond, A. M. (1997) Inconsistency between a Beginning Elementary School Teacher's Mathematics Beliefs and Teaching Practice. *Journal for Research in Mathematics Education.*, 28(5), 550–576. <https://doi.org/10.2307/749691>

Rock, M. L., Gregg, M., Ellis, E., & Gable, R. A. (2008). REACH: A Framework for Differentiating Classroom Instruction. *Preventing School Failure: Alternative Education for Children and Youth*, 52(2), 31-47. <https://doi.org/10.3200/PSFL.52.2.31-47>

Roy, A., Guay, F., & Valois, P. (2013). Teaching to Address Diverse Learning Needs: Development and Validation of a Differentiated Instruction Scale. *International Journal of Inclusive Education*, 17(11), 1186-1204. DOI:10.1080/13603116.2012.743604

- Ryniker, D. H., & Shoho, A. R. (2001). Student Perceptions of Their Elementary Classrooms: Montessori vs. Traditional Environments. *Montessori Life*, 13(1), 45-48. <https://eric.ed.gov/?id=EJ626896>
- Şimşek, N., & Çakır, Ö. (2009). Effect of Personalization on Students' Achievement and Gender Factor in Mathematics Education. *International Journal of Social Sciences*, 4(4), 278-282. <https://doi.org/10.5281/zenodo.1333154>
- Shane, H. (1962). The School and Individual Differences. *Teachers College Record*, 63(9), 44-61. <https://doi.org/10.1177/016146816206300904>
- Slavin, R. E., & Karweit, N. L. (1985). Effects of Whole Class, Ability Grouped, and Individualized Instruction on Mathematics Achievement. *American Educational Research Journal*, 22(3), 351-367. <https://doi.org/10.3102/00028312022003351>
- Slavin, R. E., Leavey, M. B., & Madden, N. A. (1984). Combining Cooperative Learning and Individualized Instruction: Effects on Student Mathematics Achievement, Attitudes, and Behaviors. *The Elementary School Journal*, 84(4), 409-422. <http://www.jstor.org/stable/1001367>
- Snow, E.R., and D.F. Lohman (1984), "Toward a Theory of Cognitive Aptitude for Learning from Instruction", *Journal of Educational Psychology*, 76(3), 347-376. <https://doi.org/10.1037/0022-0663.76.3.347>
- Strong, R., Thomas, E., Perini, M., & Silver, H. (2004). Creating a Differentiated Mathematics Classroom. *Educational Leadership*, 61(5), 73-78. <https://eric.ed.gov/?id=EJ716727>
- Superfine, A. M. C. (2008). Planning for Mathematics Instruction: A model of Experienced Teachers' Planning Processes in the Context of Reform Mathematics Curriculum. *The Mathematics Educator*, 18(2). <https://eric.ed.gov/?id=EJ841570>
- Swan, M., Lacey, P. & Mann, S. (2008) Mathematics Matters: Final Report. <https://hdl.handle.net/20.500.12365/17304>

- Takahashi, A. (2016). Recent Trends in Japanese Mathematics Textbooks for Elementary Grades: Supporting Teachers to Teach Mathematics through Problem Solving. *Universal Journal of Educational Research*, 4(2), 313-319. <https://doi.org/10.13189/ujer.2016.040201>
- Tieso, C. (2005). The Effects of Grouping Practices and Curricular Adjustments on Achievement. *Journal for the Education of the Gifted*, 29, 60– 89. <https://doi.org/10.1177/016235320502900104>
- Tomlinson, C. A. (1999). *The Differentiated Classroom: Responding to the Needs of All Learners Association for Supervision and Curriculum Development*. Alexandria, Virginia.
- Tomlinson, C. A. (2001). *How to Differentiate Instruction in Mixed-ability Classrooms*. 2nd ed. ASCD, Virginia.
- Tomlinson, C. A. (2014). *The differentiated classroom: Responding to the needs of all learners*. 2nd Edition, ASCD, Alexandria: Pearson.
- Tomlinson, C. A., Brighton, C., Hertberg, H., Callahan, C. M., Moon, T. R., Brimijoin, K., Conover, L. A. & Reynolds, T. (2003). Differentiating Instruction in Response to Student Readiness, Interest, and Learning Profile in Academically Diverse Classrooms: A Review of Literature. *Journal for the Education of the Gifted*, 27(2-3), 119-145. <https://doi.org/10.1177/016235320302700203>
- Vygotsky, L. S. (1978). Zone of Proximal Development: A New Approach. *Mind in Society: The development of Higher Psychological Processes*, 84-91.
- Wallach, M. A., & Kogan, N. (1965). *Modes of Thinking in Young Children: A Study of the Creativity-Intelligence Distinction*. New York: Holt, Rinehart & Winston. [https://doi.org/10.1002/1520-6807\(196604\)3:2<190::AID-PITS2310030226>3.0.CO;2-C](https://doi.org/10.1002/1520-6807(196604)3:2<190::AID-PITS2310030226>3.0.CO;2-C)
- Warnock, M. (1978). *Special Educational Needs: Report of the Committee of Enquiry into the Education of Handicapped Children and Young People*. HM Stationery Office.
- Wasson, W. L., & Boyles, D. R. (1998). A democratic Phenomenon: Emerging Adolescent Programs in Montessori Schools. *Philosophy of Education Archive*, 465-472. <https://educationjournal.web.illinois.edu/archive/index.php/pes/article/view/2150.pdf>

- Wittmann, E. C. (2001). Developing Mathematics Education in a Systemic process. *Educational Studies in Mathematics*, 1-20. <https://doi.org/10.1023/A:1015538317850>
- Wood, T. (1997). Alternative Patterns of Communication in Mathematics Classes: Funneling or Focusing: In Steinbring H., Bartolini-Bussi M. & Sierpiska A. (Eds.), *Language and communication in the mathematics classroom*. Reston, VA: NCTM. 167–178.
- Yenmez, A. A., & Özpınar, I. (2017). Pre-Service Education on Differentiated Instruction: Elementary Teacher Candidates' Competences and Opinions on the Process. *Journal of Education and Practice*, 8(5), 87-93. <https://eric.ed.gov/?id=EJ1133107>
- Yin, R. K. (2003). Designing case studies. *Qualitative Research Methods*, 5, 359-386.
- Zuckerman, O. (2010). Designing digital objects for learning: lessons from Froebel and Montessori. *International Journal of Arts and Technology*, 3(1), 124-135. <https://doi.org/10.1504/IJART.2010.030497>

Appendix list

Appendix A – Teachers' orientations in Bray's and McClaskey's (2015) framework

Appendix B – The description of a situation from a lesson (L2) in Maria's classroom.

Appendix A

Teachers' orientations in Bray's and McClaskey's (2015) framework

I analysed interviews (a priori and a posteriori) and lessons with each teacher using the descriptors in Appendix A and marked where “they” fell into each category and type: Individualization (I), Differentiation (D), and Personalization (P).

Personalization vs Differentiation vs Individualization Chart

Personalization	Differentiation	Individualization
starts with the learner	starts with groups of learners	starts with the need of an individual learner
connects with interests, passions, and aspirations	adjusts to learning needs of groups of learners	accommodates learning needs of the individual
learners actively participate in the design of their learning	explicit instruction based upon the learning needs of groups of learners	explicit instruction based upon the learning needs of an individual learner
learners have a voice and choice on what they learn	teachers create or adapt instruction and choose roles for learners based on different needs of learners	teachers customize lessons and tasks for learners based on individual needs
different objectives for each learner	same objectives for groups of learners	same objectives for learners with specific objectives for individuals who receive one-on-one support
learner selects appropriate technology and resources to support their learning	technology and resources are selected to support the learning needs of groups of learner	technology and resources are selected to support the learning needs of an individual learner
learners build a network of peers, experts, teachers, and paraprofessionals to guide and support their learning	learners are reliant on the guidance of teachers to support their learning	learners are dependent on individual teachers or paraprofessionals to support their learning
competency-based models where the learner demonstrates mastery	based on Carnegie unit (seat time) and grade level	based on Carnegie unit (seat time) and grade level
assessment AS learning	assessment FOR learning	assessment OF learning
teachers develop capacity to create independent learners who set goals, monitor progress, and reflect on learning and summative assessments based on student mastery	assessment involves time-based testing and teachers provide feedback to advance learning	summative assessment is grade-based and involves time-based testing which confirms what learners know and don't know

Appendix B

The description of a situation from a lesson (L2) in Maria's classroom.

- Maria L2 98:* *Now, we will do something interesting (constructing a building) So, I will... out of these blocks... something is missing. What would fit here?*
- Learner L2 81:* *A "Two" block and two "one" blocks.*
- Maria L2 99:* *Thus, either this will fit or...*
- Learner L2 82:* *Half of red.*
- Maria L2 100:* *Or two red ones, we will put red ones...*
- Learner L2 83:* *Two halves of red are one red.*
- Maria L2 101:* *This is your building plot; you just will not build the house elsewhere and it cannot protrude anywhere, and it cannot be taller either ... see? Thus, when you have checked that you have built a house as you wanted a two-story house with a roof terrace ... and I do not know what. Therefore, you lay out the house and determine exactly what components you must buy. (Dismantles the building) So we write down what the individual piece looks like. Let's start with the smallest one.*
- Learners L2:* *One time one... times, times...*
- Maria L2 102:* *We know that we had 27 cubic centimetres volume (writing on the board). Now (showing the smallest cube).*
- Learner L2 84:* *1 time 1, time 1*
- Maria L2 103:* *It has three dimensions. How many times it contains this cube?*
- Learner L2 85:* *Two times, times two.*
- Maria L2 104:* *We will not count it yet because we could have much more complicated sizes. You will write just the parts. So, the second one. (Showing the next one).*
- Learners L2:* *2 times 2, times 1*
- Maria L2 105:* *Thus, 2 times 2 times 1. How many do we have? 1, 2, 3, 4... we have four of them all together and then the last one.*
- Maria L2 106:* *This one, I have just...*
- Learners L2:* *Once.*
- Maria L2 107:* *Once. How many pieces do we have in the brackets? How much is 2 times 1 times 1 times 1?*
- Learner L2 86:* *One. (Laughing)*
- Learner L2 87:* *No, two.*
- Maria L2 108:* *How about the second one? 4 times 2 times 2 times 1?*
- Learner L2 88:* *2 times 2 is 4 and times 4 is 16.*
- Maria L2 109:* *Yes... how about the third one? 3 times 3 is...*
- Learners L2:* *Nine and times 1 is 9.*

Maria L2 110: Then...
Learner L2 89: Add it up.
Maria L2 111: Addition... 9 + 6 is 15 + 2 is
Learners L2: 17
Maria L2 112: And one...
Learner L2 90: 27...
Maria L2 113: Now, just look if you have counted it right. It should be the same...
Learner L2 91: Prism..., no.
Learners L2: Volume.
Maria L2 114: Thank you, it is up to you now.